Giant strain-sensitivity of acoustic energy dissipation in solids containing dry and saturated cracks with wavy interfaces

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Mechanisms of acoustic energy dissipation in heterogeneous solids attract much attention in view of their importance for material characterization, nondestructive testing, and geophysics. Due to the progress in measurement techniques in recent years, it has been revealed that rocks can demonstrate extremely high strain sensitivity of seismoacoustic loss. In particular, it has been found that strains of order 10^{-8} produced by lunar and solar tides are capable of causing variations in the seismoacoustic decrement on the order of several percent. Some laboratory data (although obtained for higher frequencies) also indicate the presence of very high dissipative nonlinearity. Conventionally discussed dissipation mechanisms (thermoelastic loss in dry solids, Biot and squirt-type loss in fluid-saturated ones) do not suffice to interpret such data. Here the dissipation at individual cracks is revised taking into account the influence of wavy asperities of their surfaces quite typical of real cracks, which can drastically change the values of the relaxation frequencies and can result in giant strain sensitivity of the dissipation without the necessity of assuming the presence of unrealistically thin (and, therefore, unrealistically soft) cracks. In particular, these mechanisms suggest interpretation for observations of pronounced amplitude modulation of seismo-acoustic waves by tidal strains. © *2012 Acoustical Society of America*. [DOI: 10.1121/1.3664079]

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I. INTRODUCTION

In recent years, much attention has been paid to the so-called mesoscopic nonlinear elasticity[1] of solids containing such structural features as cracks, contacts, intergrain aggregates of dislocations, etc. that are small in the scale of the elastic wave length. Quite often the quadratic nonlinear-elastic parameter β for such materials can be $10^3 - 10^4$ in contrast to $\beta \sim 10^0$ typical of ideal crystals of homogeneous amorphous solids. The common feature of the above-mentioned structural features defects is their very high relative softness compared with that of the surrounding homogeneous material. Thus the local strains at the defects are strongly (often by several orders of magnitude) enhanced, which results in strongly increased macroscopic elastic nonlinearity. This mechanism of strongly increased structurally induced nonlinear elasticity can be instructively elucidated using distributed rheological-level models.[2,3]

In many cases, adhesion or frictional effects are also localized at those soft defects; this makes the resultant nonlinearity hysteretic.[1] In recent years, increasing attention has also been paid to nonlinear-dissipative properties of mesoscopic solids, which manifest themselves in rather pronounced variations in the dissipation of one elastic wave in the presence of another wave[4,5] or an applied (quasi)static stress (although the very fact of pressure dependence of dissipation or its dependence on the acoustic wave amplitude in rocks has been known for years[6,7]). The most striking feature is that quite moderate strains $\varepsilon \sim 10^{-6} - 10^{-5}$ are able to cause variations in the decrement up to tens of percentages or even several times,[5,8–10] whereas the accompanying variations in the elastic moduli are on the order $\beta\varepsilon$ and do not exceed $10^{-2} - 10^{-3}$.

In addition to laboratory measurements, even more giant strain sensitivity of the dissipation is indicated by some field data. For example, in experiments[11] on long-range (357 and 430 km) propagation of monochromatic elastic wave produced by high-stability vibration sources operating at frequencies of $5 - 7$ Hz, the accuracy of the measurements was sufficient to single out periodic variations in the received-signal parameters that were well-correlated with the periodicity of the lunar-solar tides. For tidal strains in the Earth crust, the characteristic amplitude[12] is $\sim 10^{-8}$, whereas in observations in Ref. 11, the variations of the received-signal amplitude amounted to $2 - 4$ percent and $1 - 2°$ for the signal phase. These values correspond to the path-averaged *relative* variations in the elastic modulus $\Delta E/E \sim 10^{-5}$ and *absolute* variations in the decrement $\Delta\theta \sim 3 \cdot 10^{-5}$, which are of the same order of magnitude. Assuming a reasonable[6,7] for moderately cracked rocks decrement $\theta \sim (0.3 - 1) \cdot 10^{-2}$ (i.e., the quality factor $Q = \pi/\theta \sim 100 - 300$), we estimate that the *relative* variation in the decrement is $\Delta\theta/\theta \sim 0.3 \cdot 10^{-2}$ and is over two orders of magnitude greater than $\Delta E/E \sim 10^{-5}$.

Taking into account that the tidal strain $\varepsilon_0 \sim 10^{-8}$, we obtain the path-averaged estimate for the quadratic nonlinear parameter $\beta = (\Delta E/E)/\varepsilon_0 \sim 500 - 700$, which is not so high as $\beta \sim 10^5$ reported, for example, in the pioneering observations[13,14] of the tidal variations in the elastic-wave velocities. The above-estimated smaller value of β is not surprising because in the long-range experiments,[11] the wave

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path reached depths of several tens of kilometers where the high pressure of the overburden rock layers closed soft cracks and significantly reduced the path-averaged nonlinearity of the rocks.

Much higher (comparable with the data in Refs. 13 and 14) values of β were observed in cross-well experiments[15] with a high-stability downhole seismo-acoustic source operating at a frequency of 167 Hz, the propagation distance 360 m with the estimated wave velocity along the path about 3000 m/s. In other work,[15] the tide-induced variations for the wave phase were about 0.05 rad, and elastic-modulus variations $\Delta E/E \sim 10^{-3}$, which indicated the nonlinearity parameter $\beta = (\Delta E/E)/\varepsilon_0 \sim (1-2)\cdot 10^5$ like in Refs. 13 and 14. For the wave amplitude, its tide-induced variations were about 10% and corresponded to the absolute variations in the decrement $\Delta\theta \sim (2-5)\cdot 10^{-3}$, which means that the strain-sensitivity of the rocks was so giant that the tidal strains 10^{-8} were able to produce the relative variations in the decrement $\Delta\theta/\theta \sim 10^{-2} - 10^{-1}$.

Similar estimates for $\Delta\theta/\theta$, although less directly, are confirmed by the field observations of the tidal modulation of the intensity of received endogenous seismic noises at several observation sites at the Kamchatka peninsula and in Japan.[16,17] Normally, the weak-amplitude noise with strains $10^{-10} - 10^{-12}$ was recorded by a sensitive narrow-band receiver around a frequency of 30 Hz. Coherent averaging (from several weeks to several months) of the noise envelope made it possible to reliably single out periods of individual solar and lunar-tide components in the noise-intensity modulation with a typical depth ranged from $2-3$ to $6-8\%$. Taking into account that for rather weak tidal strains $\varepsilon_0 \sim 10^{-8}$, their direct influence on the rock fracturing and the accompanying seismo-acoustic emission does not look very probable, the observed modulation can readily be explained[18] by the tidal modulation of the effective size of the region from which the signal at the receiver is collected. This size is determined by the characteristic damping length for the noise. Thus the relative variations in the intensity of the received noise should be proportional to $\Delta\theta/\theta$. The above-obtained estimate $\Delta\theta/\theta \sim 10^{-2} - 10^{-1}$ based on the independent direct measurements[15] well agrees with the depth of the tidal modulation of the noise.[16,17]

Thus various experimental data require an explanation of the giant value of strain-sensitivity of dissipation in mesoscopic solids. Because those strain-induced variations in the dissipation are observed for very small amplitudes of probing acoustic waves, for which absolute displacements at the microstructural defects (cracks and contacts) fall into essentially sub-atomic range, the responsible mechanism(s) should not involve an activation threshold (unlike hysteretic mechanisms of frictional[6,19] or adhesion origin[20]).

As discussed in previous papers,[4,21] pronounced strain-dependent dissipation in mesoscopic solids should arise due to combined action of purely elastic nonlinearity of the soft defects and conventional linear (i.e., viscous-like) dissipation that is also localized at the same defects because of the locally strongly enhanced strain rate. This mechanism does not require a finite threshold (unlike essentially super-atomic displacements required for activation of adhesion/frictional phenomena), although it can act in parallel with hysteretic mechanisms.

In what follows, we consider physical realizations of this mechanism that are relevant to solids containing dry and fluid saturated cracks. In both cases, the dissipation is threshold-less in amplitude: of thermoelastic origin in dry and viscous in fluid-filled cracks. In both cases, the key role is played by the same geometrical features of real cracks for which corrugated surfaces (having wavy asperities) are typical rather than smooth nearly plane-parallel form often used in the dissipation models. The analysis will be performed in the style of physical argumentation used by Landau and Lifshitz[22] in the discussion of thermoelastic loss in polycrystalline solids and in work.[23] Such an asymptotic approach gives clear representation of the physics of the discussed phenomena and ensures quantitative estimates with a reasonable accuracy comparable with that for formally exact solutions obtained for idealized (and thus approximate) models like elliptical cracks, etc.

