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Elastic nonlinear parameter as an informative characteristic in problems of prospecting seismology

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Abstract. The paper discusses the character of high nonlinearity of so-called structural inhomogeneous media, to which the majority of rocks is assigned. The quantitative characteristics of such structural nonlinearity were analyzed by the example of a fluid-saturated granular medium. The given examples demonstrate that the nonlinear elastic parameters may be sensitive to changes of layer structure and conditions of its loading much more than conventionally determined velocities of elastic waves.

Introduction

Seismoacoustic methods are widely and successfully applied in geologic prospecting [*Dobrovol'skiy and Preobrazhenskiy*, 1991; *Gurvich and Nomokonov*, 1981; *Petkevich and Verbitskiy*, 1970; *Sheriff and Geltard*, 1987; *White*, 1986]. Interpreting observational data, we try to find, first, the distribution of longitudinal and (or) transverse waves as well as their absorption coefficient in a region studied. Furthermore, to reach conclusions on structure of rocks in the region requires the use of empirical or theoretically based relations between the above mentioned seismoacoustic parameters and interesting features of the rock structure. The problem of selection of a determining parameter that is most sensitive to studied structural peculiarities is formulated depending on which structural characteristics are of immediate interest in the given concrete case (for example, porosity, jointing, and presence of fluid saturation) [*Petkevich and Verbitskiy*, 1970].

If we restrict ourselves to the approach of linear acoustics (seismics), then such a selection will have limited possibilities. The list of possible variants is practically exhausted by the parameters indicated above (velocities of seismic waves and coefficients of absorption and scattering). Furthermore, the values of changes in these parameters themselves appear to be negligible compared to the background of random variation in many situations that are of interest in geologic prospecting. In connection with this, considerable recent attention has been focused on the studies of nonlinear seismoacoustic parameters of rocks which gives the hope of gaining additional evidence of rock behavior. For instance, the definite relation of the nonlinear parameters to structural properties of a medium was found experimentally, and the range of change in the nonlinear parameters proved to be essentially more (sometimes, by many orders) than that of simultaneous changes in values of the linear characteristics [*Groshkov et al.*, 1990, 1991; *Jonson and Shankland*, 1990; *Nazarov*, 1993; *Nikolayev*, 1987]. The theoretical studies of the relation of elastic nonlinear parameters to the internal fabric of rocks is still at the initial stage; for example, we can refer to the works [*Bogdanov and Skvortsov*, 1992; *Dunin*, 1989; *Nesterenko*, 1983] concerned with the analysis of nonlinear elastic properties of the simplest models of granular media.

The present work presents the theoretical analysis of nonlinear properties using the example of a granular medium that may be saturated with a gas-liquid mixture. Then, based on the conducted analysis, we discuss some geologic prospecting situations (in connection, for example, with oil exploration) in which variation in the nonlinear parameter of rock may exceed simultaneous variations of the linear velocity of waves. In similar cases, the use of the nonlinear parameter may be more valuable than the traditional use of the linear parameters.

The question of whether or not the parameter can be measured represents in itself an independent problem. For this, for example, the modulation technique can be applied which is similar to that used in the usual acoustic well logging or interwell sonic sounding [*Groshkov et al.*, 1990]. Of more interest may be methods of the tomographic type (discussed, for example, in the work by *Dunin* [1989]) that are similar to the methods of tomography of nonlinear parameters offered for medical-

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biological objectives [*I. Yu. Belyaeva and A. M. Sutin*, unpublished manuscript, 1992]. We do not address this aspect of the problem in the present paper but focus on the relation itself between the nonlinear parameter and structure of a substance.

Character of Elastic Nonlinearity in Various Media: Quantitative Characteristics of Nonlinearity

Nonlinearity of elastic properties in common homogeneous materials is determined by the form of interatomic (intermolecular) potential that is very close to parabolic in the vicinity of equilibrium of the atoms (molecules). Since the nonlinear effects in such materials are fairly weak at strain amplitudes characterizing elastic waves, it is widely believed that they become marked in solids only at sufficiently large strain (on the order of $10^{-3} - 10^{-2}$) that are close to the breaking point. Essentially another situation may exist in materials with inhomogeneous structure (granular, jointy, porous, etc.), including components with contrasting elastic properties. We note that such a medium is just the rule rather than the exception for the seismic problems. The presence of "soft" components (joints, intergranular contacts, pores under special conditions, etc.) in such structural, inhomogeneous materials leads to the fact that the local values of strain can be anomalously high even at small, on the average, amplitudes of strain (the corresponding stresses are concentrated in these places); because of this, the stress-strain relation in such regions is essentially nonlinear by its nature. As a consequence, we may observe a high nonlinearity of the medium as a whole which is noticeable already at comparatively small amplitudes of disturbance (on the order of 10^{-5} and smaller) that are typical for seismoacoustic waves.

