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Microstructure induced nonlinearity of unconsolidated rocks as related to seismic diagnostics problems

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Abstract. Manifestations of the so-called structure-induced nonlinearity are considered for the case of a granular medium, the latter being a generally accepted model of nonconsolidated rocks in seismics. The consideration is carried out using the medium model in the form of the "ideal" random packing of spherical elastic granules in which the interparticle space can be filled with a fluid. A physical equation of such a medium is derived; the dependencies of nonlinear parameters on the grain material elastic moduli, the fluid compressibility and the initial medium strain are analyzed. The influence of defects in nonideal grain packings (that is, the presence of a fraction of unloaded intergranular contacts) upon the nonlinear properties of the medium is investigated. It is shown that the packing nonideality has the stronger effect on higher-order nonlinear properties. It is demonstrated that the nonlinear parameters may be used in exploration seismology as a much more sensitive and informative characteristic compared with conventionally used linear moduli.

Introduction.

Seismoacoustic methods of surveillance are widely used in modern geology [Sheriff and Geldart, 1987; Gurvich and Nomokonov, 1981]. In most cases these methods are active in the sense that a probing wave source is needed, although passive methods based on the location of seismic noise are also known (see, e.g., [Gurvich and Nomokonov, 1981]). Typical informative characteristics are the propagation time and signal amplitude, knowledge of which permits one to retrieve spatial distribution of wave propagation velocity and its damping rate, for both longitudinal and transverse wave modes. This, in turn, opens a possibility for some conclusions concerning the structure of a rock, provided the theoretical or empirical relations between the measured parameters and the material structural properties are available. Depending on details of the structure (porosity, cracks, fluid filling, etc.), different parameters could be most sensitive to these details. In the classical linear seismoacoustics, an option is practically restricted by few parameters mentioned above (wave velocity, absorption and scattering coefficients). Moreover, in a number of important cases, variations of these parameters often turn to be practically unresolvable (e.g., layers with different structures may have close linear impedances [Gurvich and Nomokonov, 1981]).

New possibilities are associated with the measurements of nonlinear seismoacoustic parameters of Earth materials which are attracting a growing interest in the last decade. The fact of a relationship between nonlinear parameters of a medium and its structural characteristics is experimentally established now. It is important that the variations of parameters characterizing the nonlinearity turn to be significantly (up to several orders) larger than variations of linear parameters in the same experiments [Groshkov et al., 1990: Nikolaev, 1987; Meegan et al., 1993]. Theoretical models of medium nonlinearity based on traditional, for the elasticity theory, expansions of the stress-strain relations with keeping quadratic and qubic terms [Nikolaev and Galkin, 1991; McCall, 1993], in some cases permit one to explain experimental data by choosing proper values of expansion coefficients. In other cases, however, nonlinear effects in complex media appear to have a qualitatively different character (predominance of the 3rd harmonic over the second one, quadratic dependence of the 3rd harmonic amplitude on the first harmonic amplitude, etc.). Such effects can be sometimes described by phenomenological models for stress-strain dependence taking into account hysteretic phenomena [Nazarov and Sutin, 1989; Nazarov, 1991].

However, phenomenological models have only a restricted use because of the lack of understanding of the pro-

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cesses occurring in the medium, and of a possibility to relate measured parameters to the internal medium structure. There exists a need to construct theoretical models based on the "microstructure" of the medium ("micro" means here just that the scales of this structure are much smaller than the wavelength of the seismoacoustic field, but a larger than atomic or molecular scales). A similar approach is well known for liquids with bubbles where continual models describing the bubble-liquid system as a continuous medium are widely used [Kobelev and Ostrovsky, 1980]. For structured solids, only few nonlinear models of such a kind were considered. The models of water-like porous media [Ostrovsky, 1988], grainy media [Nesterenko, 1983; Bogdanov and Skvortsov, 1992; Dunin, 1989], and a bimodular model for cracks [Nazarov, 1994] should be mentioned in this connection. However, only the simplest variants were studied even for these cases.

Here we consider the nonlinear models of grainy media which can be applicable to the description of some non-consolidated Earth materials. We start from the model of an ideal , fluid-saturated grain packing, and then consider non-ideal packing (that is, the intergrain contacts are subjected to strongly different initial loading). These models are shown to describe adequately many of experimentally observed effects in grainy media. After this, some realistic situations are discussed when the measurements of nonlinear parameters for seismic diagnostics (e.g., for the oil searching) can give much more information than traditional linear methods. Main attention will be paid to the relation between nonlinear parameters of the medium and its structure rather than to the technical aspects of the problem.

1 On the origin of elastic nonlinearity in structurally inhomogeneous media. Quantitative characteristics of nonlinearity.

It is well known that in "ordinary" homogeneous materials the elastic nonlinearity is determined by interatomic (intermolecular) potential profile, which in the vicinity of the equilibrium state is close to the parabolic type. Due to this fact at small amplitudes characteristic of elastic waves, nonlinear effects in such materials are rather weak, and, therefore, it is a widely accepted opinion that the nonlinearity may become noticeable only at large enough strain (of the order of $10^{-3}...10^{-2}$), that is close to the breakdown threshold.

In the case of structurally inhomogeneous Earth rocks (grainy, crack-containing, porous, etc.) the situation is quite different. A dependence of medium physical properties on its structural features is well known for a number of various effects. One may mention, for example, the anomalously high thermo-absorption of sound in polycrystalline materials, compared with the absorption in the material of singular crystallite [Landau and Lifshits, 1986]. It is the appearance of a small scale (of the order of the singular crystalline grain size) in the medium that leads to appearance of high local temperature gradients and increases thermal sound dissipation. In a somewhat analogous manner the existence of a small scale in the medium elastic properties may lead to the appearance of high local values of strain which belongs to considerable nonlinear region of the "stress-strain" dependence, and results in anomalously high nonlinear response of the whole sample even at small mean strains.

The existence of components (inclusions) with contrast elastic properties in a material structure may lead to a large growth of the medium nonlinearity. In seismics the microinhomogeneity of rocks is just the rule rather than the exception. The presence of "soft" component in structurally - inhomogeneous rocks leads to the appearance of high (in comparison with the spatially averaged value) magnitude of strain and stress, that correspond to considerably nonlinear form of the local "stress-strain" relation.