II. GENERAL CONSEQUENCES OF WAVY ROUGHNESS OF SURFACES IN REAL CRACKS FOR STRAIN SENSITIVITY OF DISSIPATION

We emphasize the fact that unlike often assumed near-parallel geometry, for interfaces of real cracks, wavy (curved) forms are quite typical,[24] which is confirmed by images of crack-like defects[25] in rocks obtained by various methods and is in agreement with known models of crack initiation. Such initially coinciding surfaces often are not simply separated in the normal direction but also exhibit certain tangential displacement and create inside the crack elongated "waists" (either nearly contacting or already contacting) as shown in Fig. 1. Inside liquid-saturated cracks, the so-created narrow waist can act for fluid flows as a kind of valve near which pressure gradients, flow velocities, and the corresponding viscous dissipation in the crack should be localized. For dry cracks, the thermo-elastic dissipation should also be strongly localized and enhanced at the inner contacts. In the vicinity of such wavy asperities, the local separation (or interpenetration) $\tilde{h}$ of the crack surfaces is significantly smaller

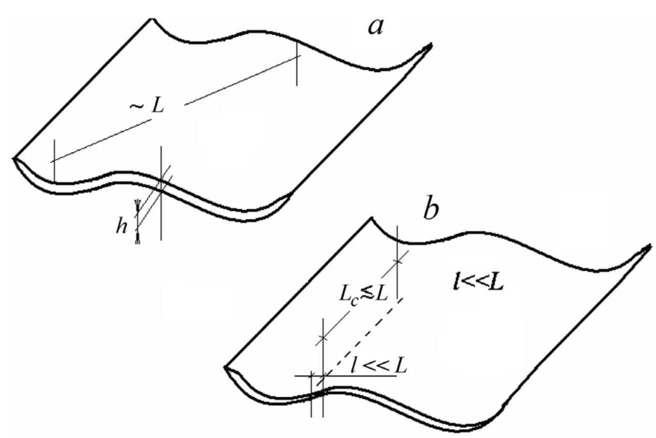

FIG. 1. Schematically shown crack with wavy roughness at the interface (a) that results in the creation of elongated (strip-like) contacts or waists (b) inside the crack that may act as a kind of valve for the fluid flow if the crack is liquid-saturated.

than the average opening h of the crack. In contrast, the absolute variation in the average opening h and in the local separation $\tilde{h}$ of loosely contacting crack surfaces are practically the same, $\Delta h \approx \Delta\tilde{h}$. Due to this fact, the variations of the contact pre-strain (or the narrow waist opening) can be $h/\tilde{h} \gg 1$ times stronger perturbed than the average opening that determines the loss at the whole crack.

After finding the acoustic loss at one crack the macroscopic logarithmic decrement θ can be found as the ratio

$$\theta(\omega) = \frac{W_0}{2W_{ac}}, \tag{1}$$

where the energy W_0 is dissipated during one period in a unit volume and $W_{ac} = K\varepsilon^2/2$ is the acoustic energy density stored in the material with the modulus K. In the order-of-magnitude estimates, it is not critical to specify which particular modulus is chosen and to take into account the correction to the denominator related to the modulus reduction (typically,[26] of the order 10^{-1}) due to the presence of cracks.

III. THERMOELASTIC LOSS AT INNER CONTACTS IN DRY CRACKS

Unlike homogeneous materials for which thermoelastic dissipation of elastic waves is often negligible, in solids with microstructure (e.g., polycrystalline[22]), thermoelastic dissipation significantly increases due to the presence of small (compared with the elastic-wave length) heterogeneities that strongly increase temperature gradients. For crack-containing solids, there is another factor that additionally strongly enhances the thermo-elastic coupling: the stress- and strain concentration at the crack perimeter as considered in work.[27] For the thermoelastic loss at the entire crack with characteristic diameter L, the thermoelastic dissipation exhibits maximum in the vicinity of the relaxation frequency $f_L \approx \kappa/(2\pi\rho C L^2)$, where ρ is the material density, C the specific heat per unit mass, and κ the thermal diffusivity. For millimeter-size cracks in rocks, this maximum corresponds to frequencies in the range $10^{-3} - 10^{-1}$ Hz. The analysis[27] was based on exact solutions for the stress-field distribution near two-dimensional cracks represented as narrow elliptical cavities. However, even without specifying details of a particular crack model and estimating temperature gradients determined by the crack size in the low-frequency limit and by the temperature-wave length in the high-frequency limit, it is possible to evaluate the elastic energy loss using the approach similar to that used by Landau and Lifshitz[22] for polycrystalline solids.

We start from the thermal diffusivity equation for temperature variations $\tilde{T}$ with respect to the mean value T_0

$$\frac{\partial\tilde{T}}{\partial t} + \frac{\kappa}{C\rho}\Delta\tilde{T} = \gamma T_0 \frac{\partial\varepsilon}{\partial t}, \tag{2}$$

where $\gamma = \mu_T K/(\rho C)$ is the Gruneisen parameter of the thermoelastic coupling, μ_T is the thermal expansion coefficient, K is the bulk modulus, ε is the material dilatation, which for the present approximate consideration can be identified with strain in the field of a compressional wave. The acoustic energy loss due to irreversible heat flows can be found from the integral[22]

$$\frac{\partial W}{\partial t} = -\frac{\kappa}{T_0}\int \left(\nabla\tilde{T}\right)^2 dV. \tag{3}$$

Estimating the gradients in the crack from Eq. (2) and evaluating Integral (3), we obtain the following approximate expressions for the elastic energy dissipated by the crack during one oscillation period, which well agree with the asymptotic forms of the results presented in Ref. 27:

$$W_{\text{crack}}^{LF} \approx 2\pi\omega T_0(\mu_T^2 K^2/\kappa)L^5\varepsilon^2, \quad \text{for} \quad \omega \ll \omega_L \approx \frac{\kappa}{\rho C L^2}, \tag{4}$$

$$W_{\text{crack}}^{HF} \approx 2\pi T_0 \frac{\mu_T^2 K^2}{\rho C}\left[\frac{\kappa}{\rho C\omega}\right]^{1/2} L^2\varepsilon^2, \quad \text{for} \quad \omega \gg \omega_L, \tag{5}$$

$$W_{\text{crack}}^{\max} \approx 2\pi T_0(\mu_T^2 K^2/\rho C)L^3\varepsilon^2, \quad \text{for} \quad \omega \approx \omega_L, \tag{6}$$

where ω is the circular frequency and ω_L is the characteristic circular frequency of thermal relaxation determined by the characteristic diameter L of the crack.

Because we need Eqs. (4) to (6) only for comparison with similar equations for thermoelastic loss at inner contacts in cracks, here, we only briefly outline their derivation. First, note that for sufficiently low frequencies, the crack size L is much smaller than both the elastic wave length and the thermal wave length $\lambda_{th} = [\kappa/(C\rho\omega)]^{1/2}$. Therefore, it is the scale L that determines the gradients of the temperature variations in the elastic-wave field, $\nabla\tilde{T} \sim \tilde{T}/L$. Thus in Eq. (2), we can estimate that $\Delta\tilde{T} \sim \tilde{T}/L^2$ and $\partial\tilde{T}/\partial t \sim \omega\tilde{T}$. The condition $L \ll \lambda_{th}$ is equivalent to $\omega \ll \omega_L = \kappa/(\rho C L^2)$ and thus $\partial\tilde{T}/\partial t \ll \Delta\tilde{T}$, so that the first term in Eq. (2) can be omitted. Because $\partial\varepsilon/\partial t \sim \omega\varepsilon$, Eq. (2) yields the estimate for the temperature variation $\tilde{T} \sim \omega\gamma T_0 C\rho L^2/\kappa$. Then we estimate integral (3) taking into account that the temperature gradient $\tilde{T}/L$ is localized in the crack vicinity within the characteristic volume L^3. Then one obtains $\partial W/\partial t \sim \omega^2 T_0\gamma^2 C^2\rho^2 L^5\varepsilon^2$. Finally, for the energy dissipated over one oscillation period $2\pi/\omega$, we recover Eq. (4).

In the high-frequency limit when $\omega \gg \omega_L = \kappa/(\rho C L^2)$, in contrast, the first term $\partial\tilde{T}/\partial t$ becomes dominant in Eq. (2), so that the the temperature variations are adiabatic: $\tilde{T} \approx \gamma T_0\varepsilon$ and $\lambda_{th} \ll L$. In this case, the dissipation is mainly localized in the near vicinity of the crack edge in which fairly universal asymptotic behavior of strain[28] has the form $\varepsilon_{cr} \sim \varepsilon(r/L)^{-1/2}$. Here, radius r is counted from the crack edge (and the angular factor in this order-of-magnitude estimate is neglected). When estimating Integral (3), we have to consider two regions $r \geq \lambda_{th}$ and $r < \lambda_{th}$. In the latter region, the formally infinite strain $\varepsilon_{cr} \sim \varepsilon(r/L)^{-1/2}$ and strain gradient at $r \to 0$ does not cause divergence of the integral because the amount of the generated heat is finite and is smeared within the region $r < \lambda_{th}$. As a result, both subregions $r \geq \lambda_{th}$ and $r < \lambda_{th}$ give functionally identical and approximately equal contributions, so that $\partial W/\partial t \sim \kappa T_0 \gamma^2 L^2/\lambda_{th}$, and for the loss over one period, we recover Eq. (5). Finally, Eq. (6) can be found as the crossover between Eq. (4) and Eq. (5) at $\omega = \omega_L$.