We briefly dwell on the question of how to describe nonlinear elastic properties of a medium having the equation of state $\sigma = \sigma(\varepsilon)$, where σ is stress and ε is relative strain (for simplicity, we consider the isotropic medium). Bearing in mind sufficiently small (wave) disturbances on the background of some initial strain ε_0, we can expand the equation of state in series with respect to $\tilde{\varepsilon} = \varepsilon - \varepsilon_0$

$$\tilde{\sigma} = \sigma'_{\varepsilon}(\varepsilon_0)\tilde{\varepsilon} + \frac{1}{2!}\sigma''_{\varepsilon\varepsilon}(\varepsilon_0)\tilde{\varepsilon}^2 + \\ + \frac{1}{3!}\sigma'''(\varepsilon_0)\tilde{\varepsilon}^3 + \ldots \quad (1)$$

where $\tilde{\sigma} = \sigma(\varepsilon) - \sigma(\varepsilon_0)$.

It is common practice in nonlinear acoustics to introduce the linear and nonlinear parameters defined as follows:

$$M = \sigma'_{\varepsilon}(\varepsilon_0) \quad (2)$$

$$\Gamma^{(2)} = \sigma''_{\varepsilon\varepsilon}(\varepsilon_0)/\sigma'_{\varepsilon}(\varepsilon_0) \quad (3)$$

$$\Gamma^{(3)} = \sigma'''_{\varepsilon\varepsilon\varepsilon}(\varepsilon_0)/\sigma'_{\varepsilon}(\varepsilon_0) \quad (4)$$

We note that the quantity M is the linear modulus of longitudinal deformation that determines the velocity of longitudinal sound waves $v_p = [M/\rho]^{1/2}$, and the quantities $\Gamma^{(2)}$ and $\Gamma^{(3)}$ determine the quadratic and cubic nonlinear terms, respectively [*Ostrovsky*, 1991]. A typical value, for example, of the quadratic nonlinear parameter for common homogeneous materials (air, water, melted quartz, and many metals) lies within 3–15 [*Zarembo and Krasil'nikov*, 1966], whereas this parameter reaches the values of $10^3 - 10^4$ for media of inhomogeneous structure (water with gas bubbles [*Kobelev and Ostrovskiy*, 1980], porous plastisols [*Belyaeva and Timanin*, 1991], and some hard rocks [*Jonson and Shankland*, 1990]).

Nonlinear Acoustoelastic Properties of a Granular Medium With Gas-Liquid Saturation

A medium with granular structure characteristic of many sedimentary rocks is still one of the examples of a structural, inhomogeneous medium important for seismic application (additionally, it allows a sequential theoretical consideration). The porous space in these rocks may be fluid-saturated in a number of cases, in connection with which the study of seismoacoustic properties of such media is of great interest for seismic prospecting of valuable deposits. Many authors have used the model of granular medium in their analyses of the linear elastic characteristics (above all, longitudinal and transverse sound velocities) [*Gurvich and Nomokonov*, 1981; *Sheriff and Geltard*, 1987; *White*, 1986]. As for the nonlinear effects in such media, it is clear that the presence of soft intergranular contacts obeying Hertz's law [*Landau and Lifshits*, 1965] must give rise to an appearance of strong nonlinear properties in these media which were discussed in several papers [*Bogdanov and Skvortsov*, 1992; *Dunin*, 1989; *Nesterenko*, 1983].