Let us elucidate briefly how the nonlinearity of a medium may be characterized quantitatively. Suppose, that the case of longitudinal deformation stress-strain relation is $\sigma = (\varepsilon)$, where σ is the stress, ε is the strain, and for the sake of simplicity the material is supposed to be isotropic. Usually it is possible to consider small vibrational perturbations on the background of some initial strain, ε_0 , as a power series of variation $\widetilde{\varepsilon} = \varepsilon - \varepsilon_0$:

$$\widetilde{\sigma} = \sigma'_{\varepsilon}(\varepsilon_0) \cdot \widetilde{\varepsilon} + \frac{1}{2!}\sigma''_{\varepsilon\varepsilon}(\varepsilon_0)\widetilde{\varepsilon}^2 + \frac{1}{3!}\sigma'''_{\varepsilon\varepsilon\varepsilon}(\varepsilon_0) + \ldots \qquad (1)$$

where $\sigma = \sigma(\varepsilon) - \sigma(\varepsilon_0)$.

It is typical in nonlinear acoustic to define the following linear and nonlinear parameters

$$M = \sigma'_{\varepsilon}(\varepsilon_0) \qquad (2)$$

$$\Gamma^{(2)} = \sigma''_{\varepsilon\varepsilon}(\varepsilon_0)/\sigma'_{\varepsilon}(\varepsilon_0) \qquad (3)$$

$$\Gamma^{(3)} = \sigma'''_{\varepsilon\varepsilon\varepsilon}(\varepsilon_0)/\sigma'_{\varepsilon}(\varepsilon_0) \qquad (4)$$

Note that M corresponds to linear elastic modulus which determines the value of the longitudinal wave velocity $v = [M/\rho]^{1/2}$, and the coefficients $\Gamma^{(2)}$, $\Gamma^{(3)}$ determine quadratic and qubic nonlinear effects, respectively [Ostrovsky, 1990; Belyaeva et al., 1993]. It is known that the real stress-strain relation for many structurally inhomogeneous media actually cannot be approximated by the power expansion (1), as their state equation has more complex form, for example, it may be of the so-called bi-modular type (that is to have different elasticity with respect to stretch and compression) or may have a hysteresis character. However, even in these cases the nonlinear parameters $\Gamma^{(2)}$, $\Gamma^{(3)}$ may be introduced in some effective sense using the relation of magnitudes of higher harmonics and that at the fundamental frequency (however the parameters defined in such a way may appear to be amplitude-dependent unlike of the parameters introduced by (1)-(4)). Therefore, the nonlinear parameters defined above allow to compare nonlinear properties of different media. For example, to illustrate quantitatively the statements made above, the following typical values of the quadratic nonlinear parameter may be noted: $\Gamma^{(2)} \sim 3...10$ for most part of "ordinary" homogeneous materials (such as water, melted quartz, most metals) [Zarembo and Krasil'nikov, 1966], whereas for microinhomogeneous media such as bubbly water, porous rubber-like plastics, Earth rocks, and some polycrystalline metals it may be as large as 10^3 -10^4.

2 Nonlinear properties of grainy media with ideal packing.

From the viewpoint of seismic applications one of the most important examples of structurally inhomogeneous media is the medium with grainy structure that is typical of many kinds of sedimentary rocks. In some cases the intergrain space in such rocks may be filled with gas or liquid that is important for the seismo- prospecting of mineral resources.

Then, the grainy structure is one of a few structure types that allows for consistent analysis of its elastic properties. The model of the grainy material was used in numerous works for studying linear elastic properties of rocks (first of all, the longitudinal and the transversal sound velocity), as a basis of conventional linear methods of seismoprospecting [Sheriff and Geldart, 1987].

Bearing in mind the qualitative consideration given in the previous section, it is evident that the presence of soft intergrain contacts should lead to highly increased nonlinearity of grainy media, that was discussed in a number of papers [Nesterenko, 1983; Bogdanov and Skvortsov, 1992; Dunin, 1989; Belyaeva et al.,1993; Belyaeva and Timanin, 1992].

Let us show schematically how the nonlinear "stress-strain" relation for such a medium may be derived in the assumption that the motion of fluid and elastic components is joint, as it is usually accepted in analysis of grainy medium models [Gurvich and Nomokonov, 1981]. In this section we restrict our analysis by the types of grain packing, that is called below as the "ideal" packing. The latter means that the intergrain contacts are supposed to be loaded with approximately equal extent. The packing may be either regular or random, but it should not contain contacts which are deformed with considerably different extent. The discussion of the effects of such contacts will be performed in section 4, where the corresponding packings will be called as "non-ideal" ones.

Consider a sample composed of a large number N of particles of radius R. The properties of particle material are described by the constants K_s (the bulk compression modulus) and E_s (the Young modulus). Let α be the void content (porous space) per unit volume of the aggregate and $\overline{n}$ the average number of contacts between particles. The values of α and n for a system of randomly packed identical particles have been found experimentally: $\alpha = 0.392, \overline{n} = 8.84$ [Deresewicz. 1958]. (These values range from $\alpha = 0.477$ in the case of a simple cubic packing to $\alpha = 0.23$ in the case of tightest packing of identical granules). Let the porous space between particles be filled with a fluid with bulk modulus K_f under pressure P_f . The total volume of the "particle + fluid" system amounts to

$$V_t = \frac{4}{3}\frac{\pi N R^3}{(1-\alpha)}$$

Suppose that the whole aggregate of N particles is subjected to quasistatic compression due to pressure P_{ext} that causes particle deformation. Therefore, the contact point transforms to a circle and the particle centers draw closer to a distance $2 \cdot \widetilde{\Delta}$ which is related to the force F, acting at the contact, by the Hertz formula [Landau, 1986]:

$$\widetilde{\Delta} = \left(\frac{3\left(1-\nu_s^2\right)F}{4ER_s^{1/2}}\right)^{2/3} \tag{5}$$

that is valid at relatively small strains, $\widetilde{\Delta}/R \ll 1$. Here, ν_s is the Poisson coefficient of the particle material. The variation of total volume V_t to the first order in $\widetilde{\Delta}$ is equal to:

$$\delta V_t = -b\frac{4\pi N R^2\widetilde{\Delta}}{1-\alpha} \tag{6}$$

where $b = 1$ in the case of bulk compression and $b = 1/3$ for longitudinal strains.

