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Acoustic dither injection in a medium with hysteretic quadratic nonlinearity

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Abstract

The influence of a weak high-frequency acoustic signal (a dither) on the absorption and velocity of a stronger harmonic wave in a medium with hysteretic quadratic nonlinearity is analysed. It is demonstrated that, depending on the relative phase of the stronger wave and the weak dither, either the effects of the induced absorption or of the induced transparency are possible. A physical explanation of the effects proportional to the dither amplitude is proposed. It is predicted that the absorption of the acoustic wave by a noise in the materials with hysteretic quadratic nonlinearity is proportional to noise intensity.

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1. Introduction

There are currently two important tendencies in the evaluation of the mesoscopic materials (such as rocks, polycrystalline metals, ceramics, etc.) by the methods of the nonlinear acoustics. First, the researchers use more and more elaborated methods and principles for the analysis of the acoustic nonlinearity of these materials. The experiments on the self-action of harmonically pumped acoustic wave in the resonators [1–3] are replaced by the experiments on mixing the acoustic waves of different frequencies [4–10]. The latter experiments provide, in particular, an access to the information on the dispersion (i.e., the dependence on frequency) of the acoustic nonlinearity [10,11]. Second, a consensus among the researchers on the importance of the dissipative nonlinearity in the mesoscopic materials is growing [4–11]. Different manifestations of the interaction and/or of the self-action of the acoustic waves due to possible nonlinear absorption has been reported in rocks [3,12,13], polycrystalline metals [1,4–10], sand [11–14] and even in the homogeneous materials containing individual cracks [15–17].

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However, there is no consensus on the physical mechanisms of the dissipative nonlinearity in the mesoscopic materials. There are purely phenomenological models [12–14,18], there are models introducing nonlinear dissipation in the acoustically induced motion of the dislocations [10], there are models attributing nonlinear dissipation to soft mechanical elements in the mesoscopic materials [9,19]. In particular, it might be expected that thermoelastic absorption of sound by soft contacts between the crack lips or between the grains (in polycrystalline materials) contributes to nonlinear dissipation. The nonlinearity of the latter mechanism is due to the modulation of the contact dimensions by the acoustic waves. Finally, it is well established that in the mesoscopic materials the hysteretic quadratic nonlinearity plays an important role. In particular, the variation of the acoustic wave decrement proportionally to the wave amplitude is commonly attributed to the hysteretic quadratic nonlinearity [1,3]. Evidently the hysteretic nonlinear absorption should contribute to the processes of frequency mixing as well.

In the analysis of each particular experiment the different contributions to nonlinear dissipation listed above should be compared. For this comparison the role of the different mechanisms of the dissipative nonlinearity in various possible processes should be studied. This research is currently in progress. The absorption of a weak ultrasonic pulse under the action of an intense low-frequency pumping wave has been analysed both in the framework of the modified Granato–Lucke theory [10] and as being caused by the hysteretic quadratic nonlinearity [20]. General theoretical predictions for the induced absorption or amplification of a small amplitude acoustic wave of an arbitrary frequency in the field of a large amplitude harmonic pump wave (traveling or standing) in the materials with hysteretic quadratic nonlinearity has been formulated [20]. In the following a complementary theory of a weak signal influence on a strong wave (due to the presence of the hysteretic nonlinearity) is developed.

2. Theory

The hysteretic quadratic nonlinearity is known to be even (quadratic) in acoustic wave amplitude but have a character (symmetry) of an odd nonlinearity in its physical manifestations [1,3,20–23]. An elementary scattering process due to the interaction of a weak signal at a cyclic frequency ω_s and of an amplitude a with a strong wave at a frequency ω_p and of an amplitude A is controlled by the following conservation law [20,25]