Let us consider a sample of the granular medium consisting of a great number N of randomly packed identical elastic spheres having the radius R and made from a material with the density ρ_s and the elastic constants, the Young's modulus E_s, the bulk modulus K_s, and the corresponding Poisson's ratio v_s. To characterize this packing, we introduce the porosity coefficient α and the mean number of contacts $\bar{n}$ of an individual sphere with neighboring particles (the values $\alpha = 0.392$ and $\bar{n} = 8.84$ [*Deresiewicz*, 1958] were found experimentally for a random packing). We assume that the intergranular space is filled, under pressure ρ_{f0}, with a gas-bearing liquid where the gas volume content is β. The bulk modulus of the liquid is K_f, and its density is ρ_f. Sound velocity c_g and density ρ_g are used as gas parameters. The relation of the force F on an individual contact with the distance between the centers of adjacent grains $\tilde{\Delta}$ is described by the known Hertz's law [*Landau and Lifshits*, 1965]

$$\tilde{\Delta} = \left(\frac{3(1-v_s^2)F}{4ER^{1/2}} \right)^{2/3} \tag{5}$$

On finding the relation between the external pressure p_{ext} (stress) and resulting average relative strain $\varepsilon = \Delta/(2R)$ of the aggregate, we use the approach applied in the work by *Belyaeva et al.* [1993] and based on the equation of energetic balance

$$\delta W_{ext} = \delta W_m + \delta W_s + \delta W_c \tag{6}$$

where W_{ext} is the energy required for a quasi-static compression of the aggregate, W_m is the energy stored in the porous gas-liquid mixture due to its compression, W_s is the elastic energy of material particles acquired due to their confining squeezing by fluid, and W_c is the energy stored by the particles along contacts. The characteristic frequency of an external disturbance is assumed to be sufficiently small so that the motion of the fluid and the granular skeleton (frame) can be treated as common [*Gurvich and Nomokonov*, 1981; *White*, 1986].

The expenditure of energy for decreasing the volume of the aggregate by the value of dV_t is given by the expression

$$\delta W_{ext} = p_{ext} dV_t \tag{7}$$

There is a fairly clear relation for the change of the total volume in the case of plane deformation [*Belyaeva et al.*, 1993]

$$V_t = 4\pi N R^2 \tilde{\Delta}/(1-\alpha) \tag{8}$$

The energy accumulated in the porous mixture is

$$\delta W_f = p_f dV_f \tag{9}$$

where $p_f = p_{f0} + \delta p_f$, and δp_f is an additional pressure due to the increment of the fluid volume dV_f.

The energy accumulated in the volume of granules compressed by the fluid is

$$\delta W_s = p_f dV_s \tag{10}$$

where dV_s is the increment of volume of spherical granules under the confining pressure p_f from the porous fluid.

The energy related with the work on the contacts of deformed particles can be written in the form

$$\delta W_c = b\bar{n} N F(\tilde{\Delta}) d\tilde{\Delta} \tag{11}$$

where $\tilde{\Delta}$ and F are related by Hertz's law. In the given case, the quantity b characterizes an effective number of "active" contacts (accumulating the energy). As was shown by *Belyaeva et al.* [1993], for an isotropic medium under confining compression, $b = 1$, and in the case of plane deformation (which is considered below), we have $b = 1/3$, since, on the average, one third of the total number of contacts in the aggregate volume contributes to the elastic energy E_c.

Changes of the pressure and volume of the gas-liquid mixture and granules are interrelated through the elastic constants of materials (neglecting an intrinsic nonlinearity of grains and liquids)

$$\delta p_f = -K_s \frac{dV_s}{V_s} \tag{12}$$

$$dV_f = -(a_1(\delta p_f) - a_2(\delta p_f)^2 + a_3(\delta p_f)^3 + \ldots) \tag{13}$$

where the designations are used:

$$\begin{aligned} a_1 &= V_f(1-\beta)/K_f + V_f\beta/\rho_g c_g^2 \\ a_2 &= \beta V_f(\gamma+1)/2\rho_g^2 c_g^4 \\ a_3 &= \beta V_f(\gamma+1)(\gamma+2)/2\rho_g^4 c_g^6 \end{aligned}$$

On deriving (13) we used Taylor's expansion of the equation of state of gas, $p\rho_g^{-\gamma} = const$ (γ is the isentrope exponent). It was also assumed that the gas content is small ($\beta \ll 1$), and the characteristic time of deformation is much greater than the period of free oscillations of gas bubbles (the standard quasi-homogeneous approximation for a gas-liquid mixture [*Kobelev and Ostrovskiy*, 1980]).