This variation includes the decrease of the porous volume as the particle centers draw together. the decrease of the spherical granule volume due to the all-round compression by the fluid, and, finally, the particle volume variation in the contact region. It can be shown that the latter correction is of the order of $\widetilde{\Delta}^2$ (while other items have the order of $\widetilde{\Delta}$) and is therefore negligible.

To ascertain the relation between the force F acting at the contacts between particles and the given external pressure P_{ext} , we make use of the energy balance equation:

$$\delta W_{ext} = \delta W_f + \delta W_s + \delta W_c \tag{7}$$

where W_{ext} is the energy required for the quasistatic compression of the aggregate by external pressure, W_f

is the energy of pore fluid , W_s is the elastic energy of the particle matter due to the bulk compression by the fluid, and W_c is the energy stored in the contacts.
The energy spent for reducing the aggregate volume by δV_t is determined by the expression:

$$\delta W_{ext} = P_{ext} dV_t \quad (8)$$

The energy accumulated by a pore fluid is given by:

$$\delta W_f = (P_f + \delta P_f)\, dV_f \quad (9)$$

where δp_f is the excess pressure due to the variation of the fluid volume δV_f .
The energy stored in the volume of fluid- compressed granules is given by:

$$\delta W_s = (P_f + \delta P_f)\, dV_s \quad (10)$$

where δV_s is the variation of the spherical granule volume under the action of compressing excess pressure δP_f from the pore fluid side.
The energy due to the deformation of contact areas can be written as:

$$\delta W_c = b\overline{n} N F\left(\widetilde{\Delta}\right)\widetilde{\Delta} \quad (11)$$

where the relation between $\widetilde{\Delta}$ and δV_t is given by (6). The quantity b in this formulae describes the effective number of "active" (those that accumulate energy) contacts. It is obvious that in the case of all- round compression the particle centers are shifted with respect to each other only along the normal to the contact surface. Note that all contacts have "equal rights" owing to the isotropy of the problem and one should put $b = 1$ in (11). In the case of plane deformation granule centres are also shifted along contact normales (i.e., in radial directions). However, due to the lack of granule centre shift along two coordinate axes (which are orthogonal to the plane deformation axis), because of the isotropy in contact orientation, and independence of each contact deformation work the average effective share to the elastic energy of contact W_c is paid by one third part of the total contact number. Therefore one should put in (11) the factor $b = 1/3$.
The pressure and volume variations of the fluid and the granules are related via elastic constants by:

$$\delta p_f = -K_f \frac{\delta V_f}{V_f} \quad (12)$$

$$\delta p_f = -K_s \frac{\delta V_s}{V_s} \quad (13)$$

Thus, for the variables $\delta p_f, \delta V_f$,and δV_s , we have the set of linear equations (6), (12) and (13) that gives the expressions of all the quantities in terms of $\widetilde{\Delta}$:

$$\delta p_f = \frac{\frac{3\widetilde{\Delta}}{R} b K_f}{\alpha + (1-\alpha)\eta} \quad (14)$$

$$\delta V_f = -\frac{\alpha b}{\alpha + (1-\alpha)\eta} \cdot \frac{4\pi N R^2}{1-\alpha}\widetilde{\Delta} \quad (15)$$

$$\delta V_s = -\frac{\gamma b}{\alpha + (1-\alpha)\eta} 4\pi N R^2 \widetilde{\Delta}^2 \quad (16)$$

where $\eta = \frac{K_f}{K_s}$.
In these expressions $\widetilde{\Delta} = (\Delta - \delta R_s)$ where $2 \cdot \delta R$ is the variation of the distance between the granule centres due to the all- round compression of granules by a pore filler (note that $\delta R_s / R_s \simeq (\delta V_s / V_s)\ /3$, i.e.:

$$\widetilde{\Delta} = \Delta\left(1 - \frac{\eta b}{\alpha + (1-\alpha)\eta}\right) \quad (17)$$

Substituting these expressions into the energy balance equation (7), we obtain:

$$\frac{4\pi R^2}{1-\alpha}(P_{ext} - P_f) - \pi F - \frac{12\pi R K_f b \widetilde{\Delta}}{(1-\alpha)\left[\alpha + (1-\alpha)\eta\right]} = 0 \quad (18)$$

where according to (5) F is equal to:

$$F = \frac{4E_s R^{1/2}}{3(1-\nu_s^2)} \widetilde{\Delta}^{3/2} \quad (19)$$

Eq. (18) relates, in fact, the effective pressure and the strain in the system. Hence, assuming b = 1 in the case of bulk compression and linearizing (18) with respect to strain $\left(\widetilde{\Delta}/R\right)$ variations we can obtain the linear modulus of bulk compression.
Consider in more detail the case of pure longitudinal strain, which corresponds to the longitudinal wave propagation, and put $b = 1/3$ in (18). Introducing the longitudinal strain $\varepsilon = \delta l_{11}/l_{11} = \delta V_t / V_t$, we obtain the "nonlinear Hooke's law" for a fluid saturated granular medium:

$$\sigma_{eff} = \frac{K_f}{\alpha + (1-\alpha)\eta}\varepsilon + \frac{\overline{n}(1-\alpha)E_s}{3\pi(1-\nu_s^2)}\varepsilon^{3/2} \quad (20)$$

Here we take into account that the external pressure is shared between the fluid and the grain skeleton, thus in (20) the effective stress is defined as $\sigma_{eff} = P_{ext} - P_f$. However real sedimentary rocks have usually partially cemented grains, so the pore fluid provides a less extent of the solid skeleton unloading. To take this fact into account it is accepted to introduce the so-called unloading coefficient k in the definition of effective pressure:

$$\sigma_{eff} = P_{ext} - kP_f \quad (21)$$

The value of k usually lay in the band $0.85 \leq k \leq 1$ and is dependent on the relation of compressibilities of the pore fluid, the grain bulk and the extent of the grain cementing [Gurvich and Nomokonov, 1981].
We emphasize that the nonlinear term appears exclusively due to the presence of Hertz contacts rather than

the nonlinearity of the aggregate component materials which we neglect in our consideration.
Using the definition introduced in equations (1) - (4) we can obtain the elastic modulus for longitudinal strain of the aggregate $M_{\|}$:

$$M_{\|} = \sigma'_{\varepsilon}(\varepsilon_0) = K_f \frac{1}{\alpha + (1-\alpha)\eta} + \frac{\overline{n}(1-\alpha)E_s}{2\pi(1-\nu_s^2)K_f}\varepsilon_0^{1/2} \quad (22)$$

and for nonlinear parameters:

$$\Gamma_{agr}^{(2)} = \frac{\sigma''_{\varepsilon\varepsilon}(\varepsilon_0)}{\sigma'_{\varepsilon}(\varepsilon_0)} = \frac{\frac{\overline{n}(1-\alpha)E_s}{4\pi(1-\nu_s^2)}\varepsilon_0^{-1/2}}{K_f\left(\frac{1}{\alpha+(1-\alpha)\eta} + \frac{\overline{n}(1-\alpha)E_s}{2\pi(1-\nu_s^2)K_f}\varepsilon_0^{1/2}\right)} \quad (23)$$