$$\omega = \pm\omega_s - 2m\omega_p, \tag{1}$$

where $m = 0, \pm 1, \pm 2, \ldots$ is the integer number and ω denotes the frequency of the scattered wave. The selection rule in Eq. (1) indicates that nonlinear scattering in a medium with hysteretic quadratic nonlinearity is a multi-phonon process. The scattered phonon is combined of a phonon from a weak signal wave and of $2m$ phonons from a strong wave. The amplitude of the wave scattered in process (1) is proportional to the product of the weak wave and the strong wave amplitudes ($\propto aA$). The selection rule in Eq. (1) is valid for the self-action of the strong wave as well. In the latter case $\omega_s = \omega_p$ and for $m = \pm 2$ the scattered wave frequency ω is equal to the stronger wave frequency ω_p: $\omega = \pm\omega_s \mp 2\omega_p = \mp\omega_p$. This provides a direct mechanism for the self-action due to a single act of the nonlinear scattering [20,21]. The variation ΔA of the stronger wave amplitude due to the self-scattering is proportional to the square of its own amplitude ($\propto A^2$) [21,24]. Consequently for $a \ll A$ the influence of a signal on the stronger wave is at least a factor of $a/A \ll 1$ weaker in comparison with the strong wave self-action.

However the effects of the order of $a/A \ll 1$ are possible not for all relative values of the weak-wave and strong-wave frequencies. Actually it is impossible to get a scattered wave at the frequency of the stronger wave (i.e., to satisfy Eq. (1) with $\omega = \pm\omega_p$) if the weak-wave frequency ω_s differs from the odd harmonics of the strong-wave frequency. The processes of the first order in $a/A \ll 1$ are allowed only for

$$\omega_s = (2m+1)\omega_p, \quad m = 0, \pm 1, \pm 2, \ldots. \tag{2}$$

The latter requirement is a one more manifestation of the odd character (symmetry) of the hysteretic quadratic nonlinearity. A signal wave at a frequency that differs from Eq. (2) might influence the stronger wave only through the scattering processes, which are less effective than those described by Eq. (1). For example if the signal wave is an even harmonic of the strong wave ($\omega_s = 2m\omega_p$) then modifications of the strong wave will be at least of the order of $(a/A)^2 \ll a/A \ll 1$.

In the following, we analyse the pump wave modifications caused by the presence of a weak wave at one of the pump wave harmonics ($\omega_s = n\omega_p$, $n = \pm1, \pm2, \ldots$). The results obtained confirm the presented above qualitative ideas. For the odd harmonics ($n = 2m + 1$, $m = \pm1, \pm2, \ldots$) the variations of the stronger wave amplitude and velocity proportional to weak-wave amplitude are possible. For the even harmonics ($n = 2m$, $m = \pm1, \pm2, \ldots$) the variations in the stronger wave induced by the weaker one are proportional to the square of the signal amplitude. These latter weak effects are evaluated just for comparison with the recent predictions obtained both numerically and analytically for the fundamental wave interaction with its second harmonic [26].

Both the induced variations of the fundamental wave absorption and velocity are evaluated. For $\omega_s = n\omega_p$ the total acoustic field composed of the stronger and the weaker (high-frequency) waves is periodic with a period $2\pi/\omega_p$. In this case there is no need to apply the mathematical formalism of successive approximations developed in [21,25] for the mixing of the incommensurate frequencies (that treats the interaction process at the time interval from $t = -\infty$ to $t = \infty$). A simpler iterative procedure at a single period of the acoustic field ($-\pi/\omega_p \leqslant t < \pi/\omega_p$) is applied.

It should be mentioned that a high-frequency signal acting on a nonlinear system is commonly called the "dither" [27,28]. The dithers have been used to modify the behaviour of different systems (to compensate for the effects of the Coulomb friction, dead zones in hydraulic valves and hysteretic effects, as well as for the stabilisation of the systems [27,28]). Commonly the injection of a dither is supposed to smooth the discontinuities in the interactions (in particular the discontinuity of friction at low velocity) [28]. In this terminology the analysis presented below for $|n| \geqslant 2$ might be called the theory of "dithering" of a material with hysteretic quadratic nonlinearity.