To estimate the expected relaxation frequency, we can take parameters $\kappa \sim 0.015$ W/cm/K and $\rho \sim 2.6$ g/cm^3 typical of quartz-like rocks and consider cracks with $L \sim 1$ mm, which yields $f_L = \omega/(2\pi) \sim 10^{-1}$ Hz. It has been estimated in works[23,27] that thermoelastic losses described by Eqs. (4) to (6) can account for the observed values of Q-factor in dry rocks, especially for low seismic frequencies. On the other hand, it is seen from Eq. (6) that the height of the relaxation maximum is proportional to the cube of the characteristic crack size L. Therefore, to explain the typically observed in dry rocks values $Q \sim 300 - 1000$ for ultrasonic frequencies and even for frequencies of the order of $10^2 - 10^3$ Hz, it is necessary to assume very high densities of very small cracks (of micrometer scale); this, however, does not look realistic.

In this context, the above-mentioned narrow inner contacts have much higher relaxation frequencies. However, the question arises whether such contacts (the volume of which is much smaller than that for the entire crack) can dissipate an appreciable amount of the the elastic-wave energy. The expressions for thermoelastic energy loss at the inner contacts in cracks can be obtained much like Eqs. (4) to (6) by taking additionally into account the local concentration of strain at the contacts (see Refs. 29 and 30):

$$W_{\rm cont}^{LF} \approx 2\pi\omega T_0(\mu_T^2 K^2/\kappa) l^2 L_c L^2 \varepsilon^2, \quad \text{for} \quad \omega \ll \omega_l \approx \frac{\kappa}{\rho C l^2}, \tag{7}$$

$$W_{\rm cont}^{HF} \approx (2\pi/\omega)\kappa T_0 (\mu_T K/C\rho)^2 L_c (L/l)^2 \varepsilon^2, \quad \text{for} \quad \omega \gg \omega_l, \tag{8}$$

$$W_{\rm cont}^{\max} \approx 2\pi T_0(\mu_T^2 K^2/\rho C) L_c L^2 \varepsilon^2, \quad \text{for} \quad \omega \approx \omega_l, \tag{9}$$

Here, L_c in the length of the strip-like contact, l is its width and the other notations are the same as for Eqs. (4) to (6), but the relaxation frequency ω_l is determined by the width $l \ll L$ of the strip-like contact. It is seen from Eqs. (7) to (9) that for frequencies much lower that the relaxation frequency ω_l, the loss is growing as a linear function of ω, and for frequencies much greater than ω_l, the loss is inversely proportional to ω. The asymptotic law ω^{-1} is similar to the high-frequency asymptotic for spheroidal voids[27] rather than flat cracks.

The most striking conclusion, which is seen from Eqs. (7) to (9), is that for a strip-like contact with a length $L_c \sim L$, the magnitude of loss in the vicinity of the maximum located at $\omega_l \gg \omega$ is determined by the size of the whole crack (because $L_c L \sim L^3$). Consequently the maximum loss at such contact is of the same order as for the conventionally considered maximum for the entire crack [compare Eqs. (6) and (9)]. In contrast, the positions of the maxima $\omega_l \approx \kappa/(\rho C l^2)$ and $\omega_L \approx \kappa/(\rho C L^2)$ on the frequency axis can differ in orders of magnitude. Thus one crack of size L with a strip-like contact $L_c \sim L$ of width $l \ll L$ can produce near the relaxation peak ω_l the thermoelastic dissipation comparable with the contribution of $(L/l)^3 \gg 1$ small cracks with the diameter l. Taking again an example of quartz-like rock, this difference is illustrated in Fig. 2 for $L/l = 10^2$, which means that *one* strip-like inner contact produces the same dissipation as 10^6 small cracks of size l.

Matching the asymptotic Eqs. (7) and (8) and assuming that the density of such cracks with inner contacts equals n_0, one can approximate the decrement defined by Eq. (1) in the entire frequency range by the the frequency dependence typical of the classical relaxator:

$$\theta^{\rm cont} = \frac{2\pi T_0 \mu_T^2 K L_c L^2}{\rho C} \frac{\omega/\omega_l}{1 + (\omega/\omega_l)^2} n_0. \tag{10}$$

To find the overall decrement determined by the contributions of contacts with different parameters L, L_c, and l, we have to integrate Eq. (10) over the distribution $n(L, L_c, l)$, for which it is reasonable to assume that the distribution over the width l should be essentially independent of the distribution over L and L_c, so that $n(L, L_c, l) = n(L, L_c) n(l)$ and the averaging takes the form

$$\theta^{\rm cont} = \frac{2\pi T_0 \mu_T^2 K}{\rho C} \int \frac{L_c L^2 \omega/\omega_l}{1 + (\omega/\omega_l)^2} \; n(L, L_c) \; n(l) \; dL dL_c dl. \tag{11}$$

We note that a wide distribution $n(l)$ can strongly smooth the frequency dependence of the dissipation resulting in nearly constant Q-factor even without assuming very high densities of tiny cracks.

It is seen from the structure of Eq. (11) that the height of the relaxation peak and its location on the frequency axis are quite independent because they are determined by essentially different parameters (L and L_c for the peak height and l for the position). Evidently, for sufficiently small variation in the average strains and stresses, the width of the contacts can

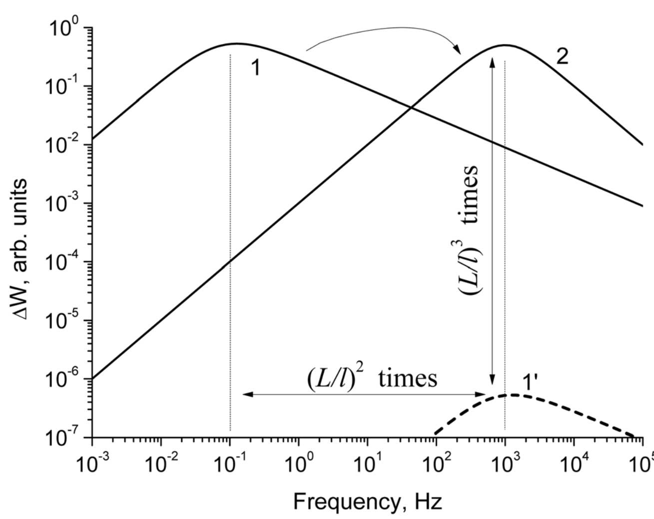

FIG. 2. Schematically shown relative positions and heights of the thermoelastic relaxation peaks for the crack of size L as a whole (curve *1*), for the inner strip-like contact of width $l \ll L$ and length $L_c \sim L$ (curve *2*), and a peak similar to curve *1*, but for a small crack of size l (curve $1'$). It is assumed that $L = 1$ mm and $l/L \sim 10^{-2}$.

already be significantly perturbed, whereas the length of the contact and the average opening of the crack can be only slightly affected. This can result in the displacement of the relaxation maximum on the frequency axis almost without affecting its height as illustrated in Fig. 3 for the case of identical contacts. To estimate how strongly the frequency $\omega_l \approx \kappa/(\rho C l^2)$ of the maximum of the thermoelastic loss at such a contact can be perturbed by variation in the mean strain in the material, let us use the presented in Landau and Lifshitz[22] solution for the width of the contact between two compressed aligned cylinders:

$$l \approx 2\left(\frac{16DF}{3\pi}\frac{RR'}{R+R'}\right)^{1/2}, \tag{12}$$

where $D=(3/4)[(1-\sigma^2)/E+(1-\sigma'^2)/E]$, σ, and σ' are Poisson's coefficients of the materials of the cylinders, E and E' are the Young's moduli of the cylinders, and F is the specific force per unit length of the contacts. In the case of identical materials with the same accuracy that corresponds to Eqs.(7) to (10), we can consider that $D \approx 3/(2E)$ and the force F per unit length for a contact of length L_c can be expressed as $F = F_c/L_c$, where F_c is the total force applied to the contact. Then from Eq. (12) we obtain

$$l^2 \approx (16F_cR)/(\pi E L_c). \tag{13}$$

For the force F_c applied to the inner contact in a crack, an approximate expression $F_c/L_c \approx \varepsilon E L^2/(L+L_c)$ can be obtained,[30] where ε is the mean strain in the material. Then taking into account that $1/2 \le L/(L+L_c) \le 1$, one obtains from Eq. (12) the following approximate expressions:

$$l^2 \approx \frac{16}{\pi}\frac{L}{L+L_c}\varepsilon RL \approx \frac{8}{\pi}\varepsilon RL, \left|\frac{\Delta\omega_l}{\omega_l}\right| = \frac{\Delta(l^2)}{l^2} \approx \Delta\varepsilon\frac{RL}{l^2}. \tag{14}$$

This expression clearly elucidates the reason of the extremely high sensitivity of l^2 (and, consequently, variations in ω_l) with respect to the variation $\Delta\varepsilon$ in the mean strain. Indeed, the wavy asperities at the crack interfaces often have radius comparable with the characteristic diameter of the entire crack ($R \sim L$), whereas the width of the inner strip-like contact often does not exceed the average opening h of the crack. Therefore, taking into account the above-discussed characteristic values of the crack aspect ratios, we conclude that the factor RL/l^2 in Eq. (14) can easily be as great as 10^6–10^8. This means that even variations in the mean deformation (which are conventionally considered unable to appreciably influence acoustical parameters of solids) actually can be able to significantly change the width of inner contacts and affect the near-contact acoustic loss (see Fig. 3).