In the case of small static initial strains and much less compressible pore filler compared to the contact compressibility, the second term in the denominator can be neglected. Thus, the expression for the nonlinear parameter is simplified:

$$\Gamma_{agr}^{(2)} = \frac{\overline{n}(1-\alpha)(\alpha+(1-\alpha)\eta)E_s}{4\pi(1-\nu_s^2)K_f}\frac{1}{\varepsilon_0^{1/2}} \quad (24)$$

Using the same procedure we introduce the cubic nonlinearity parameter. We thus obtain:

$$\Gamma_{agr}^{3} = \frac{\sigma'''_{\varepsilon\varepsilon\varepsilon}(\varepsilon_0)}{\sigma'_{\varepsilon}(\varepsilon_0)} = \frac{\frac{\overline{n}(1-\alpha)E_s}{8\pi(1-\nu_s^2)}\varepsilon_0^{-3/2}}{K_f\left(\frac{1}{\alpha+(1-\alpha)\eta} + \frac{\overline{n}(1-\alpha)E_s}{2\pi(1-\nu_s^2)K_f}\varepsilon_0^{1/2}\right)} \quad (25)$$

For a granular medium without a pore filling Eq.(18) has a simpler form:

$$\frac{4\pi R^2}{1-\alpha}P_{ext} - \overline{n}F = 0 \quad (26)$$

and in (14)-(16) we should put $\widetilde{\Delta} = \Delta$. Consider the plane deformation in such a medium. Introducing the stress $\sigma = P_{ext}$ and the relative longitudinal strain $\varepsilon = dV_t/V_t$, we write (20) in the form of the "nonlinear Hooke's law":

$$\sigma = \frac{\overline{n}(1-\alpha)E_s}{3\pi(1-\nu_s^2)}\varepsilon_0^{3/2} \quad (27)$$

Linearizing this equation with respect to the strain variation $\widetilde{\varepsilon}$ near the static initial compression ε_0, we obtain an expression for the elastic modulus of pure longitudinal strain:

$$M_{\|s} = \frac{\overline{n}(1-\alpha)E_s}{2\pi(1-\nu_s^2)}\varepsilon_0^{1/2} \quad (28)$$

In a manner similar to that in (29) we write the quadratic nonlinear parameter $\Gamma_s^{(2)}$:

$$\Gamma_s^{(2)} = \frac{1}{2\varepsilon_0} \quad (29)$$

where the initial strain ε_0 can be connected with the value of external pressure P_0,using (28), then for $\Gamma_s^{(2)}$ equation (29) yields:

$$\Gamma_s^{(2)} = \frac{1}{2}\left(\frac{3\pi(1-\nu_s^2)P_0}{\overline{n}(1-\alpha)E_s}\right)^{2/3}$$

For the cubic nonlinearity parameter :

$$\Gamma_s^{(3)} = \frac{1}{6\varepsilon_0^2} \quad (30)$$

Thus, the nonlinear parameters of a granular medium without a pore filling is exclusively determined by the initial static strain.
Model experiments [Belyaeva and Timanin, 1992] with grainy samples that consisted of random, but close to "ideal" packing of monoradial grains of lead shot. showed rather good qualitative and quantitative agreement of experimental results on measurement of linear and nonlinear coefficients with expressions (29), (30). In particular, measured levels of the quadratic nonlinear parameter are in the range from $1.3 \cdot 10^2$ to $1.5 \cdot 10^3$, and the qubic nonlinear parameter - from $4 \cdot 10^4$ to $5.5 \cdot 10^5$ under prestrain value ε_0 from $3 \cdot 10^{-4}$ to $3.4 \cdot 10^{-3}$.

3 The effects of nonideal character of the grain packing.

Let us remind, that the expressions in the previous section were derived in the assumption of "ideal" character of grain packing, that is the conditions of the grains loading were supposed to be almost the same. However, the comparison of the consequences from the ideal packing model with the results of known experiments with real non-consolidated media shows both quantitative and qualitative discrepancies between the theory and the experiments. For example, papers [Tsareva, 1956; Antsiferv et al., 1964] report on investigations of the dependence of longitudinal sound velocity in sand upon the initial static pressure. According to these data for different sand samples the observed power-law exponents in most cases appeared to be greater than 1/6 that followed from the "ideal" model, and the estimated number of contacts per grain was from 2 to 5 instead of 8..12 for packings close to the most dense one. These facts may be explained by the existence of intergrain contacts which are initially weakly loaded (or unloaded totally), that is by the non-ideal character of the grain packing.
Next, in our experimental work [Belyaeva et al., 1994(a)] on harmonic generation in grainy media we paid special attention to the difference of measured values from the theoretical conclusions which followed from the ideal

packing model. In particular, the following main features were mention in paper [Belyaeva et al., 1994(a)], which we attribute to the manifestations of nonideality of the grain packing.

For the fundamental harmonic the sample linear elastic modulus appeared to be somewhat less than the predicted by the ideal packing model; then, there was mentioned more intensive growth of the sample elasticity modulus with the increase of initial static compression; and, third, the variations of the effective elastic modulus (both negative and positive for different measurement series) depending on the excitation amplitude were observed, that also could not be explained in the frames of the ideal packing model.

For the second harmonic of the elastic force the level was by 50 to 100% higher compared with the ideal model predictions, and the dependence on the given drive displacement amplitude at the fundamental frequency was noticeably different from the ideal quadratic-power law. For the third harmonic the level appeared to be $10 \div 100$ times higher, the amplitude dependence was of considerably non-cubic type (sometimes it was more like the quadratic one), and the third harmonic phase in some experiment even changed its sign with the increase of the driving amplitude.