Let us assume that in the presence of a stronger wave and a weaker dither the local strain s in a material is described by

$$s = A\cos\theta + a\cos(n\theta + \varphi). \tag{3}$$

Here $\theta = \omega_p t$ is the nondimensional time variable and φ is the phase shift of the dither relative to the stronger-wave phase. The materials with the hysteretic quadratic nonlinearity are known to exhibit a property of the end-point memory [29,30]. For the purposes of the current analysis this property can be formulated as follows. The material remembers the maxima and the minima in the strain loading history and the nonlinear contribution to elastic modulus depends on how the current strain value s is positioned relative to the different memorised extrema s_e. It should be mentioned here that the information on the extremum could be erased in the process of subsequent loading [26,29–31] when the strain, reaching this memorised extremum, does not change the sign of the strain rate. In other words, the memory of an extremum is erased if the loading passes value s_e, but does not have at this former extremum a turning point. In practice, the nonlinear contribution E_{H} to the modulus due to hysteretic quadratic nonlinearity abruptly diminishes to zero when the strain exhibits an extremum in the loading history. The subsequent variation of the nonlinear contribution to the modulus is proportional to the deviation of the strain from its value in the latest extremum

$$E_{\mathrm{H}} \equiv \frac{\partial \sigma_{\mathrm{H}}}{\partial s} = -h_{\mathrm{H}} E |s - s_e|. \tag{4}$$

Here E is the linear elastic modulus, h_{H} is the characteristic nondimensional parameter of the hysteretic quadratic nonlinearity [21,22,32], and σ_{H} denotes the corresponding nonlinear contribution to stress. Note that Eq. (4) predicts softening of a mesoscopic material ($E_{\mathrm{H}} < 0$ for $h_{\mathrm{H}} > 0$) in accordance with multiple experimental observations [1–3]. The relation (4) is valid until the loading reaches the subsequent extremum in its history unless the system passes one of the previously memorised extrema. In the latter case the relation in Eq. (4) should be modified after the moment when the strain passes this former extremum (i.e., when the strain reaches a memorised maximum and keeps increasing or it reaches a memorised minimum and keeps decreasing). See, for example, Refs. [26,31].

In order to get with the help of Eq. (4) a description of the modulus variation over a complete wave period it is necessary to find the extrema in the loading history (3). The end points θ_m corresponding to these extrema are the solutions of the equation

$$\frac{\partial s}{\partial \theta} = -A\left[\sin\theta + \frac{a}{A} n \sin(n\theta + \varphi)\right] = 0. \tag{5}$$

It can be verified that under the condition $a/A < 1/n^2$ Eq. (5) has only two roots over a period (corresponding to an absolute maximum and an absolute minimum of strain). For $a/A > 1/n^2$ additional roots may appear for some values of the phase shift φ. This corresponds to the appearance of the local extrema in addition to the absolute ones (to the formation of the internal, minor loops in the hysteretic stress/strain relationship). The influence of the minor loops formation on the stronger wave has been studied recently both numerically and analytically in the case $n = 2$, $\varphi = 0, \pi/4$ [26]. The loop in the latter case is formed when the amplitudes of the stronger wave and the weaker one are not very different ($a/A \propto 1$). The present analysis is devoted to the influence of a weak signal on a strong wave ($a/A \ll 1$) in the absence of the minor loops induced by dithering. Only the loading histories with a single maximum and a single minimum over a wave period are considered. Moreover we assume the condition

$$\frac{a}{A} n^2 \ll 1 \tag{6}$$

to be fulfilled. The inequality in Eq. (6) keeps the system sufficiently far from the regime of minor loops formation. Importantly under the condition (6) the positions of the strain extrema can be evaluated approximately as

$$\theta_m = m\pi + \Delta\theta_m \approx m\pi - (-1)^{m(n-1)} \frac{a}{A} n \sin\varphi + \left(\frac{a}{A}\right)^2 n^3 \sin\varphi \cos\varphi. \tag{7}$$

Here $m = 0, \pm 1, \pm 2, \ldots, \theta_{m+2} - \theta_m = 2\pi$, and $\Delta\theta_m$ denotes the shift of the extremum induced by the dither. The strain values $s_e^{(m)}$ in the extrema are also modified by the dither

$$s_e^{(m)} = s(\theta_m) \approx A\left\{(-1)^m + (-1)^{mn} \frac{a}{A} \cos\varphi + (-1)^m \left(\frac{a}{A}\right)^2 \frac{n^2}{2} \sin^2\varphi\right\}. \tag{8}$$

Only the terms up to the second order in the small parameter $a/A \ll 1$ are retained in Eqs. (7) and (8).