Estimating the overall sensitivity of the thermoelastic dissipation to the mean strain it is necessary to take into account that conventionally considered thermoelastic losses at the crack as a whole[27] also contribute to the total decrement. We have, however, to recollect that the characteristic

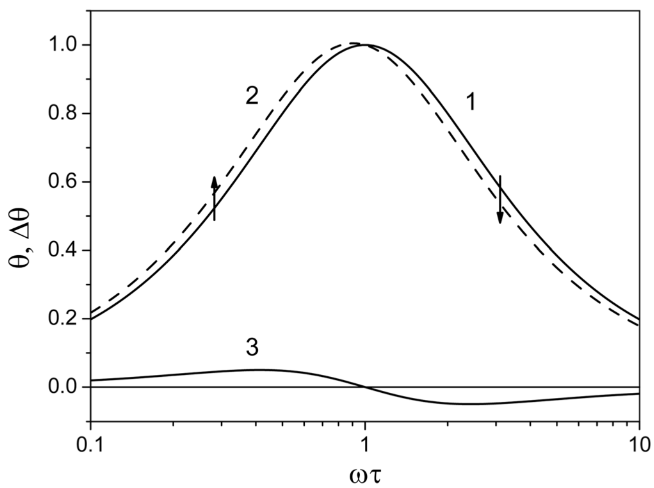

FIG. 3. Variation in the shape $\theta(\omega)$ of the normalized relaxation peak for an inner contact (curves *1* and *2*) due to 15% variation in the contact width. The variation $\Delta\theta$ in the decrement has opposite signs at the opposite sides from the maximum and peak-to-peak excursion about 15% (curve *3*).

relaxation frequencies $\omega_l \approx \kappa/\rho C l^2$ and $\omega_L \approx \kappa/\rho C L^2$ are related as $(L/l)^2$ and the high-frequency thermoelastic loss at the whole crack decreases as $(\omega/\omega_L)^{-1/2}$ [see Eq. (5)] as illustrated in Fig. 2. Then near the relaxation maximum ω_l we found that the ratio of the contributions of the loss $\theta_{\rm loc}(\omega=\omega_l)$ at the inner contacts to the conventionally considered "global" loss $\theta_{\rm glob}(\omega=\omega_l)$ at the cracks as a whole has the form

$$\frac{\theta_{\rm loc}(\omega=\omega_l)}{\theta_{\rm glob}(\omega=\omega_L)} \approx \frac{\tilde{n}_{cr}}{n_{cr}}\left(\frac{\omega_l}{\omega_L}\right)^{1/2} \approx \frac{\tilde{n}_{cr}}{n_{cr}}\frac{L}{l}, \tag{15}$$

where $\tilde{n}_{cr}$ and n_{cr} are the densities of cracks with and without inner contacts, respectively. Because $L/l \gg 1$, it is clear that even a small portion (e.g., a few percentages) of cracks with narrow strip-like inner contacts can ensure near $\omega=\omega_l$ the same contribution to the decrement as the conventional global mechanism of thermoelastic dissipation at whole cracks. Therefore, even quite a small portion of cracks with contacts can provide the above-discussed extremely high sensitivity of the dissipation to small variations in the mean strain in the material.

Even putting aside somewhat exotic manifestations of the giant stress-sensitivity of the dissipation and considering only its mean value, it can be emphasized that in the audible and ultrasonic ranges typical of laboratory studies, inner contacts can ensure a fairly strong contribution to dissipation comparable with the estimates obtained in works[23,27] for the global thermoelastic loss at a crack as a whole for lower frequencies, which showed a reasonable agreement with the experimental data.

IV. VISCOUS LOSS AT FLUID-SATURATED CRACKS WITH INNER STRIP-LIKE WAISTS

Now let us consider viscous dissipation in cracks containing liquids. There is general agreement that along with the dissipation due to global fluid flows in pore channels (Biot's mechanism), an important role in the elastic-wave energy dissipation in rocks belongs to the local (or "squirt") flows inside cracks.[31,32] For the Biot flows, sizes of pore channels with aspect ratios about unity weakly depend on the average strain and stress (except of the case[33] of Poisson's ratio close to 0.5, which is irrelevant to rocks). It is the squirt-type dissipation in relatively soft narrow cracks that demonstrates much higher sensitivity to the variation in the mean strain in the material. In what follows, we focus on new important features of strain sensitivity of the squirt mechanism in the case of cracks with wavy asperities of the interface.

To estimate the viscous loss in narrow cracks with nearly parallel surfaces, we will use the integral expression[34] for the rate of kinetic energy dissipation in a flow of a viscous fluid between parallel solid planes:

$$\frac{\partial W_{\text{kin}}}{\partial t} = -\frac{\eta}{2}\int\left(\frac{\partial \upsilon_x}{\partial y}\right)^2 dV. \tag{16}$$

In Eq. (16) over the volume of the flow, y axis is orthogonal to the interface, υ_x is the X component of the liquid-flow velocity, and η is the viscosity. The integration should be made over the volume of the flow. Using Eq. (16), we first estimate the amount of energy W_0 dissipated in a unit volume over one period. The form of $W_0(\omega)$ can again be understood by using asymptotic estimates by analogy with in the previous section considering now the linearized Navier–Stokes equation for υ_x:

$$\rho\frac{\partial \upsilon_x}{\partial t} = -\frac{\partial p}{\partial x} + \eta\frac{\partial^2 \upsilon_x}{\partial y^2}, \tag{17}$$

where p is pressure. In view of similar structures of Eqs. (17) and (2), the roles of the diffusive term and the term with the time derivative can be estimated in a very similar way.

To better delineate the place of our mechanism in the context of conventionally discussed ones, we first recall how the well-known properties of the global Biot loss in fluid-filled channels can be derived from very simple arguments similar to those used in Ref. 35. For low acoustic frequencies, the role of the first inertial term in Eq. (17) is negligible, so that $|\partial p/\partial x| \approx |\eta\partial^2\upsilon_x/\partial y^2|$, which corresponds to a fluid motion as a parabolic Poiseuille flow in the channel. Thus the velocity gradient is determined by the entire channel thickness H, $\partial^2\upsilon_x/\partial y^2 \sim \upsilon_x/H^2$, and is frequency independent. The fluid velocity υ_x in the Poiseuille flow is induced by the gradient of the acoustic pressure p_{ac}, which is determined by the acoustic wave length $\lambda_\omega \propto \omega^{-1}$, so that $\upsilon_x \propto \partial p/\partial x \sim p_{ac}/\lambda_\omega \propto \omega$. Taking into account that Integral (16) is evaluated over a frequency-independent volume of the flow, the corresponding loss over one period $2\pi/\omega$ is $W_0 \propto (2\pi/\omega)\cdot\omega^2$, so that $\theta(\omega)_{LF} \propto \omega$.

With increasing frequency ω, the first inertial term in Eq. (17) increases and becomes comparable with (and even greater than) the viscous term. This happens when $|\omega\rho\upsilon_x| \approx |\eta\upsilon_x/H^2|$, i.e., for the characteristic frequency

$$\omega_c \sim \eta/(\rho H^2), \tag{18}$$

which is often expressed in a more indirect way via the ratio of porosity to permeability instead of the use of the characteristic width of the channels. The representation of ω_c in Form (18) is especially physically clear and means that the viscous wave length $\lambda_{\text{visc}} = (2\eta/\rho\omega)^{1/2}$ becomes comparable with H. For $\omega \gg \omega_c$, the velocity gradient becomes localized near the walls within the viscous layer $\lambda_{\text{visc}} \ll H$, which determines the flow-velocity gradient, and we have $(\partial\upsilon_x/\partial y)^2 \propto (\omega^{1/2})^2 \propto \omega$. Next, because the last viscous term in Eq. (17) now can be neglected, we see that the velocity υ_x becomes proportional to the acoustical pressure amplitude p_{ac} without any frequency-dependent factor because $\partial\upsilon_x/\partial t \propto \omega\upsilon_x$ and $\partial p/\partial x \sim p_{ac}/\lambda_\omega \propto \omega$ both are proportional to ω. Besides, in integral Eq. (16), the intergration should be made only within the viscous layer with the thickness $\sim \lambda_{\text{visc}} \propto \omega^{-1/2}$ (where the gradient is localized) rather than over the entire volume of the fluid flow. The resultant frequency dependence of Integral (16) has the form $\omega^{-1/2}(\omega^{1/2})^2 = \omega^{1/2}$. The latter estimate should be multiplied by the factor $2\pi/\omega$ to find the energy loss W_0^{HF} during one period, so that for $\omega \gg \omega_c$, we obtain the asymptotic frequency dependence for the decrement: $\theta(\omega)^{HF} \propto (2\pi/\omega)\omega^{1/2} \propto \omega^{-1/2}$. Thus we recovered the well-known asymptotic dependences of the decrement $\theta(\omega)$ due to the Biot flows in the pore channels.

