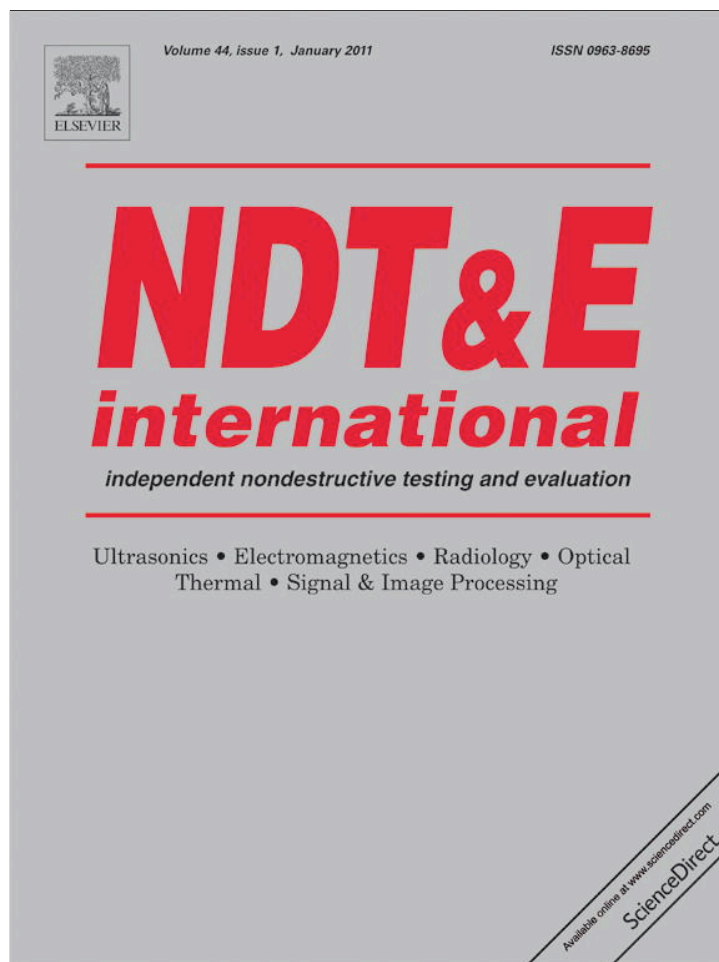


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## Elastic-wave modulation approach to crack detection: Comparison of conventional modulation and higher-order interactions

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### ABSTRACT

Comparison of recent theoretical estimates with experiments has indicated that the ultimate sensitivity of the conventional modulation technique of crack detection is mainly determined by the background modulation produced by the quadratic component of the atomic nonlinearity of the matrix material. Much smaller level of masking nonlinear effects is typical of higher-order interactions due to cubic and higher-order components in the power-series expansion of the background nonlinearity of the solid. In contrast, the level of formally higher-order components originated due to nonlinearity of crack-like defects can be comparable with that of the first-order components. Such strongly increased efficiency of higher-order interactions is due to the fact that crack-like defects often demonstrate non-analytic (non power-law) nonlinearity even for moderate acoustic amplitudes. Besides the increased level, the higher-order components arisen due to non-analytic nonlinearity of cracks can demonstrate significantly different functional behavior compared to manifestations of the atomic nonlinearity. This difference can also help to discriminate the contributions of the defects and the background atomic nonlinearity. Here, we focus on the main differences between the modulation components arisen due to cubic terms in the power-series expansion of the atomic nonlinearity and similar components generated by clapping Hertzian nonlinearity of inner contacts in cracks. We also examine experimental examples of higher-order modulation interactions in damaged samples. These examples clearly indicate non-analytical character of the defects' nonlinearity and demonstrate that the use of higher-order modulation effects can significantly improve the ultimate sensitivity and reliability of the modulation approach to detection of crack-like defects.