With the information on the extrema in hands the description of the nonlinear modulus behaviour is straightforward. During the period $\theta_{-1} \leqslant \theta \leqslant \theta_1$ Eq. (4) predicts the following dynamics of E_{H}

$$E_{\mathrm{H}} = -h_{\mathrm{H}} E \begin{cases} s - s_{-1}, & \theta_{-1} \leqslant \theta \leqslant \theta_0, \\ s_0 - s, & \theta_0 \leqslant \theta \leqslant \theta_1. \end{cases} \tag{9}$$

The integration of Eq. (9) over strain provides the description of the hysteretic stress/strain relationship

$$\sigma_{\mathrm{H}} = \begin{cases} \sigma_{\mathrm{H}}(\theta_{-1}) - \frac{1}{2} h_{\mathrm{H}} E (s - s_{-1})^2, & \theta_{-1} \leqslant \theta \leqslant \theta_0, \\ \sigma_{\mathrm{H}}(\theta_0) + \frac{1}{2} h_{\mathrm{H}} E (s - s_0)^2 = \sigma_{\mathrm{H}}(\theta_{-1}) - \frac{1}{2} h_{\mathrm{H}} E\left[(s_0 - s_{-1})^2 - (s - s_0)^2\right], & \theta_0 \leqslant \theta \leqslant \theta_1. \end{cases} \tag{10}$$

In the derivation of the second part in Eq. (10) the continuity of stress in the end-points (at $\theta = \theta_0$) has been taken into account.

The hysteretic nonlinear absorption is characterised by the energy losses $\Delta W = \oint \sigma_{\mathrm{H}}(s)\, ds$ over a wave period in a unit volume. Note that the term $\sigma(\theta_{-1})$ in Eq. (10) (which does not vary over a wave period) does not contribute to dither-induced modifications of the energy $\Delta W = \oint \sigma_{\mathrm{H}}(s)\, ds$ dissipated in the material. Thus for the purposes of the current study the evaluation of term $\sigma(\theta_{-1})$ is not necessary. To evaluate the hysteretic losses ΔW_1 of the stronger wave $A\cos\theta$, the total stress $\sigma_{\mathrm{H}}(s)$ in Eq. (10) should be considered, but only the contribution of the stronger wave to ds should be included ($ds = -A\sin\theta\, d\theta$)

$$\Delta W_1 = -A\int_{\theta_{-1}}^{\theta_1} \sigma_{\mathrm{H}}\big(A\cos\theta + a\cos(n\theta+\varphi)\big)\sin\theta\, d\theta. \tag{11}$$

To arrive to the traditional form of the losses presentation the energy density in the strong wave averaged over a wave period is found

$$W_1 = \frac{E}{2\pi}\int_{\theta_{-1}}^{\theta_1} (A\cos\theta)^2\, d\theta = \frac{EA^2}{2}$$

and the acoustic decrement of the stronger wave is defined in the conventional sense by $D_1 = \Delta W_1/(2W_1)$. The result of the decrement calculation can be presented in the form

$$\frac{D_1}{D_1(a=0)} \approx 1 + \big[1-(-1)^n\big]\frac{3(n^2-3)}{2(n^2-4)}\cos\varphi\left(\frac{a}{A}\right) - \frac{3}{4}\left\{1 - \frac{\cos(2\varphi)}{4n^2-1} + 2\frac{(-1)^n n^2+1}{n^2-1}\cos^2\varphi - 2n^2\sin^2\varphi\right\}\left(\frac{a}{A}\right)^2, \tag{12}$$

where $D_1(a=0) = (4/3)h_{\mathrm{H}}A$ is the amplitude-dependent decrement of the strong wave in the case of its self-action (i.e., in the absence of a dither).

The modification by a dither of the modulus for the stronger wave can be evaluated as [33,34]

$$\Delta E_1 = \frac{1}{A}\frac{2}{\pi}\int_{\theta_{-1}}^{\theta_1} \sigma_{\mathrm{H}}\big(A\cos\theta + a\cos(n\theta+\varphi)\big)\cos\theta\, d\theta. \tag{13}$$