Let us now turn to narrow cracks, which are much softer objects and can exhibit stronger stress-dependence. The above-considered arguments based on the Poiseuille-flow approximation in the low-frequency limit remain very similar for flows in cracks and predict the same asymptotic behavior of the decrement $\theta(\omega)_{LF} \propto \omega$. As was mentioned in Ref. 32, the localization of the shear-flow gradients within a narrow viscous layer $\sim \lambda_{\text{visc}}$ for sufficiently high frequencies may also be applied to cracks, but the characteristic frequency is much higher because the crack opening h is much smaller than the diameter H of the pore channels. However, in contrast to rigid pore channels, cracks are much softer objects[14] with a characteristic own modulus $\alpha K \ll K$. For cracks, the aspect ratio $\alpha \sim h/L$ plays the role of their softness parameter: Due to their planar geometry cracks are roughly α^{-1} times more compliant than the surrounding matrix with respect to both compression and shear.

Thus for soft crack-like pores, another characteristic relaxation frequency appears that physically corresponds to the condition that under sufficiently high-frequency oscillations, the viscous resistance to crack shearing becomes comparable with the elastic response of the crack to shearing. Namely, under tangential displacement Δx of the center of the crack, its shear elastic reaction $\alpha\mu\Delta x/h$ should be compared with the tangential viscous stress $\eta\omega\Delta x/h$. This yields a characteristic frequency

$$\omega_d \sim \alpha\mu/\eta, \tag{19}$$

which was pointed out, for example, in Ref. 36. Now we note that for geophysical materials, typically the values of the shear modulus μ and compression modulus K are of the same order, and the corresponding ω_d is relatively high. For example, for viscosity $\eta = 10^{-3}$ Pa · s typical of water, modulus $\mu \sim 10^{10}$ N/m^2 typical of rocks and quite narrow cracks with $\alpha \sim 10^{-3} - 10^{-4}$ we obtain $\omega_d \sim 10^9 - 10^{10}$ rad/s relevant only to ultrasonic laboratory studies but not to the discussed seismo-acoustic data.[11,15–17]

One more characteristic frequency can be obtained in a similar way but considering the viscous resistance of a crack-like defect to compression in the direction normal to its plane. This viscous resistance should be compared with the elastic reaction of the crack also in the normal direction. To estimate the normal viscous resistance, we have to supplement Eq. (17) with the continuity equation

$$\frac{\partial \upsilon_x}{\partial x} + \frac{\partial \upsilon_y}{\partial y} = 0, \qquad (20)$$

because in the first approximation, the liquid can be considered incompressible. For the discussed nearly Poiseuille flow, $\partial \upsilon_x/\partial x \sim \upsilon_x/L$ and $\partial \upsilon_y/\partial y \sim \upsilon_y/h$, so that from Eq. (20) we obtain $\upsilon_x \sim (L/h)\upsilon_y$. Because the variation Δh of the crack opening in the acoustic field corresponds to $\upsilon_y \sim \omega\Delta h$, we find that $\upsilon_x \sim (L/h)\omega\Delta h$. Next, in Eq. (17), we can similarly estimate that $\partial p/\partial x \sim p/L$ and $\eta\partial^2\upsilon_x/\partial y^2 \sim \eta\upsilon_x/h^2$. Neglecting the first inertial term in Eq. (17) for the Poiseuille flow and equating the above-estimated terms in the right-hand side of Eq. (17), we obtain the estimate for the pressure p^{visc} induced by the viscous flow of the liquid in the narrow gap:

$$p^{\text{visc}} \sim \eta\upsilon_x(L/h^2) \sim \eta\omega(L/h)^2(\Delta h/h). \qquad (21)$$

For the application to real cracks, it is important to understand how critical is imperfect parallelism of the crack surfaces (which for real cracks is rather a rule). Assuming that the crack opening has the values h_1 and h_2 at the opposite sides, we conclude that, in the extreme case when at one side the $h_1 = 0$, the maximal non-parallelism angle is of the order $h/L \sim \alpha$, where h is the average crack opening. This small non-parallelism is not important for the above-obtained estimates of the terms entering Eqs. (17) and (20) and consequently estimate Eq. (21).

On the other hand, one may argue that this non-parallelism leads to the fact that the shear viscous force acting along the inclined wall has a component normal to the crack plane. The viscous stress τ^{visc} acting along the inclined plane is readily estimated as $\tau^{\text{visc}} \sim \eta\upsilon_x/h \sim \eta(L/h)\omega\Delta h/h$, so that its normal component is $\tau_n^{\text{visc}} \sim \tau^{\text{visc}}\alpha \sim \eta\omega\Delta h/h$. Comparing the estimated normal component τ_n^{visc} and Eq. (21) for the pressure due to the viscous flow, we see that p^{visc} is $1/\alpha^2 >> 1$ times greater than τ_n^{visc}. Thus the contribution τ_n^{visc} of the eventual non-parallelism to the normal viscous stress is not significant compared with p^{visc}, and we can use Eq. (21) for the normal viscous reaction even for cracks with not perfectly parallel surfaces.

Now we have another question: What is the elastic reaction that should be taken for comparison with the precedingly found value [Eq. (21)] for the viscous normal resistance. In literature different variants can be found. One variant is by analogy with obtaining Eq. (19) to compare Eq. (21) with the normal elastic reaction $\sim \alpha K \cdot \Delta h/h$ of the crack related to its effective compression modulus $\sim \alpha K$. The latter value corresponds to the well known rule of thumb[24] that the effective softness relatively to the matrix material for a crack approximately equals its aspect ratio. This comparison gives us some characteristic frequency often discussed in literature[25]

$$\omega_K \sim \alpha^3 K/\eta. \qquad (22)$$

Less formally, it can be said that the viscous flow pressure p^{visc} arisen due to approaching of the crack surfaces by Δh [see Eq. (21)] becomes so high that the near-crack region of the size L (where the elastic stress was initially nearly released) experiences compression by the value $\Delta h^* \sim p^{\text{visc}}/(KL)$. For $\omega \sim \omega_K$, the value of Δh^* becomes comparable with the initial approaching Δh and strongly compensates the latter. If the crack is "instantaneously" compressed, initially the liquid remains almost "frozen" and then is redistributed during the characteristic time $1/\omega_K$ when the elastic compression Δh^* of the near-crack region gradually releases. For sufficiently high frequency $\omega \gg \omega_K$, the actually displaced volume of the liquid is equal to a small fraction ω_K/ω of the volume that would be quasistatically displaced, so that if the flow structure remains of the Poiseuille type, the amount of the dissipated energy W_0 during one period decreases as $(\omega/\omega_K)^{-1}$. This observation will be used in the following text to estimate the absorption above the relaxation maximum.

It can also be pointed out that besides the relaxation frequency given by Eq. (22), another relaxation frequency for cracks filled by compressible fluids is also discussed.[31] Namely, in view of the fact that modulus K_f of the filling fluid typically is at least an order of magnitude smaller than modulus K, it can be argued that the surrounding rock in the first approximation compresses the fluid as "absolutely rigid" body. Thus comparing the pressure of the compressed fluid $K_f\Delta h/h$ with the precedingly found normal viscous reaction [Eq. (21)] of the fluid flow, we obtain another characteristic frequency $\omega_r \sim K_f\alpha^2/\eta$, which is determined by the compressibility of the fluid and in contrast to Eq. (22) does not depend on the modulus of the rock.[37] This new relaxation frequency differs from ω_K given by Eq. (22) by a factor of $K_f/(K\alpha)$. For narrow cracks with $\alpha \ll 1$ and typical liquids like water or oil, for which $K_f/K \sim 10^{-1}$, the factor $K_f/(K\alpha)$ is quite large, so that $\omega_r \gg \omega_K$. In such a case, long before the frequency ω_r would be attained, the fluid flow should already be strongly damped for $\omega > \omega_K$ because of the finite rigidity of the near-crack region as discussed in the preceding text. Therefore the initial low-frequency asymptotic $\propto \omega$ cannot be extrapolated till the frequency $\omega_r \gg \omega_K$. Consequently, the new (and potentially stronger) relaxation maximum near ω_r simply will not be formed for typical liquids (like water and oil), for which $K_f/(K\alpha) \gg 1$. Only for sufficiently high-compressible (gaseous) fluids with $K_f/(K\alpha) < 1$ this mechanism should form the relaxation maximum near ω_r. But in this case, the relaxation frequency ω_r should necessarily lie lower than ω_K.

Less formally, the physical meaning of this type of relaxation is that a finite time is required for a compressible fluid squeezed inside a crack to form the flow that redistributes the fluid and equalizes the internal pressure. In any case, the character of such flows can noticeably be perturbed only if the mean opening of the entire crack is perturbed. This means that the mean perturbing strain should already be comparable with α like for the above-considered global thermoelastic loss.