Thus using (7)–(13) and making cumbersome calculations, we derive from the equation of energetic balance (6) the following stress-strain relation:

$$\sigma_{eff} = \frac{1}{A}\left\{ \frac{\alpha B}{A} + \frac{1-\alpha}{AK_s} - \left(\frac{\alpha BC}{A^2} + \frac{C}{2} \right) p_{f0} \right\} \varepsilon + \\ + \frac{C}{A^3} \left\{ \frac{2\alpha B}{A} + \frac{2(1-\alpha)}{AK_s} - \frac{C}{A} p_{f0} - \frac{1}{2} \right\} \varepsilon^2 + \\ + \frac{n(1-\alpha)E_s}{3\pi(1-v_s^2)} \varepsilon^{3/2} \tag{14}$$

The numerical coefficients A, B, and C introduced above are equal:

$$A = (1-\alpha)/K_s + \alpha(1-\beta)/K_f + \alpha\beta/\rho_g c_g^2,$$

$$B = (1-\beta)/K_f + \beta/\rho_g c_g^2,$$

$$C = \alpha\beta(\gamma+1)/\rho_g^2 c_g^4$$

Relation (14) represents in itself the "nonlinear Hooke's law" (equation of state) for the fluid-saturated granular medium in the case of pure plane deformation corresponding to the case of propagation of longitudinal waves. An external pressure induces the stress in the medium which is distributed between the frame and the fluid. That is why we introduced the effective stress in (14),

$$\sigma_{eff} = p_{ext} - p_{f0}$$

characterizing the loading fraction on the granular frame; in the case when the porous filler is absent, this fraction is merely equal to p_{ext}. The real granular medium (usually cemented together in part) is commonly [*Gurvich and Nomokonov*, 1981; *White*, 1986] described by the quantity

$$\sigma_{eff} = p_{ext} - kp_{f0} \tag{15}$$

where the value of the coefficient k called the unloading coefficient lies within the interval $0.85 \le k \le 1$ [*Gurvich and Nomokonov*, 1981] and depends on the ratio of compressibilities of granules and porous fluid as well as on the coherence of the grains. We note that in the considered model of completely nonconsolidated medium k is equal to unity.

Using (1)–(4), from (14) we obtain the following expressions for the linear and nonlinear elastic moduli of the medium:

$$M = \frac{1}{A}\left\{\frac{\alpha B}{A} + \frac{1-\alpha}{AK_s} - \left(\frac{\alpha BC}{A^2} + \frac{C}{2}\right)p_{f0}\right\} \tag{16}$$

for the longitudinal elastic modulus, and

$$\Gamma^{(2)} = \left[\frac{2C}{A^3}\left\{\frac{2\alpha B}{A} + \frac{2(1-\alpha)}{AK_s} - \frac{C}{A}p_{f0} - \frac{1}{2}\right\}\varepsilon_0 + \right.$$
$$\left. + \frac{n(1-\alpha)E_s}{2\pi(1-v_s^2)}\varepsilon_0^{-1/2}\right]/M \tag{17}$$

$$\Gamma^{(3)} = \left[\frac{n(1-\alpha)E_s}{8\pi(1-v_s^2)}\varepsilon_0^{-3/2}\right]/M \tag{18}$$

for the parameters of quadratic and cubic nonlinearity.

It should be pointed out that the nonlinear terms in (14) are due to the presence of the Hertz's contacts and the nonlinear deformation of gas bubbles rather than the nonlinearity of material of grains and liquid. Given in the expression for $\Gamma^{(3)}$ is only the term responsible for the nonlinearity of contacts, because estimates show that the contribution of the gas component to the cubic nonlinearity is negligible.