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### 1. Introduction

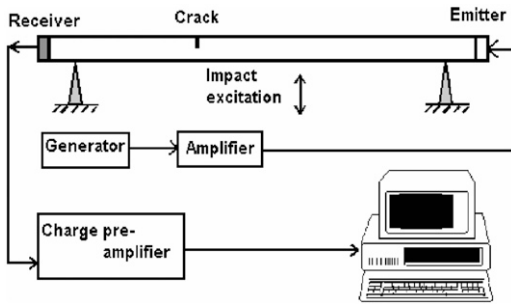
High interest to the nonlinear-modulation acoustic method of crack detection is significantly motivated by expectations to achieve superior detection sensitivity compared to other methods of nondestructive testing [1–10]. To ensure this goal, the main attention in the development of the nonlinear-modulation technique is usually paid to the reduction of various technical nonlinearities, whereas the masking effect of the atomic nonlinearity of the intact material is neglected. Apparently, this view is supported by the notion that the dimensionless quadratic nonlinear parameter  $\beta$  for homogeneous solids is on the order of  $10^0$  [11,12]. Therefore, for typical acoustic strains  $\varepsilon \leq 10^{-5}$ , the corresponding nonlinear correction  $\beta\varepsilon$  should have the relative level about  $-100$  to  $-80$  dB, which is indeed practically negligible. Furthermore, contributions of terms higher than quadratic seem to be beyond reasonable measurable values.

It has recently been argued [13] that under resonant conditions, such simplest quasistatic arguments can drastically underestimate the modulation level due to the atomic nonlinearity. More accurate resonant estimates of conventionally measured modulation components  $\omega \pm \Omega$  (where  $\omega$  and  $\Omega$  are the frequencies of the interacting weak probe and the intense pump waves, respectively) indicate that the contribution of technical nonlinearities of the modern equipment can be equal to or even less than the contribution of the quadratic atomic nonlinearity. Therefore, the atomic nonlinearity becomes the main factor, which limits the ultimate sensitivity of the conventional nonlinear-modulation approach. If the difference between the levels of the  $\omega \pm \Omega$  sidelobes for the studied samples is comparable with the natural variability of the background modulation (typically, 10–15 dB and even more [13]), it can be attributed to many factors other than the sample damage, which complicates the early detection of cracks.

As a natural way to improve the sensitivity and reliability of the nonlinear-modulation approach to crack detection, the possibility to intentionally use higher-order nonlinear interactions was mentioned in Ref. [13]. Indeed, for higher-order

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**Fig. 1.** Schematically shown configuration of nonlinear-modulation experiments [13–15] with sinusoidal probe wave at frequency  $\omega$  tuned in the range 50–70 kHz and impact-excited intense eigenmodes with frequencies  $\Omega_i$  typically ranged from a few hundreds Hz to a few kHz.

interactions, one can expect a drastic decrease in the masking signal caused by the atomic nonlinearity for which the power-series law is typical, such that each additional order in strain ensures  $\sim 80$ – $100$  dB additional reduction of the masking components due to the atomic nonlinearity. In contrast, the nonlinearity of cracks can significantly differ from the power-law type even for moderate acoustic amplitudes, so that the level of higher-order and lower-order modulational components induced by crack-like defects can be comparable. Therefore, one may expect that the observation of higher-order modulational interactions can significantly enhance the contrast between intact and crack-containing samples.