In what follows, we consider a modified form of the viscous loss in cracks with wavy roughness of the surface. This mechanism relates to the conventionally considered viscous loss at cracks much like the thermoelastic loss at the inner contacts relate to the thermoelastic loss at a crack as a whole. We show that even for "normal" liquids, the new strongly shifted to lower frequencies and very intense relaxation peak is related to the fluid compressibility rather than compressibility of rock in the crack vicinity. Besides, which is especially interesting, the stress sensitivity of such a mechanism can be additionally enhanced orders of magnitude and can readily explain the experimental data discussed in the introduction.

Consider a crack with a waist' created by wavy asperities as shown in Fig. 4. The notations used are clear from the figure. Here, quantities P_i characterize the pressure in the respective cross sections of the crack. Assuming again the Poiseuille character of the flow for sufficiently low frequencies, the nearly plane fluid flow $q = L_z \, _v x(y)dy$ in the main part of the crack and in the narrower region of the waist should be equal:

$$q = \frac{h^3 L_z}{12\eta}\frac{P_2 - P_1}{L} = \frac{\tilde{h}^3 L_z}{12\eta}\frac{P_3 - P_2}{\tilde{l}}, \tag{23}$$

where the meaning of the pressure gradients $(P_2 - P_1)/L$ and $(P_3 - P_2)/\tilde{l}$ is clear from Fig. 4. Then Eq. (23) yields

$$\frac{P_2 - P_1}{P_3 - P_2} \sim \frac{\tilde{h}^3}{h^3}\frac{L}{\tilde{l}}, \tag{24}$$

so that for sufficiently narrow waist in the crack, $(\tilde{h}^3/h^3)(L/\tilde{l}) \ll 1$, we conclude that $(P_2 - P_1) \ll (P_3 - P_2) \approx \Delta P_{\mathrm{tot}}$, that is, the total drop ΔP_{tot} of pressure is mainly localized in the region of the narrow waist. Now imagine an instantaneous slight compression of the crack under an external action after which the fluid compressed inside the crack flows through the narrowing into the outer region to equalize the pressure. This flow will exponentially decrease with a characteristic relaxation time τ. Consequently the total volume q_τ of the leaked fluid during this relaxation process is given by the integral[31]

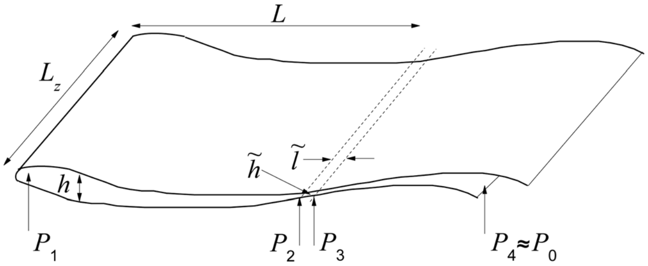

FIG. 4. Schematically shown crack with a narrow strip-like waist created by wavy asperities. The dominant part of the total difference in the pressure $|P_1 - P_4|$ in the fluid flow is localized near the narrow waist, so that $|P_1 - P_4|/|P_3 - P_2| \approx 1$. P_4 is close to the equilibrium pressure P_0 in the outer pore channel.

$$q_\tau = q\int_0^\infty exp(-t/\tau)dt = q\tau, \tag{25}$$

where q is the flow value corresponding to the initial excess of pressure inside the crack over the pressure in the outer space. Because the rigidity of the outer pore space is much greater than that of the thin crack, the relative change in the volume of the crack is much greater than that for the outer rigid pore channels. Consequently, the outer pore pressure remains close to its equilibrium value P_0, and we can consider that $P_3 - P_2 \approx P_1 - P_4 \approx P_1 - P_0$. Thus from Eq. (23) the initial value of the flow is

$$q = \frac{\tilde{h}^3 L_z}{12\eta}\frac{P_3 - P_2}{\tilde{l}} \approx \frac{\tilde{h}^3 L_z}{12\eta}\frac{P_1 - P_0}{\tilde{l}}. \tag{26}$$

Because the variation χ in the volume of the liquid and the pressure P are related via the bulk modulus K_f of the liquid, $P = \chi K_f$, then the total liquid volume $q_\tau = q\tau$ displaced in order to equalize the pressure can be written via the initial volume C_0 of the fluid-filled crack as

$$q_\tau = C_0(\chi_1 - \chi_0) = C_0 K_f^{-1}(P_1 - P_0). \tag{27}$$

Equations (26) and (27) yield

$$q_\tau = \frac{\tilde{h}^3 L_z}{12\eta}\frac{K_f(\chi_1 - \chi_0)}{\tilde{l}}\tau \approx C_0(\chi_1 - \chi_0). \tag{28}$$

Assuming that the sizes of cracks along x and z directions are of the same order $L_z \sim L$, the crack volume can be estimated as $C_0 \approx hL^2$, so that from Eq. (28) we readily obtain the relaxation time $\tilde{\tau}$ and its inverse value, i.e., the relaxation frequency $\tilde{\omega}_r$ (tilde denotes that we consider the crack with a narrowing):

$$\tilde{\omega}_r = \frac{\alpha^2 K_f}{12\eta}\left(\frac{\tilde{h}}{h}\right)^3\frac{L}{\tilde{l}}, \quad \text{and} \quad \tilde{\tau} = \frac{12\eta}{\alpha^2 K_f}\left(\frac{h}{\tilde{h}}\right)^3\frac{\tilde{l}}{L}. \tag{29}$$

Here we intentionally singled out the squared aspect ratio α^2 for convenience of comparison with the crack without waists.[31] The absence of the narrow waist can be interpreted as $\tilde{l} \sim L$ and $\tilde{h} \sim h$, i.e., $(\tilde{h}/h)^3 \tilde{l}/L \sim 1$, so that Eq. (29) reduces to

$$\omega_r = \frac{\alpha^2 K_f}{12\eta}, \quad \text{and} \quad \tau \approx \frac{12\eta}{\alpha^2 K_f}. \tag{30}$$

Within the accuracy of its derivation, Eq. (30) coincides with the result $\tau \approx 8\eta/\alpha^2 K_f$ obtained in Ref. 31 for narrow cracks without the waist.

The relaxation times in Eqs. (29) and (30) do not depend on the elastic modulus of the solid matrix. However, we have already mentioned that for cracks with uniform opening, the relaxation peak [Eq. (3)] can actually be formed only for very highly compressible fluids (i.e., gases rather than liquids), for which $K_f/K \leq \alpha$, otherwise the compressibility of rock in the crack vicinity should dominate and form the relaxation peak near ω_K given by Eq. (22).

In contrast to this, the presence of the narrow waist creates a strong obstacle for the fluid leaking. Therefore, for a given frequency, this increases the role of the liquid compression, whereas the pressure p^{visc} and the deformation of the rock in the crack vicinity is strongly reduced. All this results in a very strong reduction of the relaxation frequency $\widetilde{\omega}_r$ related to the compressibility of the fluid [compare Eqs. (30) with Eq. (29) containing the additional small factor $(\tilde{h}/h)^3\tilde{l}/L \ll 1$]. Thus even for "normal" liquids like water and oil, the frequency $\widetilde{\omega}_r$ of the new relaxation maximum becomes comparable and even lower than ω_K, that is, shifts from ultrasonic frequencies to the seismoacoustic range directly relevant to the experiments[11,13–16] discussed in the Introduction.

Compared with the mean crack opening h, the local opening $\tilde{h}$ of the waist is much more sensitive to variations in the mean strain in the medium.

Therefore it is not necessary to assume the existence of unrealistically thin cracks to ensure the same sensitivity for the mean opening of the crack. The conditions ensuring the Poiseuille type of the flow are even better fulfilled in the vicinity of the waist, so that we can apply the considered in the beginning of this section arguments concerning the low- and high-frequency asymptotic behavior of the energy loss W_0 per one period (proportional to ω and $1/\omega$, respectively). This means that the dissipation due to the discussed mechanism can be well approximated by the frequency dependence for a standard relaxator:

$$\theta \approx \theta_0 \frac{\omega/\widetilde{\omega}_r}{1+(\omega/\widetilde{\omega}_r)^2}, \tag{31}$$

where the relaxation frequency $\widetilde{\omega}_r$ is given by Eq. (29) for a crack with a narrow waist.

Now, using Eq. (16) we will determine the prefactor θ_0 in two cases: for a crack with the narrow waist and for a similar in size crack with uniform opening, although in the latter case, the formation of the relaxation peak ω_r can be possible only if $K_f/K \le \alpha$, that is, for gaseous fluids (or for unrealistically rigid solid matrix in the case of normal liquids). Nevertheless, such an expression will be useful for comparison.