We attribute the above mentioned distinctions to the influence of weakly loaded inter-grains contacts which occur in real grain packing. Let us consider the main differences from the ideal model, that should be caused by such unloaded contacts. The relation between the elastic stress force and the displacement of the grainy sample boundary in the considered case should be different from $F \sim x^{2/3}$ law that followed from the ideal packing model [Belyaeva et al., 1993]. In particular, for a grainy medium sample compressed in a rigid metal cup by a harmonically vibrating piston with the displacement $x = d_0 + d\cos\omega t$ the expression for the elastic force F may be written as follows:

$$F = \sum \nu_i Q_0 \mu_i^{3/2} d_0^{3/2} \left[1 + \frac{\xi}{\mu_i}\cos wt\right]^{3/2} \theta\left[\mu_i - \xi\cos wt\right],$$

$$\theta(x) = \begin{cases} 1, & \text{at } x > 0 \\ 0, & \text{at } x \le 0 \end{cases} \qquad (31)$$

where $\xi = d_\omega / d_0$ - is the normalized oscillation amplitude at the main frequency. The factor Q_0 is proportional to the number of contacts in the sample, the precompression of which is determined by the total value of initial deformation d_0 (this part of contacts is named as the "main fracture" below); the value ν_i characterizes the quantity of the contacts in the i-th fracture, which are slightly compressed, and the extent of their compression, in its turn, is determined by the value $\mu_i d_0$, ($|\mu_i| \le 1$). The value $\mu_0 = 1$ corresponds to the particles from the main fracture, that are compressed to the value d_0.

In the simplest non-trivial case we may restrict ourselves by the consideration of a two-fraction model (that includes the only one additional fraction of the unloaded contacts), and try to simulate the dependences observed in experiments [Belyaeva et al., 1994], which could not be explained on the base of ideal (that is one-fraction in terms of equation (31)) model. Some results of such a simulation are presented in Figure 1, where the elastic force harmonic amplitudes against the amplitude of the sample boundaries are shown in logarithmic scale.

As it may be seen, the results of calculation according to the model (31) are in good accordance with the experimental data already for the simple two-fraction model. The curves in Figure 1 show that the most sensitive to the presence of the unloaded contacts appeared to be the elastic moduli of higher orders. Indeed, the difference of calculation of the linear elastic modulus according to the "ideal" and "non-ideal" models are about several percents, whereas for the second harmonic the difference achieves 10 times, and for the third harmonics the discrepancy appeared to be up to 2-3 orders of magnitude. It is worth to note also a rather unusual quadratic amplitude dependence of third harmonics observed in some experiments (Fig.1a), that may be also described in the frames of the model (31). The difference in the qualitative character of the harmonics behavior (i.e. different exponents of power dependences in the cases (a) and (b)) is connected with the influence of "clapping' unloaded contacts in experiments presented in Figures 1(a), (b).

Therefore, the analysis of the effects of non-ideal packing showed that while linear elastic modulus is rather low sensitive to the packing defects (the presence of unloaded contacts), the nonlinear moduli may become different by the orders of value from the calculations based on the model of ideal grain packing, and they even may acquire qualitatively different character of the amplitude dependences. The improved model itself agrees much better with experimental data compared with the ideal packing model.

4 Comparative analysis of the linear elasticity and nonlinear parameters variation in the layers with different structure.

It was shown in the previous sections that experimentally observed levels of the nonlinear parameters for crumbly earth rocks may be explained by the action of "structural" nonlinearity. It will be demonstrated below on the base of the above model that nonlinear parameters can be used as a very sensitive characteristics in seismoprospecting problem.