In Eq. (13) in comparison with Eq. (11) $\cos\theta$ appears under the integral instead of $\sin\theta$. With the help of Eq. (13) the stronger wave velocity modification Δc_1 induced by the dither can be evaluated and presented in the form

$$\frac{\Delta c_1}{\Delta c_1(a=0)} \approx 1 + \big[1-(-1)^n\big]\left\{\frac{\cos\varphi}{2} + \frac{2}{\pi}\frac{n^2-2}{n(n^2-4)}\sin\varphi\right\}\left(\frac{a}{A}\right) + \left\{\frac{\sin\varphi}{2} + \frac{2}{\pi n}\left[\frac{2}{4n^2-1} - \frac{1+(-1)^n}{n^2-1}\right]\cos\varphi\right\}n^2\sin\varphi\left(\frac{a}{A}\right)^2. \tag{14}$$

Here $\Delta c_1(a=0) = -h_{\mathrm{H}}c_0 A$ denotes the reduction of the stronger-wave velocity due to its self-action, c_0 is the velocity of a linear acoustic wave (i.e., of a wave of an infinitely small amplitude). Both in Eq. (12) and in Eq. (14) only the terms up to the second order in a small parameter $a/A \ll 1/n^2 \ll 1$ are retained.

The analytical results (12), (14) for the decrement and the phase velocity confirm the conclusion derived earlier on the basis of the selection rule (1). Only the dithering at frequencies equal to the odd harmonics of the stronger wave leads to its modification proportional to the dither amplitude. Neglecting in Eq. (12) and in Eq. (14) for the case $n = 2k+1$ ($k = 0, \pm1, \pm2, \ldots$) the terms of the order of $(a/A)^2$ this effect is described by

$$\frac{D_1}{D_1(a=0)} \approx 1 + 6\frac{2k^2+2k-1}{(2k-1)(2k+3)}\cos\varphi\left(\frac{a}{A}\right), \tag{15}$$

$$\frac{\Delta c_1}{\Delta c_1(a=0)} \approx 1 + \left\{\cos\varphi + \frac{4}{\pi}\frac{(2k+1)^2-2}{(2k-1)(2k+1)(2k+3)}\sin\varphi\right\}\left(\frac{a}{A}\right). \tag{16}$$

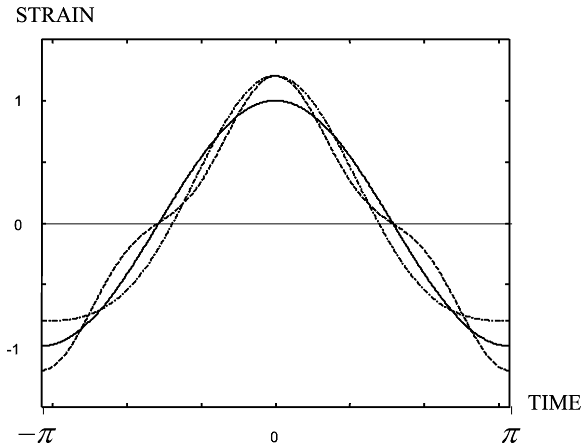

Fig. 1. The normalized total strain field s/A in the absence of any dithering (continuous curve, the strong wave $s/A = \cos\theta$), when dithering is accomplished at the second harmonic of the strong wave (dash-dotted curve, $s/A = \cos\theta + (1/5)\cos(2\theta)$), and when the dithering is accomplished at the third harmonic of the strong wave (dashed curve, $s/A = \cos\theta + (1/5)\cos(3\theta)$). The modulation of the total acoustic field amplitude proportional to the dither amplitude is achieved only in the latter case. This leads to the variation of the strong wave decrement and the phase velocity proportional to the dither amplitude.

The physical interpretation of the result in Eq. (15) can be obtained by noting that in accordance with Eq. (7) a dither at frequency $\omega_s = (2k+1)\omega_p$ does not modulate the time interval between the subsequent extrema ($\Delta\theta_m - \Delta\theta_{m-1} = 0$ for $n = 2k+1$), but it strongly influences the amplitude of the total acoustic field. Actually, from Eq. (8) it follows that in the case $n = 2k+1$ the acoustic wave amplitude is equal to