The partial cases of (16)–(18) were comprehensively considered by *Belyaeva et al.* [1993] (for a dry medium and a medium with a high-elastic porous filler as compared to elasticity of contacts). For example, the equation of state for the medium without filler has the particularly simple form

$$\sigma(\varepsilon) = \frac{\bar{n}(1-\alpha)E_s}{3\pi(1-v_s^2)}\varepsilon^{3/2} \tag{19}$$

and the nonlinearity parameters expressed in terms of the initial strain ε_0 are determined by this strain alone:

$$\Gamma^{(2)} = 1/(2\varepsilon_0), \quad \Gamma^{(3)} = 1/(6\varepsilon_0^2) \tag{20}$$

Hence it is evident that at small values of ε_0 (on the order of 10^{-3} and smaller) these parameters of "structural" nonlinearity reach anomalously high values that are several orders higher than nonlinearity of grain material, which was supported by laboratory experiments [*Belyaeva and Timanin*, 1992].

Before proceeding to estimating the real cases on the basis of the derived relations, we make some remarks on the applicability of the model considered. We note that on calculating the linear characteristics of nonconsolidated media, the similar models demonstrated good qualitative agreement (increase of sound velocity with depth and its change depending on fluid saturation) and, sometimes, also quantitative agreement with the results of field measurements [*Gurvich and Nomokonov*, 1981; *Sheriff and Geltard*, 1987; *White*, 1986]. As to the nonlinear characteristics, good numerical agreement of (19) and (20) was found when comparing the results of laboratory experiments on the model media from metallic grains of the same size and from grains of tuff (including the grains of different sizes) [*Belyaeva and Timanin*, 1992]. The data of full-scale measurements of nonlinear parameters in loose rocks (loams) [*Groshkov et al.*, 1990; 1991] also agree satisfactorily with the theoretical estimates. Such correspondence of the calculated parameters with the experimental data obtained for the media with size distribution of grains would be expected, despite the fact that the grain sizes in the model considered were assumed to be equal. Indeed, as follows from the logic of derivation of the equation of state, a difference in the grain sizes must affect only the value of the numerical factor ahead of $\varepsilon^{3/2}$ in (19) which appears due to the elastic properties of individual granular contacts whose behavior obeys Hertz's law (5). Hence it is clear that in line with (2) and (3) relating the quantities $\Gamma^{(2)}$ and $\Gamma^{(3)}$ with the equation of state $\sigma = \sigma(\varepsilon)$, a

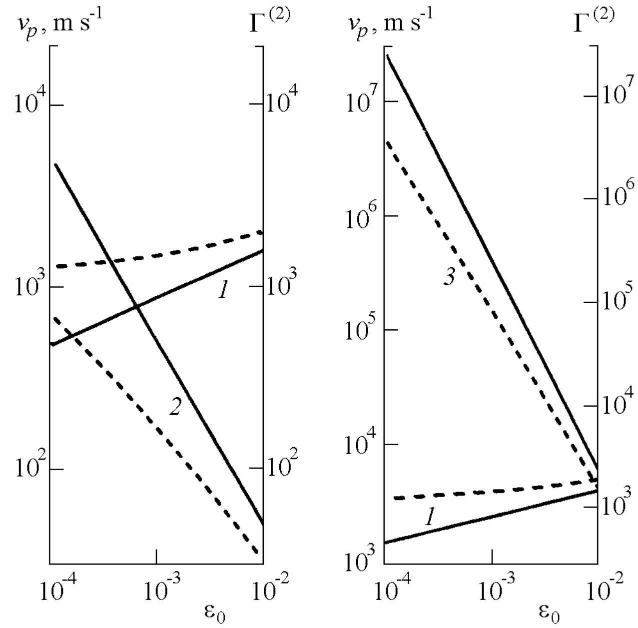

Figure 1. Velocity of longitudinal waves and nonlinearity parameters versus initial strain ε_0; 1, velocity of longitudinal waves v_p; 2, parameter of quadratic nonlinearity $\Gamma^{(2)}$; and 3, parameter of cubic nonlinearity $\Gamma^{(3)}$. Solid line represents the dry medium, and dashed line represents the water-saturated medium.