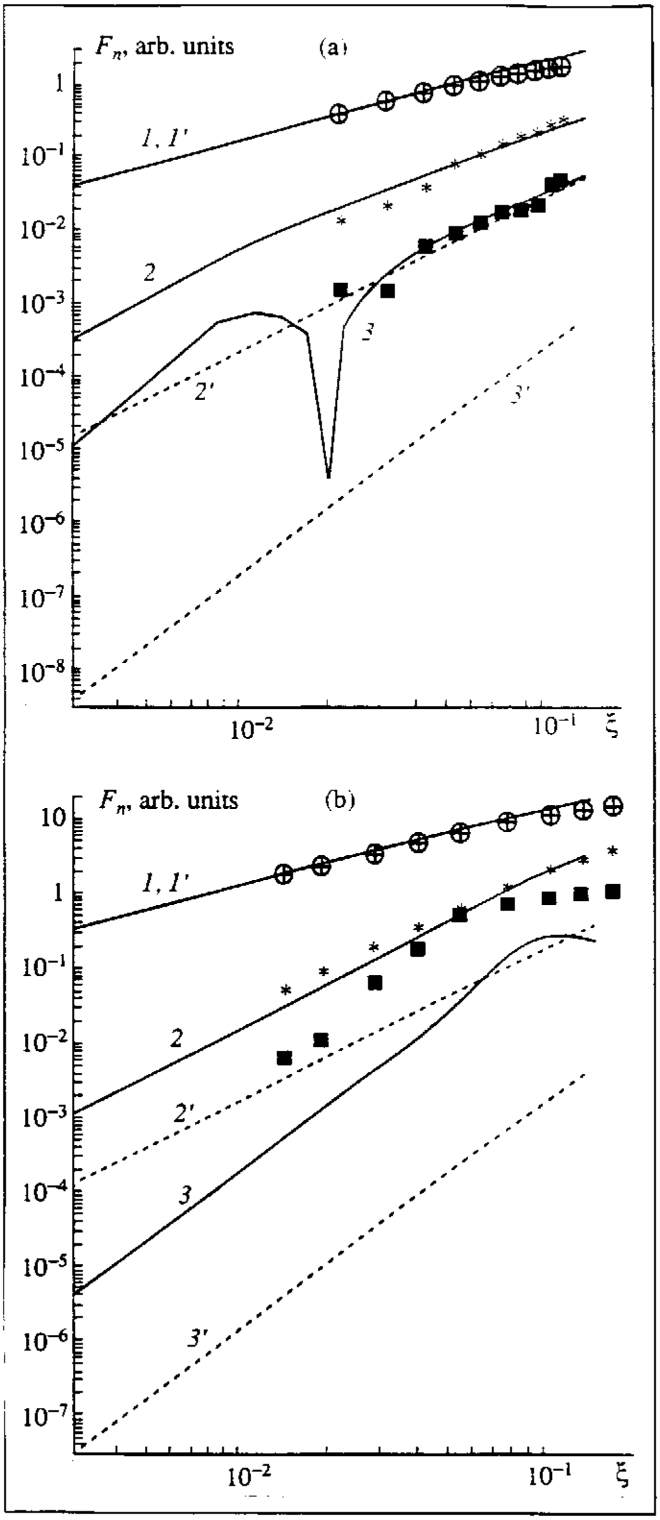

FIG. 1. Log plots of the first - third harmonics of the elastic force F_n as functions of the normalized displacement ξ of the boundary of the sample. ⊕ is the fundamental. * is the second harmonic, and ■ is the third harmonic. The solid lines represent a computation for nonideal packing (the two- component approximation); the dashed curves represent calculation for the ideal- packing model (the one-component approximation). The numerals 1 - 3 and 1' - 3' on the curves are the orders of the harmonics. (a) Lead shot; initial compression 3 μm; (b) plastic granules; initial compression, 25 μm.

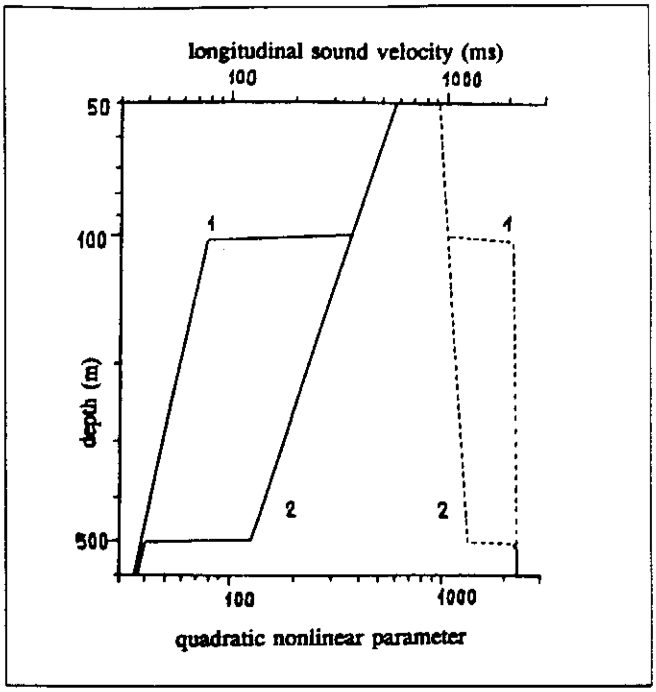

FIG. 2. Dependence of longitudinal sound velocity C_p and quadratic nonlinear parameter $\Gamma^{(2)}$ on depth h in the absence of nonpenetrable dome over fluid-saturated layer. 1 — fluid boundary H_0 = 100 m, 2 — H_0 = 500 m. Solid line — nonlinear parameter, dashed line — longitudinal sound velocity.

In the following analysis we restrict ourselves by consideration of the linear and the low-order nonlinear modulus (quadratic nonlinear parameter). As it was shown above, the nonideality of grain packing leads to especially strong increase of qubic (and of higher orders) nonlinear parameters, so for further estimations it may be used the model of ideal grain packing.

As a first example we choose the crumbly sand-like sediments with a water layer below the depth H_0 . The results of the calculations based on the derived formulas (20)-(24) are presented in Fig.2. The following model parameters were chosen for the calculation: $\alpha = 0.2$, $E_s = 5.3 \cdot 10^{10} N/m^2$, $\nu = 0.2$, $\rho_s = 2.65 \cdot 10^3 kg/m^3$, $K_f = 2.2 \cdot 10^9 N/m^2$, and it was kept in mind that the value p_{ext} in (20) is determined by the weight of overlaying medium layer. The figure shows that the sound velocity variation at the boundary H_0 of water layer is about 50-70%, while the nonlinear parameter $\Gamma^{(2)}$ decreases by $4 \div 5$ times (both changes are connected with the increase of medium compressibility due to the appearance of fluid filling). With H_0 increase the value of both characteristics changes decreases. Anyway, the variability of nonlinear parameter is significantly higher than that of sound velocity, though the difference is still not so drastic as in the following example.

Much more abrupt variations of the nonlinear parameter can be observed at the fluid boundary under anomalous pressure [Gurvich and Nomokonov, 1981]. The discovery of the fluid layers covered by nonpenetrable

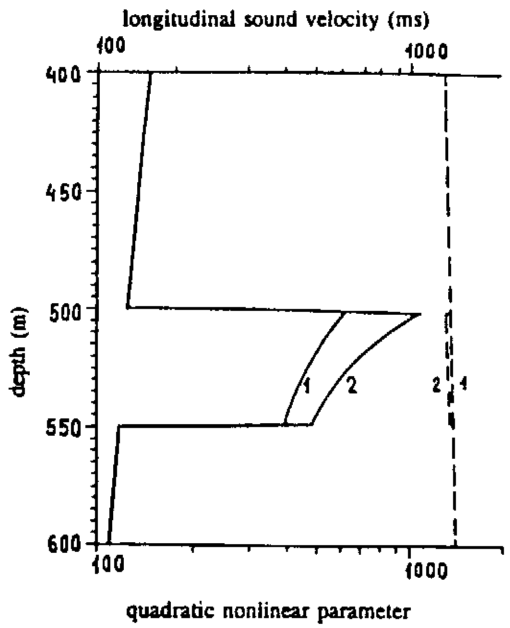

FIG. 3. Dependence of longitudinal sound velocity C_p and quadratic nonlinear parameter $\Gamma^{(2)}$ on depth h in the presence of nonpenetrable dome over fluid-saturated layer, $H_L = H_0$. 1 — unloading coefficient $k = 0.95$, 2 — $k = 0.985$. Solid line — nonlinear parameter, dashed line — longitudinal sound velocity.