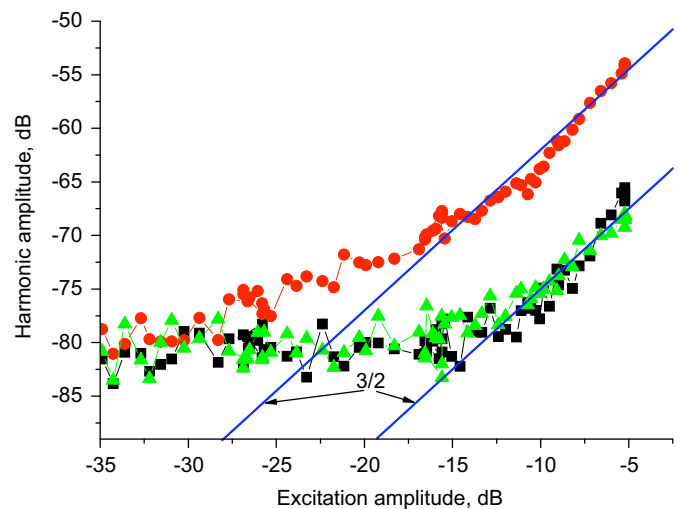
In what follows, we discuss in more detail this idea and make comparison with the same series of experiments as discussed in Refs. [13–15]. We recall that in those measurements (for which the experimental configuration is schematically shown in Fig. 1), favorable conditions for multi-wave interactions were ensured by the impact excitation of a significant number of intense (“pump”) low-number resonant eigenmodes of the sample at frequencies  $\Omega_i$ . Initially, those measurements were processed according to the conventional nonlinear-modulation approach by singling out the quadratic-type modulational components  $\omega \pm \Omega_i$  in the vicinity of the fundamental frequency  $\omega \gg \Omega_i$  of a sinusoidal probe ultrasound wave. By re-processing the experimental records, we demonstrate that more complex components of the  $\omega \pm \Omega_i \pm \Omega_j$  type can also be well observed. In what follows, we call them “cascade components” (instead of “higher-order”) to avoid the incorrect impression that their amplitude is of the next order of smallness in comparison with the conventionally discussed  $\omega \pm \Omega_i$  modulation sidelobes. We show that the cascade components can ensure noticeably higher contrast between the reference and crack-containing samples. We also show that the amplitude dependences for the cascade components  $\omega \pm \Omega_i \pm \Omega_j$  (i.e., formally cubic type) significantly differ from the scaling law expected for classical cubic nonlinearity, which can be used as an additional signature of the presence of crack-like defects. We demonstrate that the actual amplitude behavior of the cascade components can be fairly well modeled by clapping nonlinearity of inner Hertzian contacts in cracks.

**2. Preliminary note on indications of clapping Hertzian nonlinearity in acoustically driven crack-like defects**

For the further discussion, it is essential to recall that high softness of crack-like defects results in high increase of the local strain, which is the physical reason of the strongly increased nonlinearity of damaged samples. It can readily be shown that in the expansion of the defect’s equation of state in a power-series in

strain, the nonlinear terms of  $n$ th order increase as  $1/\zeta^{n-1}$ , where  $\zeta \ll 1$  is the small parameter describing the defect softness compared to the surrounding intact solid [16,17]. Formally this means that for strains  $\varepsilon \sim \zeta$ , the higher-order nonlinear terms should be of the same order as the lower-order terms. Actually, however, the power-series expansion is no more valid for such oscillation amplitudes, and the regime of the defect deformation becomes essentially non-analytical, so that nonlinear components of formally different orders can have amplitudes of the same order of smallness. For example, if Hertzian contacts at the crack interface begin to clap, then their nonlinearity can be approximated as  $\varepsilon^{3/2}H(\varepsilon)$  (where strain  $\varepsilon$  is considered positive for compression and  $H(\varepsilon)$  is a Heaviside function). Consequently, amplitudes of all higher harmonics produced by such nonlinearity under sinusoidal excitation  $\varepsilon = A \cos(t)$  exhibit the same functional dependence  $\propto A^{3/2}$ , which is easily seen by taking the Fourier transform of  $\varepsilon^{3/2}H(\varepsilon)$ .