$$\frac{1}{2}|s_m - s_{m-1}| \approx 1 + \frac{a}{A}\cos\varphi. \tag{17}$$

Here only the contribution up to the first order in small parameter $a/A \ll 1$ is retained. Comparison of Eq. (17) and Eq. (15) demonstrates that the variation in the stronger wave decrement is proportional to the variation of the total acoustic field amplitude. The most important is that both variations exhibit the same dependence on the relative phase φ between the dither and the stronger, low-frequency wave. It can be concluded that the influence of the dither on the low-frequency stronger wave can be of the order of a/A if the dither is able to induce the modulation of the strong-wave amplitude of the same order a/A. In contrast to the case $\omega_s = (2k+1)\omega_p$ a dither at a frequency $\omega_s = 2k\omega_p$ modulates the total acoustic amplitude $|s_m - s_{m-1}|/2$ only at the level $\propto (a/A)^2 \ll a/A \ll 1$ (as it follows from Eq. (8)). Although in the case $n = 2k$ the magnitude of strain in the individual extrema s_m and s_{m-1} is modulated at the level $\propto a/A$ this (in the absence of the variation in the amplitude $|s_m - s_{m-1}|/2$ of the order of a/A) contributes to the decrement variation only in the next order of smallness. In Fig. 1 the total acoustic field is presented in the absence of the dithering, and also when the system is dithered at frequencies $\omega_s = 3\omega_p$ and $\omega_s = 2\omega_p$. The phase φ is chosen to be equal to zero. In order to enlighten the discussed effects the parameter a/A in Fig. 1 is chosen to be rather large (its value does not satisfy the strong inequality (6)). However, Fig. 1 provides a clear and physically correct qualitative illustration of the difference in the influence of the dithers at odd and at even harmonics of the strong wave on the total acoustic field.

In accordance with Eq. (15) the dither at an odd harmonic can induce either additional absorption (if $\cos\varphi > 0$) or the transparency (if $\cos\varphi < 0$). For $\varphi = \pm\pi/2$ the decrement variations of the order of a/A are absent. In accordance with Eq. (16) the dependence of the induced variation in the stronger-wave velocity on the phase φ differs from that for the absorption. Because of this the modulation of the velocity can dominate over the modulation of the decrement (in the vicinity of $\varphi = \pm\pi/2$, for example). At the same time linear in a/A variation in the stronger-wave velocity disappears for the critical phase

$$\varphi_{\rm cr} = -\arctan\left\{\frac{\pi/4}{((2k+1)^2-2)/((2k-1)(2k+1)(2k+3))}\right\}.$$

Consequently, for the phases near $\varphi_{\rm cr}$ the induced variations of the decrement dominate. For the high-frequency dithering (with $k \gg 1$) the results in Eqs. (15) and (16) can be simplified

$$\frac{D_1}{D_1(a=0)} \approx 1 + 3\cos\varphi\left(\frac{a}{A}\right), \qquad \frac{\Delta c_1}{\Delta c_1(a=0)} \approx 1 + \cos\varphi\left(\frac{a}{A}\right). \tag{18}$$

The asymptotic formulas (18) (valid for $k \gg 1$, $a/A \ll 1/4k^2$) demonstrate that the influence of the high-frequency dithers on the wave velocity is comparable to its influence on the wave amplitude for all possible φ.

The influence of a dither at a frequency equal to an even harmonic of the strong wave is essentially less efficient. For $n = 2k$ $(k = 0, \pm 1, \pm 2, \ldots)$ Eqs. (12) and (14) take the form

$$\frac{D_1}{D_1(a=0)} \approx 1 - \frac{3}{4}\left\{1 - \frac{\cos(2\varphi)}{16k^2-1} + 2\frac{4k^2+1}{4k^2-1}\cos^2\varphi - 8k^2\sin^2\varphi\right\}\left(\frac{a}{A}\right)^2, \tag{19}$$

$$\frac{\Delta c_1}{\Delta c_1(a=0)} \approx 1 + \left\{\frac{\sin\varphi}{2} + \frac{2}{\pi k}\left[\frac{1}{16k^2-1} - \frac{1}{4k^2-1}\right]\cos\varphi\right\}4k^2\sin\varphi\left(\frac{a}{A}\right)^2. \tag{20}$$