The velocity profile for the Poiseuille's flow in the crack with uniform opening h has a parabolic form

$$\upsilon_x = -\frac{1}{2\eta}\frac{\partial P}{\partial x} y(h-y) = -4\upsilon_{\max}\frac{y(h-y)}{h^2}, \tag{32}$$

where $\upsilon_{\max} = -1/8\eta|\partial P/\partial x|h^2$ is the maximal velocity of the fluid in the flow. Similarly, for the narrow waist, where the losses and gradients are localized, we have

$$\tilde{\upsilon}_x = -\frac{1}{2\eta}\frac{\partial P}{\partial x} y(\tilde{h}-y) = -4\tilde{\upsilon}_{\max}\frac{y(\tilde{h}-y)}{\tilde{h}^2}, \tag{33}$$

where $\tilde{\upsilon}_{\max} = -1/8\eta|\partial P/\partial x|\tilde{h}^2$. Equations (16), (32), and (33) yield for $\partial W_{\mathrm{kin}}/\partial t$ the following expressions in the discussed two cases of the crack without and with the waist:

$$\frac{\partial W_{\mathrm{kin}}}{\partial t} = -\frac{8}{3}\eta\upsilon_{\max}^2\frac{L_x L_z}{h} \tag{34}$$

$$\frac{\partial \widetilde{W}_{\mathrm{kin}}}{\partial t} = -\frac{8}{3}\eta\tilde{\upsilon}_{\max}^2\frac{\tilde{l}L_z}{\tilde{h}}. \tag{35}$$

In the estimates in the following text, we will assume that $L_x \sim L_z \sim L$.

Let us now relate the parameters of the flow with the mean strain in the crack-containing solid. When the mean macroscopic strain ε is created in the material, the crack volume correspondingly changes, which creates the flow of the liquid inside the crack. We recall[24] that the crack is a soft object the relative (compared with the surrounding matrix) compliance ζ of which is approximately equal to the crack aspect ratio, $\zeta \sim \alpha \sim h/L$. Therefore the own strain ε_{cr} of the crack (i.e., the relative variation in its volume) can be estimated as $\varepsilon_{cr} \approx \varepsilon/\alpha \approx \varepsilon L/h$, where L is the characteristic size of the crack. For the crack volume C_0, we already used the estimate $C_0 \sim L^2 h$ in the discussion of Eq. (28). Then for variation ΔC_0, one obtains $\Delta C_0 \approx \varepsilon_{cr} C_0 \sim \varepsilon L^2 h \cdot (L/h) = \varepsilon L^3$. Under sinusoidal oscillatory variation of the crack volume with frequency ω and amplitude ΔC_0, we can readily relate the rate $\omega \Delta C_0$ of the crack-volume variation with the maximum velocity of the Poiseuille's flow of the fluid inside the cracks without and with the waist:

$$\upsilon_{\max} = \frac{3}{2}\frac{\omega\varepsilon L^2}{h} \tag{36}$$

$$\tilde{\upsilon}_{\max} = \frac{3}{2}\frac{\omega\varepsilon L^2}{\tilde{h}}. \tag{37}$$

Substituting these expressions into Eqs. (34) and (35) for cracks without and with the waist, we relate the period-averaged amounts of the dissipated energy with the amplitude of the mean strain ε:

$$\left(\frac{\partial W_{\mathrm{kin}}}{\partial t}\right)_{\mathrm{aver}} = -3\eta\omega^2\varepsilon^2\frac{L^6}{h^3}, \tag{38}$$

$$\left(\frac{\partial \widetilde{W}_{\mathrm{kin}}}{\partial t}\right)_{\mathrm{aver}} = -3\eta\omega^2\varepsilon^2\frac{L^5\tilde{l}}{\tilde{h}^3}. \tag{39}$$

In Eqs. (38) and (39), we took into account that for sinusoidal strain with amplitude ε, the period-averaged value of its square equals $\varepsilon^2/2$. We also take into account that the energy loss during one period $T = 2\pi/\omega$ is $W_0 = (2\pi/\omega)(\partial W_{\mathrm{kin}}/\partial t)_{\mathrm{aver}}$, and the density of the accumulated elastic energy is $W_{el} \approx K\varepsilon^2/2$ (because in the used approximation, we assume that the elastic modulus does not significantly change due to the presence of the cracks). Then from Eq. (1) we obtain the following asymptotic low-frequency expression for the decrement in the case of cracks without the waists:

$$\theta_{LF} = 6\pi\eta\frac{L^6}{h^3 K} n_{cr}\omega. \tag{40}$$

The low-frequency Eq. (40) is obtained neglecting the liquid compressibility and agrees with the low-frequency results earlier derived for cracks without waists.[32]

To find high-frequency expressions (for $\omega >> \omega_r$), we recall that because of the finite relaxation time, the volume of the replaced liquid is ω/ω_r times smaller than in the low-frequency limit. Thus the high-frequency expression for the cracks without the waists is

$$\theta_{HF} = 6\pi\eta \frac{L^6}{h^3 K} n_{cr} \omega \frac{\omega_r^2}{\omega^2} = \frac{\pi}{2} n_{cr} L^3 \frac{K_f}{\alpha K} \frac{\omega_r}{\omega}. \tag{41}$$

For cracks with the waists in a similar way, we obtain:

$$\widetilde{\theta}_{LF} = 6\pi\eta \frac{L^6}{\widetilde{h}^3 K} \left(\frac{\widetilde{l}}{L}\right) \tilde{n}_{cr} \omega, \tag{42}$$

$$\widetilde{\theta}_{HF} = 6\pi\eta \frac{L^6}{\widetilde{h}^3 K} \left(\frac{\widetilde{l}}{L}\right) \widetilde{n}_{cr} \omega \frac{\widetilde{\omega}_r^2}{\omega^2} = \frac{\pi}{2} \widetilde{n}_{cr} \frac{L^3 K_f}{\alpha K} \frac{\widetilde{\omega}_r}{\omega}. \tag{43}$$

Comparing Eqs. (40), (41), and (42), and (43) with the low- and high-frequency asymptotics of Eq. (31), we obtain the following relaxator-like expressions for the viscous loss at cracks without and with the waists, respectively:

$$\theta = \frac{\pi}{2} \frac{K_f L^3}{K\alpha} n_{cr} \frac{\omega/\omega_r}{1 + (\omega/\omega_r)^2}, \tag{44}$$

$$\tilde{\theta} = \frac{\pi}{2} \frac{K_f L^3}{K\alpha} \tilde{n}_{cr} \frac{\omega/\widetilde{\omega}_r}{1 + (\omega/\widetilde{\omega}_r)^2}, \tag{45}$$

where ω_r and $\widetilde{\omega}_r$ are given by Eq. (29).

A striking feature of Eqs. (44) and (45) is that they have the same maximum values determined by the characteristic size L of the whole crack. However, the relaxation frequency in Eq. (45) is strongly shifted from ultrasonic to seismoacoustic frequencies down to $10^0 - 10^2$ Hz that were used in field experiments.[11,15,16]

The next point is that much smaller average strains (e.g., the above-discussed tidal strains) can already noticeably affect the opening of the waist. This means that due to the variation in the relaxation frequency $\widetilde{\omega}_r$ for the crack with the waist, the position of the relaxation maximum can noticeably be changed, whereas the height of the relaxation maximum should remain yet practically unperturbed. Taking into account the equality of the absolute variations $\Delta\widetilde{h} \approx \Delta h$ and the relationship $\varepsilon_{cr} = \varepsilon/\alpha = \varepsilon L/h$ (which we have already used in the preceding text), we find the relative variation in the relaxation frequency $\widetilde{\omega}_r$ caused by the variation $\Delta\varepsilon$ in the mean strain:

$$\Delta\widetilde{\omega}_r/\widetilde{\omega}_r = \Delta\varepsilon \frac{d\widetilde{\omega}_r}{d\varepsilon} \frac{1}{\widetilde{\omega}_r} \approx \Delta\varepsilon \frac{3h}{\widetilde{h}} (L/h) = \Delta\varepsilon \frac{3h}{\widetilde{h}} \frac{1}{\alpha}. \tag{46}$$

The parameter $(3h/\widetilde{h})/\alpha$ can be very large, for example, $(3h/\widetilde{h})/\alpha \sim 10^6$ for quite realistic $\alpha \sim 10^{-4}$ and $(h/\widetilde{h}) \sim 20..30$. Thus the tidal strains with amplitude $\varepsilon_0 \sim 10^{-8}$ can cause the peak-to-peak variation in the relaxation frequency $2\Delta\widetilde{\omega}_r/\widetilde{\omega}_r$ of several percent and, consequently, comparable in magnitude variation $2\Delta\tilde{\theta}/\tilde{\theta}$ of the decrement at the wings of the relaxation curve.

In addition, there are known experimental indications that for flows in very narrow gaps (down to nanometer scale), the effective viscosity of the liquid can noticeably exceed the viscosity for macroscopic gaps.[38,39] This effect can additionally enhance the variations in the dissipation due to variations in the waist opening. Because small variations in the average strain practically do not yet affect the average aspect ratio for the crack, the prefactor in Eq. (45) remains almost unchanged. Thus the shift of the relaxation maximum can cause the variation in $\Delta\tilde{\theta}$ of opposite signs depending on the position of the observation frequency ω relative to the frequency $\widetilde{\omega}_r$ of the relaxation-curve maximum. The similar feature for the thermoelastic loss is illustrated in Fig. 3.