Due to high softness of crack-like defects, such a non-analytical clapping regime for inner contacts can occur for fairly moderate average acoustic strains, e.g., on the order of  $\sim 10^{-5}$  and even less. Fig. 2 shows experimental examples of amplitude dependences of the 2nd, 3rd, and 4th harmonics in a sinusoidally excited resonant sample containing an artificial crack-like defect. The same power-law behavior with the exponent 3/2 for all harmonics (see the slopes of the solid lines) corresponds to the clapping Hertzian nonlinearity. The sample excited near its first resonance represented a glass rod (1 cm in diameter and about 20 cm in length) with one free and another acoustically rigid (due to cemented massive backload) boundary. Like in Ref. [18], the defect was modeled by a transversal diamond-saw-cut (1 mm in width and 4–5 mm in depth) in which a small metal plate was inserted. By slightly changing the plate position it was possible to observe different nonlinear regimes of the defect oscillations, including the contact-clapping regime, which could be attained at fairly moderate strains of  $10^{-6}$ – $10^{-5}$ . For the classical power-law nonlinearity, the higher harmonics should exhibit the quadratic, cubic, fourth-power, etc. dependences with pronouncedly different slopes in log-log scale. Certainly, the relative levels of the harmonics in Fig. 2 are affected by resonant properties of the sample, so that the resonant odd-type 3rd harmonic is higher than the non-resonant 2nd and 4th ones. When the inset was removed (which corresponded to the disappearance of the



**Fig. 2.** Amplitudes of the 2nd (squares), 3rd (circles), and 4th (triangles) harmonics in a rod-shape resonant sample with an artificial crack-like defect as functions of the fundamental-harmonic amplitude. The maximal strain of the harmonic excitation in the defect vicinity was  $5 \times 10^{-6}$ .

crack-like defect), all harmonics were hardly noticeable against the measurement noise.

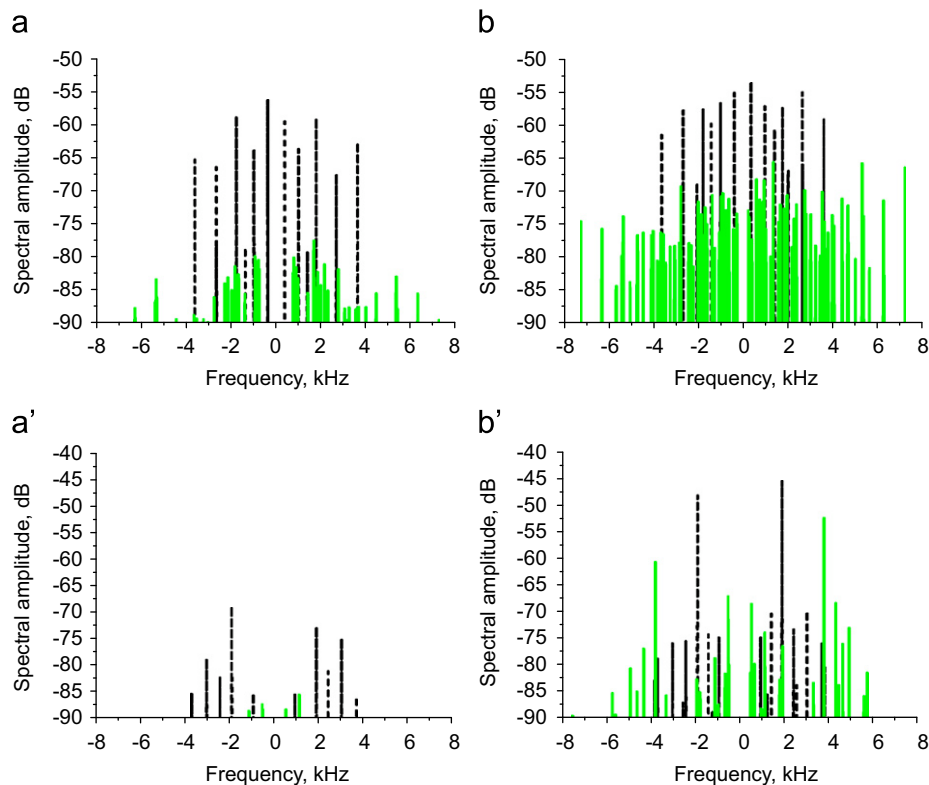
Returning to the discussed nonlinear-modulation experiments [13–15] with multi-frequency pump components  $\Omega_i, \Omega_j$ , etc., instead of different higher harmonics we compare the conventional modulation sidelobes  $\omega \pm \Omega_{ij}$  and cascade components  $\omega \pm \Omega_i \pm \Omega_j$ , which formally correspond to the second- and third-order nonlinear interactions, respectively. Thus for a power-series law typical of pure atomic nonlinearity, cubic components  $\omega \pm \Omega_i \pm \Omega_j$  should have practically immeasurable level (80–100 dB lower than the quadratic  $\omega \pm \Omega_{ij}$  components).