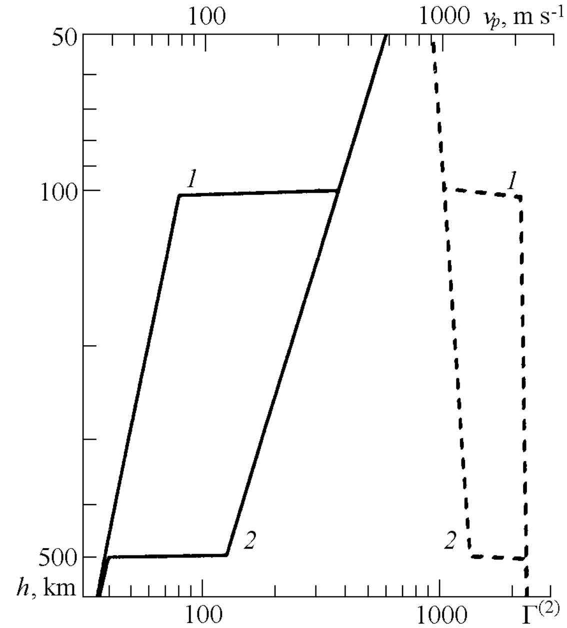

Figure 2. Velocity of longitudinal waves v_p and parameter of quadratic nonlinearity $\Gamma^{(2)}$ versus depth h; 1, for top of a fluid-bearing layer, $H_0 = 100$ m and 2, $H_0 = 500$ m. Dashed line shows velocity of longitudinal waves and solid line shows the parameter of quadratic nonlinearity.

change in the coefficient at $\varepsilon^{3/2}$ does not influence the values of the nonlinear parameters (20), but must affect the linear moduli. Above all, this explains the discrepancies between the sound velocity in loose rocks from some full-scale measurements and the calculated results of the model of the described type.

It should be particularly noted that on changing a composition of the fluid-saturated granular medium and the value of initial loading in the limits characterizing a real situation, the range of variation velocities of seismic waves (as in full-scale experiments, so in theoretical considerations) is usually not less than 50%. Also, the corresponding change in the values of nonlinear parameters may be essentially higher, as is shown below by several examples, which is of great interest to seismic prospecting problems.

Comparative Analysis of Changes in Linear and Nonlinear Parameters in Layers of Different Structure

First, we use (15)–(17) to compute the dependence of the sound velocity and the nonlinear parameters on the initial strain ε_0 in the dry and in the water-saturated media (Figure 1). In this computation, we accept the following values of the model parameters: the porosity coefficient is $\alpha = 0.2$; Young's modulus of the grain material is $E_s = 5.3 \times 10^{10}$ N m^{-2}; the Poisson ratio is $v_s = 0.2$; the density is $\rho_s = 2.65 \times 10^3$ kg m^{-3}; and the bulk modulus of water is $K_f = 2.2\times$ $\times 10^9$ N m^{-2}. As seen from the graphs, the sensitivity of the nonlinear parameters to changes in ε_0 is essentially (by orders of magnitude) higher than that of the sound velocity. These differences are exhibited in typical geologic situations discussed below where the variations in ε_0 are determined by fabric and occurrence depth of rocks.

As the first example, we consider the vertical profiles of sound velocity and the quadratic nonlinear parameter in the nonconsolidated fluid-penetrable medium (such as sands and loams), in which the level of subsoil water is located at a depth of H_0. Figure 2 shows the results of calculations for the two values of H_0. (The calculation was made with the same model parameters as in constructing the graphs of Figure 1.) To find the value of the initial static strain of the frame determined by the weight of overlying layers, we used (18) for the dry medium; because of the strength of the frame permeability, the fluid elasticity can affect only the value of the dynamic (wave) stresses. This is the cause of the jumps in the sound velocity and the acoustic nonlinear parameter which appear at the fluid boundary. We notice that an increase in the elastic modulus M of the medium related to the presence of fluid gives rise to increasing sound velocity, whereas the nonlinear parameter decreases in accord with (17). As is seen in Figure 2, the jump in sound velocity at the depth H_0 amounts to 50%, but the nonlinear parameter changes 4–5 times. The values of the jumps of both parameters decrease with growing H_0. Thus in the given case, changes in the nonlinear parameter is markedly higher than that in sound velocity.