The solution in Eq. (19) reproduces the results obtained earlier [26] both numerically and analytically for $k = 1$, $\varphi = 0, \pi/4$. In particular, the effect of the induced transparency ($\propto (a/A)^2$) is confirmed for $\varphi = 0$ and the effect of the induced absorption ($\propto (a/A)^2$) is confirmed for $\varphi = \pi/4$. In the regime of the high-frequency dithering ($k \gg 1$) the results in Eqs. (19) and (20) can be simplified:

$$\frac{D_1}{D_1(a=0)} \approx 1 + 6k^2\sin^2\varphi\left(\frac{a}{A}\right)^2, \qquad \frac{\Delta c_1}{\Delta c_1(a=0)} \approx 1 + 2k^2\sin^2\varphi\left(\frac{a}{A}\right)^2. \tag{21}$$

The asymptotics in Eq. (21) (valid for $k \gg 1$, $a/A \ll 1/4k^2$) demonstrate once again the comparable influence of the high-frequency coherent excitations on the stronger-wave absorption and velocity for all possible phases.

3. Discussion

The developed theory provides an important time-domain criteria necessary for the effective influence of a weak signal on a stronger wave. Earlier [20,21,25] the criteria of the effective frequency mixing in the materials with hysteretic quadratic nonlinearity has been formulated in the frequency-domain as an energy conservation law in multiphonon processes (see Eq. (1)). The results presented in Section 2 indicate that in the time-domain the same criteria can be formulated in terms of the amplitude modulation of the total acoustic field. The modulation of a stronger wave by a weak signal is proportional to the signal amplitude and the signal is capable to modulate the total field amplitude efficiently (i.e., also proportionally to the signal amplitude). This is a necessary condition for the efficient modulation of both the decrement and the velocity of the stronger wave.

The developed theory can be applied to derive some preliminary conclusions on the propagation of the stronger wave in the presence of the acoustic noise. It is well known that in materials with classical (elastic) quadratic nonlinearity noise induces additional absorption of acoustic waves proportional to noise spectral intensity [35,36]. From the solutions derived in Section 2 it follows that in the materials with hysteretic quadratic nonlinearity the interaction of the stronger wave with the noise can lead to absorption that is also proportional to noise spectral intensity. Assuming that in the stochastic acoustic field (in the acoustic noise) the phase of a spectral component varies chaotically (homogeneous distribution of phase in the interval $-\pi \leqslant \varphi \leqslant \pi$) it can be verified that the strong-wave modifications proportional to a/A in Eq. (12) disappear in averaging over $-\pi \leqslant \varphi \leqslant \pi$. Only the decrement variations proportional to $(a/A)^2$ survive in Eq. (12) after averaging

$$\frac{D_1}{D_1(a=0)} \approx 1 + \frac{3}{4}\left\{n^2 - \frac{(-1)^n n^2 + 1}{n^2 - 1} - 1\right\}\left(\frac{a}{A}\right)^2. \tag{22}$$

In accordance with Eq. (22) the interaction of the spectral components of the noise with the stronger wave always induces for this wave an additional absorption (the coefficient in front of the $(a/A)^2$ in Eq. (22) is positive for all n).

To finish the discussion it is worth noting that in engineering the term "dither" is also used to refer to unintentional noise which may be picked up by a system [27]. Moreover, the coherent signals injected in a nonlinear system are sometimes referred to as artificial dithers. So, the discussed above interaction of the low-frequency stronger wave with a noise is just a particular case of dithering the systems with the hysteretic quadratic nonlinearity.

4. Conclusions

The influence of an artificial weak dither on the absorption and velocity of a stronger harmonic wave in a medium with hysteretic quadratic nonlinearity is analyzed. It is demonstrated that due to odd symmetry of this acoustic nonlinearity the modifications of the stronger wave by the dither at frequencies equal to its odd harmonics is proportional to the dither amplitude. Depending on the relative phase of the low-frequency strong wave and the dither either the effects of the induced absorption or of the induced transparency are possible. The induced effects proportional to the dither amplitude are due to the capability of the dither to modulate the amplitude of the total acoustic field proportionally to the dither amplitude. The modifications of both the absorption and the velocity of the stronger wave by a dither at a frequency equal to an even harmonic of the strong wave is proportional to the square of the dither amplitude. It is predicted that the absorption of the acoustic wave by a noise in the materials with hysteretic quadratic nonlinearity is proportional to noise intensity.

An interesting perspective for the extension of the developed theory is the analysis of the dithers that are capable to induce minor loops in the nonlinear hysteretic stress/strain relationship.

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