Examples of modulation spectra observed in the experiments [13–15] for defect-containing and reference samples (a railway-wheel axle and disk) are shown in Fig. 3, where conventional  $\omega \pm \Omega_{ij}$  components are plotted in black dashed lines and cascade components  $\omega \pm \Omega_i \pm \Omega_j$  are shown in a lighter color (solid lines). The expected positions of the sidelobes were calculated using the experimentally measured frequencies of the impact-excited pump eigenmodes in the lower-frequency part of the spectrum. The expected and actual frequencies of the sidelobes were considered coinciding within the double frequency resolution determined by the inverse width of the used time window. Typically its value was 0.2 s, which corresponded to 5 Hz spectral resolution. The width of the time window was intentionally limited to ensure the observation of the time dependences of different spectral components within the characteristic decay time (typically 0.8–1.0 s) of the impact-excited eigenmodes that produced the modulation of the sinusoidal probe wave. To ensure a stabler form of the modulation spectrum, the probe-wave frequency was tuned in the range 50–70 kHz typically in steps of 500 Hz and at each step identical impacts excited the pump eigenmodes. The spectra in Fig. 3 are averaged over 10–15 steps and correspond to

the middle of the entire time window of the observation. The probe-wave component is not explicitly shown: its amplitude is normalized to zero dB and its position on the frequency axis is shifted to zero frequency for each measurement step.

For an ideally linear measurement system (assuming that the background modulation was entirely produced by the atomic nonlinearity), the cascade components would not be visible at all in the scale of Fig. 3. The actual noticeable level of cascade components in the reference samples is evidently explained by the parasite nonlinearity of the measurement system. Nevertheless, in Fig. 3, the visual contrast between the reference and damaged samples is significantly higher for the cascade components  $\omega \pm \Omega_i \pm \Omega_j$  than for the conventional  $\omega \pm \Omega_{ij}$  modulation sidelobes.

To better quantify the difference between the spectra shown in Fig. 3 for the reference and damaged samples we use a histogram representation by analogy with Ref. [19]. Figs. 4 and 5 show the histograms in which the vertical axis corresponds to the summed energy for either conventional or cascade sidelobes whose amplitudes fall into certain amplitude range corresponding to the width of individual bins on the horizontal axis (in Figs. 4 and 5 the bin width is equal to 4 dB, although the exact value is not critical). Fig. 4 corresponds to the spectra shown in Fig. 3a and b obtained for the disks and Fig. 5 is for the spectra Fig. 3a' and b' obtained for the axle with an artificial crack-like defect and without it. The right panels in Fig. 4 demonstrate that the energies of conventionally considered  $\omega \pm \Omega_{ij}$  modulation sidelobes for the reference and defect-containing samples do not change drastically (since the extent of damage is intentionally chosen not very high). Their total energies of the  $\omega \pm \Omega_{ij}$  sidelobes summed over all bins differ by 5 dB only. Therefore, such conventionally used components do not allow one to make a reliable conclusion



**Fig. 3.** Examples of normalized averaged modulation spectra for the samples without defects (panels (a) and (a')) and with a defect ((b) and (b')). The upper row is for railway-wheel disks (the defect-containing disk has naturally developed fatigue damage). The lower row is for a railway-wheel axle (in which an artificial crack-like defect could be created by inserting a small steel plate in the saw-cut 7 mm in depth). Black dashed lines correspond to conventional modulation components  $\omega \pm \Omega_{ij}$ , and lighter solid lines are for the cascade components  $\omega \pm (\Omega_i \pm \Omega_j)$ .

