We now turn our attention to the case that appears to be more interesting for seismic prospecting. Let us assume that the fluid-bearing range is separated from overlying rocks by an impermeable interlayer (this situation is typical for hydrocarbon deposits included into a trap formed by an impermeable dome). The fluid in the trap can take over part of the loading formed by the weight of overlying rocks [*Sheriff and Geltard*, 1987]. In computing the value of the nonlinear parameter, following definition (17), we found numerically the value of the static strain ε_0 at the depth h from the equation of state (14), in which the frame and fluid elasticity and the initial pressure of the fluid p_{f0} was taken into account. Unloading of the granular frame gives rise to decreasing initial strain ε_0, which leads, in turn, to increasing nonlinearity of the intergranular contacts (in accord with (17) and (18)). For the linear parameters, (16) shows that a decrease in ε_0 results in decreasing the elastic modulus of the frame. However, the presence of fluid gives a positive contribution to the aggregate elasticity (frame plus fluid) and, additionally, raises its density. Therefore the three competing parameters affect the value of sound velocity, and the sign of velocity change at the dome boundary depends on their ratios. Thus situations may be encountered where the presence of fluid has practically no effect on changes in seismic velocities, whereas variations of the nonlinear parameters may be very significant.

The curves in Figure 3 correspond to such a case, giving the vertical profiles of sound velocity and nonlinear parameter, at the two values of unloading. The computations were conducted with the same parameters of grain material as were used for the curves in Figures 1 and 2. Furthermore, we selected the value of the porosity coefficient $\alpha = 0.38$, the bulk modulus of the fluid $K_f = 1.2 \times 10^9$ N m^{-2}, and its density $\rho_s = 0.85 \times 10^3$ kg m^{-3} these are the typical values for liquid hydrocarbons). The depth of the dome top is $H_0 = 500$ m; on giving the value of the porous pressure of fluid p_{f0}, we assumed that the fluid replaces the load of all the overlying layers (i.e., the granular frame under the dome appears to be mostly unloading). It is seen from the graphs that the value of the velocity at the depth H_0 falls insignificantly (by several percents), but the nonlinear parameter sharply increases due to frame unloading, and its relative increase exceeds the relative variations in sound velocity by about two orders of magnitude.

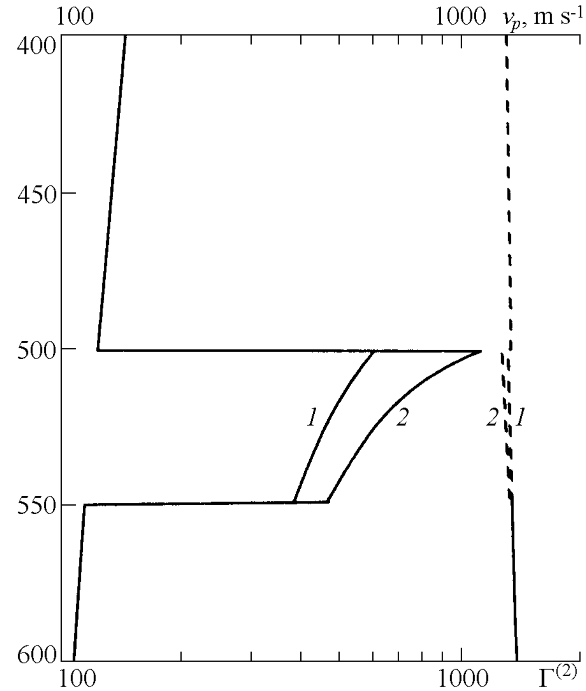

Figure 3. Velocity of longitudinal waves v_p and parameter of quadratic nonlinearity $\Gamma^{(2)}$ versus depth h in the presence of fluid-impermeable dome. Unloading of intergranular contacts by pore fluid is close to maximal. Dashed line represents velocity of longitudinal waves, and solid line represents the parameter of nonlinearity; 1, $k = 0.95$; 2, $k = 0.985$.

Furthermore, we give the estimates for the case when the liquid filling replaces only a part of the weight of the overlying rocks (this is a more typical situation [*Sheriff and Geltard*, 1987]), and unloading of the under-dome region is not as great as in the previous example. We accept that the pore pressure replaced one half of the weight of the overlying layers, and the coefficient is $k = 0.96$ for the other parameters of the model. The results are illustrated by curve 1 in Figure 4. Changes in the nonlinear parameter became smaller than in Figure 3, but in the given case they exceed the corresponding changes in the sound velocity approximately by one order of magnitude.