Because not all cracks have the strip-like waists, the key question is how many such cracks are required to ensure near the characteristic frequency $\widetilde{\omega}_r$ the dissipation $\theta \sim 10^{-2} - 10^{-1}$ typical of rocks. For $\omega = \widetilde{\omega}_r$, Eq. (45) yields the following estimate of the loss

$$\tilde{\theta}_{\max} = \frac{\pi}{4} \frac{K_f L^3}{K\alpha} \tilde{n}_{cr} = \frac{\pi}{4} \frac{K_f}{K\alpha} \tilde{\varepsilon}, \tag{47}$$

where we singled out the quantity $\tilde{\varepsilon} = L^3 \tilde{n}_{cr}$. The latter is close to the effective volume of cracks[40] (i.e., the volume of the circumscribed spheres independent of the cracks' aspect ratios). For further estimates, we will use the well known fact that the presence of cracks with the effective volume $\tilde{\varepsilon}$ results in reduction of the elastic moduli of the material by a fraction of $a\tilde{\varepsilon}$ (where factor $a \sim 1$ slightly differ for particular moduli[40]). Then taking for filling water $K_f = 2.25 \cdot 10^9$ Pa, modulus $K = 3.8 \cdot 10^{10}$ Pa typical of quartz, and $\alpha = 10^{-3} - 10^{-4}$, we find that $\tilde{\theta}_{\max} \approx (50 - 500)\tilde{\varepsilon}$. This means that for the effective crack density $\tilde{\varepsilon} = 10^{-2} - 10^{-3}$, which reduces the elastic moduli also by a fraction of order $10^{-2} - 10^{-3}$, we can already obtain $\tilde{\theta} \sim 10^{-2} - 10^{-1}$ in the vicinity of the relaxation frequency $\widetilde{\omega}_r$. On the other hand, it is well known that, e.g., in real sandstones, the modulus reduction due to soft crack-type porosity can be on the order of tens of percentages.[26] This means that even if only a small portion of all cracks (a few percent or even less) has wavy surfaces creating the narrow waists, this portion can already be sufficient to explain both the background value of dissipation observed in the seismo-acoustic frequency range $10^0 - 10^3$ Hz and its extremely high strain-sensitivity indicated by the experimental data.[11,15,16]

Therefore the considered modified mechanism of the squirt-type dissipation in cracks suggests a plausible alternative explanation to the experimentally observed rather high dissipation in the seismoacoustic frequency range. In terms of Ref. 41, our mechanism is of purely "microscopi" type and does not require the presence of larger-scale (mesoscale) heterogeneities assumed in the "patchy saturation" models to shift the frequencies of viscous relaxation towards the seismoacoustic frequency range. Certainly such mechanisms can operate simultaneously. However, among those possibilities, only the above-considered modified squirt mechanism is evidently able to ensure sufficiently high strain sensitivity to explain the observations.[11,15,16]

To better understand the ratio of contributions of the conventional and the proposed modified squirt mechanisms, let us compare the corresponding decrements given by Eqs. (44) and (45) in the vicinity of the characteristic relaxation frequency $\omega \sim \widetilde{\omega}_r$ of the cracks with the narrow waists. Because $\omega_r \ll \widetilde{\omega}_r$, for this comparison, we take the low-frequency asymptotic form of Eq. (44), such that in the vicinity of $\widetilde{\omega}_r$ we obtain

$$\theta(\omega \sim \widetilde{\omega}_r) \approx 2\frac{n_{cr}}{\tilde{n}_{cr}}\frac{\widetilde{\omega}_r}{\omega_r}\tilde{\theta}_{\max}, \tag{48}$$

where the difference in the relaxation frequencies ω and $\widetilde{\omega}_r$ [see Eqs. (29) and (30)] can be rather large: $\omega/\widetilde{\omega}_r = (h/\tilde{h})^3(\tilde{l}/L) \gg 1$. The factor $(h/\tilde{h})^3(\tilde{l}/L)$ can easily reach 10^2–10^4, so that quite a small portion of cracks with narrow waists can ensure in the low-frequency range a contribution comparable with or even strongly exceeding the contribution of the majority of other cracks of the same size but without the waists. For example, a portion $\tilde{n}_{cr}/n_{cr} \sim 10^{-2} - 10^{-4}$ of cracks with narrow waists is already able to ensure extremely high strain-sensitivity of the overall decrement in the seismo-acoustic range.

Figure 5 illustrates the relative positions and heights of the relaxation peaks corresponding to the relaxation frequency $\widetilde{\omega}_r$ [Eq. (29)] for a crack with a narrow waist, and the peaks at frequencies ω_K [Eq. (22)] and ω_r [Eq. (33)] for cracks with the same aspect ratio α and size L but without waists. We emphasize that for $\omega_K < \omega_r$, the relaxation peak at ω_r actually does not exist and is shown by the dashed line. However, it is shown for convenience of comparison with the low-frequency peak $\widetilde{\omega}_r$, for which the height is the same as for the would-be peak at ω_r. The figure demonstrates that the viscous relaxation in cracks with the waists can form

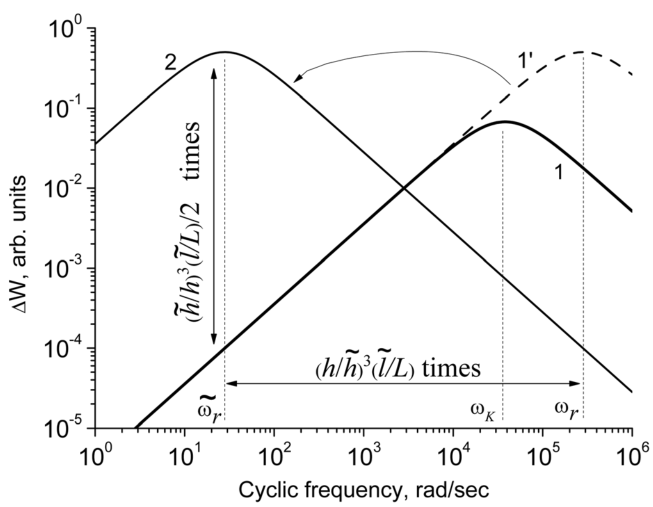

FIG. 5. Schematically shown relative positions and heights of the viscous relaxation peaks at ω_r and ω_K for the crack as a whole (curves *1* and *1'*) and the low-frequency peak for a crack of the same size L having an inner narrow waist with the local opening $\tilde{h}$ and length $\tilde{l}$ (curve *2*). The examples correspond to $(h/\tilde{h})^3(\tilde{l}/L) = 10^4$ and $\alpha = 10^{-3}$.

extremely strong peaks in the seismo-acoustic frequency range, so that even for small density of such cracks, their contribution can easily account for the typically observed levels of the dissipation in this frequency range.

Concerning the question of averaging over the distribution of real cracks over their parameters, we can put forward very similar arguments as for Eq. (11) in the above-considered case of thermoelastic loss. Namely, it is reasonable to assume that the distributions over the sizes L and h of the crack as a whole and the distribution over the local parameters $\tilde{h}$ and $\tilde{l}$ of the waist (which determine the characteristic relaxation frequency $\widetilde{\omega}_r$) are essentially independent and can be factorized. Therefore the averaging over the characteristic size L of the crack gives only a numerical factor like in integral Eq. (11), whereas the averaging over the relaxation frequencies (of, equivalently, the relaxation times) can also be performed independently. Such averaging should not radically change the conclusions obtained for the simplest case of cracks with identical parameters of the inner contacts or waists.

V. CONCLUSION

The performed analysis of the role of elongated inner contacts and waists in cracks significantly changes the conclusions based on conventionally discussed models of thermoelastic dissipation at cracks (like Refs. 23 and 27) and viscous squirt loss (like works 25, 31, 32, 36, and 41). Thus a single larger crack with a strip-like contact can ensure the same thermoelastic dissipation as $10^5 - 10^6$ small cracks of the size equal to the contact width.

For fluid saturated cracks, a rather intense maximum formed by a crack with a narrow waist can ensure the same dissipation in the seismo-acoustic frequency range of $10^2 - 10^3$ Hz as a similar crack without the waist would produce in the ultrasonic range according to conventional squirt-dissipation models (see Fig. 5). This can substantially affect some conclusions[41] on insignificant role of local viscous loss at cracks in the seismo-acoustic range.

Probably the most striking feature of the considered modified dissipation mechanisms is their giant strain sensitivity. In this context, it should be clearly understood that each group of cracks with the wavy asperities can exhibit this giant strain sensitivity only in a rather narrow strain range. When the waist becomes either completely closed or widely open, the loss at such cracks does not much differ from that at cracks without the asperities. Nevertheless, because the parameters of real cracks should have a rather wide distribution, for a current mean strain, another portion of cracks with such narrow waists or contacts can be "activated." This resolves the problem[14] of how apparently very soft defects exhibiting giant strain sensitivity can exist under very different mean pressures (in a wide range of depths in field conditions). Their giant effective compliance should be understood as differential. It cannot be directly extrapolated for significantly higher strains $10^{-6} - 10^{-4}$ quite typical of laboratory experiments or real tectonic strains. For such strains, the dissipation does not change many times, although for 10^{-8} the variations may already reach several percentages.

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