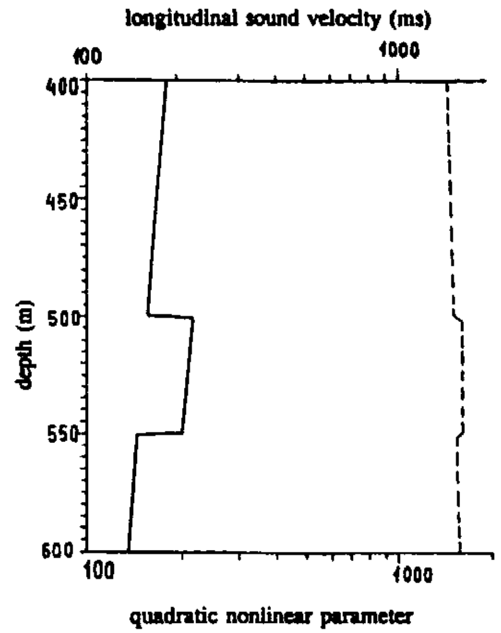

FIG. 4. Dependence of longitudinal sound velocity C_p and quadratic nonlinear parameter $\Gamma^{(2)}$ on depth h in the presence of nonpenetrable dome over fluid-saturated layer, $H_L = H_0/2$. Unloading coefficient $k = 0.96$. Solid line — nonlinear parameter, dashed line — longitudinal sound velocity.

domes is very important, e.g., for oil exploration. Consider the case of such a structure similar to that in the Ker-Girs oil field in Azerbaijan [Dobrovolsky and Preobrazhensky, 1991]. The oil boundary depth $H_0 \approx 500m$. To model the anomalous liquid pressure at the horizon H_0 suppose that it is determined by the overlaying rock layer. Consider the case when the pore fluid pressure under the dome is determined by the total weight of the upper layer with thickness $H_L = H_0$ (in this extreme case the grain skeleton immediately under the dome is unloaded at maximal extent). Therefore in the range $h \geq H_0$ one can obtain for initial fluid pressure:

$$p_{f0}(h) = (1-\alpha)\rho_s g H_L + \rho_f g (h - H_0) \tag{32}$$

where g is acceleration due to gravity.
In order to find the initial strain ε_0 , the stress-strain equations (20) were solved numerically using the expression (32) for $p_{f0}(h)$. The resulting magnitude of ε_0 is very sensitive to the choice of the unloading coefficient k in the expression (20) for σ_{eff} , because the anomalous fluid pressure at $k = 1$ and $H_L = H_0$ in (32) provides total unloading of the grain "skeleton" ($\varepsilon_0 = 0$) and, consequently, formally leads to infinite values of nonlinear parameters according to (24). We chose, therefore, more realistic unloading coefficient values $k < 1$. Other parameters used in the calculation were as follows: intergrain void content $\alpha = 0.38$, liquid compressibility $K_f = 1.2 \cdot 10^9 N/m^2$ and density $\rho_f = 0.85 \cdot 10^3 kg/m^3$ (typical of oil), the thickness of the fluid reservoir was equal to $50m$. The corresponding results are presented in Fig.3. One can see that at $k = 0.95$ (the curves marked by 1) the sound velocity changes are hardly seen, and at $k = 0.985$ (the curves marked by 2) the sound velocity variations are about 5%, while the corresponding changes of the nonlinear parameter are two orders of magnitude higher.
The analogous dependencies were calculated for the case of twice lower initial pore fluid pressure (that is $H_L = H_0/2$ in (32)) and, e.g., for the coefficient $k = 0.96$ in (20) (see the curves in Fig.4). The contrast between the changes of the nonlinear parameter and the sound velocity decreased due to smaller unloading, but the nonlinear parameter variations were still an order of magnitude higher. Note that the presence of a gas component may also be taken into account, and it can be shown that it leads (even for rather small gas cubic content of about 10^{-2}) to an additional nonlinear parameter increase of many per cent without significant influence on the sound velocity.
The above consideration is based on the model of totally non-consolidated medium (though the empirical coefficient $k < 1$ introduced in the model makes it possible to account for partial inter-cementing of the medium grains). There exist published experimental data on strong dependence of sound velocity in rocks on external pressure [White, 1965], which demonstrate that high

nonlinearity is intrinsic to consolidated rocks (e.g. sandstone) as well as to crumbly ones. The nonlinear parameter value may reach $\Gamma^{(2)} \sim 10^4$, and the nonlinearity variations may exceed the accompanying changes of the sound velocity by many times. Evidently, in such a case the nonlinearity has also contact origin, that allows to characterize nonlinear properties of such media in a similar manner as considered above, though it is necessary to perform special theoretical treatment for more accurate quantitative description of the phenomenon.

Conclusion.

The above consideration testifies for the significant role of the "structural mechanism" in appearance of anomalously high acoustic nonlinearity which is sharply different from nonlinearity of "common" intact materials. The developed models of nonlinearity in grainy media make it possible to explain high absolute values of nonlinear parameters observed in experiments and the qualitative functional behavior of nonlinear parameters. Evidently, analogous mechanisms of nonlinearity growth may be revealed for other classes of structurally inhomogeneous media (e.g. polycrystalline metals and rocks, crack-containing materials and so on), for which experimental data on anomalous nonlinear properties are also available.

The performed analysis of nonlinear properties for dry and fluid-saturated grainy materials, and the comparison of simultaneous variability of linear sound velocity and the quadratic nonlinear parameter for some geological situations shows that the nonlinear parameter, being extremely sensitive to the medium structure and its loading conditions, may be used as a rather useful informative characteristics in seismo- prospecting and seismo-monitoring.

The analysis presented above does not touch the problem of collecting and interpreting field data. This question is outside the scope of the present paper, but we discussed it elsewhere. Here we restrict ourselves to the following brief remarks. For reconstruction of the spatial distribution of the medium nonlinearity tomographic methods based on observation of the waves transmitted through the medium [Belyaeva et al., 1994(b); Sato and Fukusima, 1985; Sato, 1990] may be used. Another possible approach (that may be preferable in some cases) based on the so-called effect of nonlinear sound backscattering [Belyaeva and Zaitsev, 1993] was discussed in [Belyaeva and Zaitsev, 1996] as applied for seismic problems. Evidently the transfer of these methods from industrial and biological applications to seismics requires special experimental investigations.

As far as we know, the field data on successful separation of source and material nonlinear effects are available only for the case of transmitted signals and for the path averaged values of the nonlinear parameters [De Fazio et al., 1973; Groshkov et al., 1990]. Apparently, conventional apparatus and techniques used in routine problems of seismo-prospecting are not quite appropriate for such measurements because of intrinsic nonlinearity of existing sources (such as Vibroseis) and, evidently, nonlinearity of conventional detector electronics. On the other hand, there is an extensive successful experience in radiation of intensive and stable sound waves, and in processing weak signals (including the ones generated due to the medium nonlinearity) in hydroacoustic problems. It seems possible that extension of this hydroacoustic experience to seismics [Bogolyubov et al., 1995] may be fruitfully used for realization of new methods of seismo-acoustic sounding based on nonlinear principles.