It is also interesting to discuss the situation when the fluid is gas-saturated. As is known [*Kobelev and Ostrovskiy*, 1980], the presence of even a small gas content may give rise to a steep (by orders of magnitude) increase in the nonlinear parameter of liquid (the causes of this increase were discussed in the section 2), whereas the sound velocity changes moderately. Since the liq-

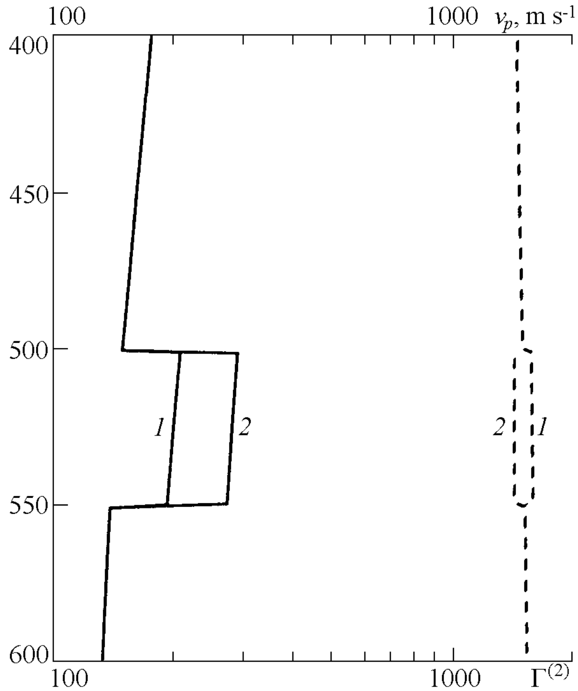

Figure 4. Velocity of longitudinal waves (dashed line) and parameter of quadratic nonlinearity (solid line) versus depth in the presence of fluid-impermeable dome and at partial unloading of intergranular contacts due to pressure in pore fluid; 1, gas content $\beta = 0$, and 2, $\beta = 0.01$.

uid (and associated gas) at these depths is under great pressure, the contrast between the compressibilities of these two components is not as great as at the normal conditions, and therefore the increase in the nonlinear parameter induced by the presence of gas is already not as sharp, although it is significant in its absolute value. Figure 4 (curve 2) gives the result of the corresponding computation at the selected values of the volume gas content $\beta = 0.01$, gas density $\rho_g = 150$ kg m^{-3}, sound velocity in the gas $c_g = 500$ m s^{-1}, and unchanged other parameters. It is seen that the presence of gas leads to some increase in velocity jump and a markedly greater increase of the jump of the nonlinear parameter.

We note that the velocity changes in Figures 3 and 4 were compared with the quadratic nonlinear parameter; as is shown in Figure 1, the corresponding variations of the cubic parameter are still greater by several orders of magnitude.

The conducted consideration of the examples with the sharp changes in the nonlinear parameter was based on the model of a nonconsolidated granular medium (although the empirical coefficient $k < 1$ allowed computations to be made taking into account partial cohesion). There are experimental data on the measured sound velocity depending on external pressure [*Petkevich and Verbitskiy*, 1970; *White*, 1986] which show that high nonlinearity ($\Gamma^{(2)} \simeq 10^2 - 10^3$) takes place not only in loose rocks, but in cohesive rocks as well; in addition, changes in the nonlinear parameter may be many times greater than the changes in the linear parameters. The nonlinear behavior in this case also seems to be explained above all by the effects of contacting intergranular nonlinearity. In this connection, such media must be characterized by the dependencies that are qualitatively similar to those obtained above, although their more rigorous quantitative treatment invites additional theoretical studies.

Conclusions

We conducted the analysis of the nonlinear parameters of the fluid-saturated media and used it for comparing the simultaneous changes in sound velocity and the quadratic nonlinear parameter in a number of typical geologic situations. Our results showed that the nonlinear parameter, being more sensitive to the presence of a porous filler and applied stresses, can serve as a useful informative characteristic in solving the problems of seismic prospecting. Thus profiling of the elastic nonlinear parameters of rocks (applying, above all, the methods that were successfully tested in biological and industrial applications [*Sato*, 1990] and recently developed as applied to seismics [*Belyaeva et al.*, 1992; *I. Yu. Belyaeva and A. M. Sutin*, unpublished manuscript, 1992] opens new possibilities in seismoacoustic prospecting methods.

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