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References

Antsiferov, M.S., Antsiferova, N.G., Kagan, Ya.Ya., Propagation of acoustic waves in the dry sand under the presure, *Izv.Akad.Nauk. SSSR (Geophysics), 12,* 1774-1781, 1964, (in Russian).

Belyaeva, I.Yu., Timanin, E.M., Experiments of garmonic generation in grainy media, *Acoust.Lett., 15,* N11, 221-225, 1992.

Belyaeva, I.Yu., Ostrovsky, L.A., Zaitsev, V.Yu. Nonlinear acoustoelastic properties of grainy media, *Akusticheskii zhurnal, 37,* N1, 17-23, 1993, (in Russian).

Belyaeva, I.Yu., Zaitsev, V.Yu., An investigation of nonlinear sound backskattering, *Acoust. Let., 16,* N11, 239-242, 1993.

Belyaeva, I.Yu., Zaitsev V.Yu., Timanin, E.M., Experimental investigation of elastic nonlinear properties of grainy media with nonideal packing, *Akusticheskii zhurnal, 40,* 893-898, 1994(a), (in Russian).

Belyaeva, I.Yu., Zaitsev, V.Yu., Sutin, A.M., Tomography of elastic nonlinear parameters of earth rocks in the seismology and seismoprospecting, *Fizika Zemli, 10,* 39-47, 1994(b). , (in Russian).

Belyaeva, I.Yu., Zaitsev, V.Yu., Method of seismo-acoustic nonlinear parameter profiling on the base of effect of nonlinear sound backscattering, *Fizika Zemli* accept for publ., (in Russian).

Bogdanov, A.N., Skvortsov, A.T. Nonlinear shear waves in grainy media, *Akusticheskii zhurnal, 38,* N2, 408-412, 1992, (in Russian).

Bogolyubov, B.N., Dubovoy, Yu.A., Zaitsev, V.Yu., Zaslavsky, Yu., M., Maryshev, A.P., Nazarov, V.E., Sutin, A.M., Talanov, V.I., Experiments on the use of high coherence hydroacoustic radiator for extitation of elastic waves and investigation of nonlinear seismo- acoustic effects in field conditions, *Preprint IAP RAS N380,* 1995. , (in Russian).

Deresewicz, H.A., Review of some recent students of the mechanical behavior of granular media., *Appl.Mech.Rev., 11,* 259-261, 1958.

Dobrovolsky, I.P., Preobrazhensky, V.P., Oscillation of pressure in the oil collector, *Izv.Akad.Nauk SSSR, 313,* N1, 63-66, 1990, (in Russian).

Dunin, S.Z., Attenuation of finite amplitude wave in grainy media, *Fizika Zemli, 5,* 106-109, 1989, (in Russian).

De Fazio, T., Aki, K., Alba, J., Solid Earth tide and observed change in the "in situ" seismic velocity, *J.Geophys.Res.*, *78*, N8, 1319-1322, 1973.

Groshkov, A.L., Shalashov, G.M., Shemagin, V.A., Nonlinear inter-borehole sounding using acoustic waves modulation by the seismic field, *Doklady Akad. Nauk SSSR*, *313*, N1, 63-65, 1990, (in Russian).

Seismo-prospecting (Handbook in Geophysics). Eds. Gurvich I.I., Nomokonov V.P., Nedra, Moscow, 1981, (in Russian).

Kobelev, Yu.A., Ostrovsky, L.A., Models of gas-liquid mixture as nonlinear dispersive medium. In: *Nonlinear acoustics*, IAP Acad.Sc.USSR, Gorky, 143-160, 1980, (in Russian).

Landau, L.D., Lifshits, E.M., *Theory of elasticity*, Pergamon Press, 1986.

McCall, K.R., Theoretical study of nonlinear acoustic wave propagation, *J.Geophys.Res.*, *99*, 2591-2600, 1993.

Meegan, G.D., Johnson, P.A., McCall, K.R., and Guyer, R., Observations of nonlinear elastic wave behavior in sandstone, *J. Acoust. Soc. Am.*, *94*, 3387-3391, 1993.

Nazarov, V.E., Sutin, A.M., The harmonic generation under propagation of elastic waves in elastic nonlinear media, *Akusticheskii zhurnal*, *35*, N4, 711-716, 1989, (in Russian).

Nazarov, V.E., Influence of copper structure on its acoustic nonlinearity, *Physics of metals*, *3*, 172-178, 1991, (in Russian).

Nazarov, V.E., Bimodular elastisity of the crack-containing media, *Akusticheskii zhurnal*, *40*, N3, 459-461, 1994, (in Russian).

Nesterenko, V.F. Propagation of nonlinear compression pulse in grainy media, *J.Prikl.Mech.Teor.Fiz.*, N5, 136-148, 1983, (in Russian).

Nikolaev, A.V., *Problems of nonlinear seismics*, Nedra, Moscow, 1987, (in Russian).

Physical backgrounds of seismic methods, Eds. Nikolaev A.V., Galkin I.N. Nauka, Moscow, 1991, (in Russian).

Ostrovsky, L.A., Nonlinear acoustics of weakly-compressed porous media, *Akusticheskii zhurnal*, *34*, N5, 908-913, 1988, (in Russian).

Sato, T., Fukusima, A., Nonlinear acoustic tomography system using counter- propagating probe and pump waves, *Ultrasonic Imaging*, *7*, 49-59, 1985.

Sato, T., Industrial and medical applications of nonlinear acoustics, *Frontiers of Nonlinear Acoustics: Proc. 12th ISNA*, London, 98-112, 1990.

Sheriff, R.E, Geldart, L.P., *Exploration Seismology, v.1,2*, Cambridge University Press, Cambridge, 1987.

Tzareva, N.V., Propagation of acoustic waves in sandstone, *Izv.Akad.Nauk SSSR (Geophysics)*, *1*, 1044-1053, 1956.

White, J., *Seismic Waves - Radiation, Transmission and Attenuation*, New York, Mc-Graw-Hill, 1965.

Zarembo, L.K., Krasil'nikov, V.A. *Introduction to nonlinear acoustics*, Moscow, Nauka, 1966, (in Russian).