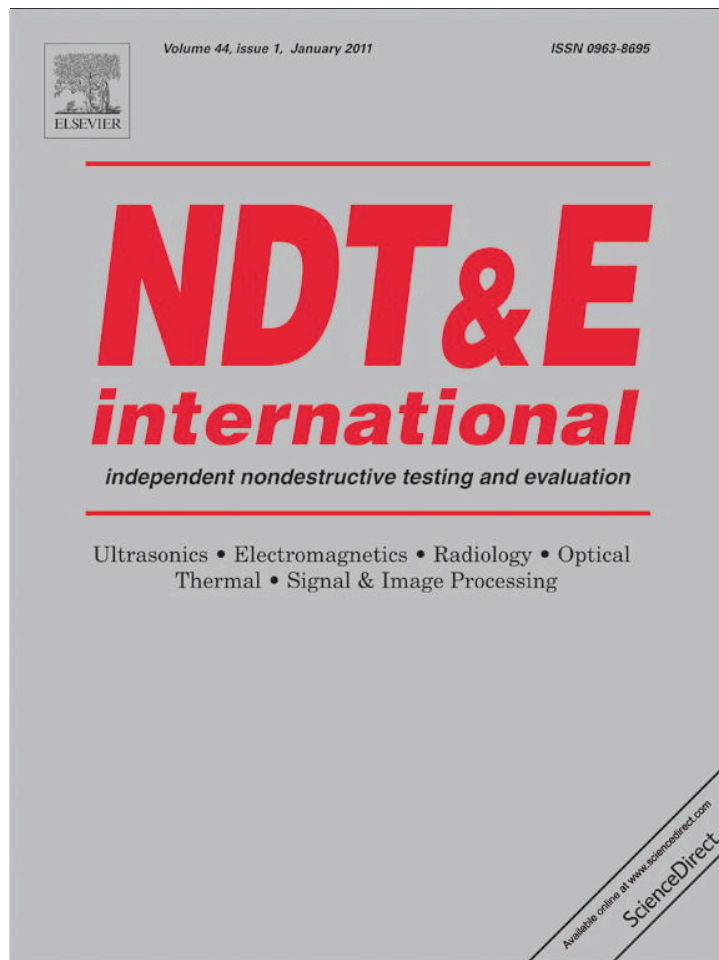


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## Elastic-wave modulation approach to crack detection: Comparison of conventional modulation and higher-order interactions

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### ABSTRACT

Comparison of recent theoretical estimates with experiments has indicated that the ultimate sensitivity of the conventional modulation technique of crack detection is mainly determined by the background modulation produced by the quadratic component of the atomic nonlinearity of the matrix material. Much smaller level of masking nonlinear effects is typical of higher-order interactions due to cubic and higher-order components in the power-series expansion of the background nonlinearity of the solid. In contrast, the level of formally higher-order components originated due to nonlinearity of crack-like defects can be comparable with that of the first-order components. Such strongly increased efficiency of higher-order interactions is due to the fact that crack-like defects often demonstrate non-analytic (non power-law) nonlinearity even for moderate acoustic amplitudes. Besides the increased level, the higher-order components arisen due to non-analytic nonlinearity of cracks can demonstrate significantly different functional behavior compared to manifestations of the atomic nonlinearity. This difference can also help to discriminate the contributions of the defects and the background atomic nonlinearity. Here, we focus on the main differences between the modulation components arisen due to cubic terms in the power-series expansion of the atomic nonlinearity and similar components generated by clapping Hertzian nonlinearity of inner contacts in cracks. We also examine experimental examples of higher-order modulation interactions in damaged samples. These examples clearly indicate non-analytical character of the defects' nonlinearity and demonstrate that the use of higher-order modulation effects can significantly improve the ultimate sensitivity and reliability of the modulation approach to detection of crack-like defects.

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### 1. Introduction

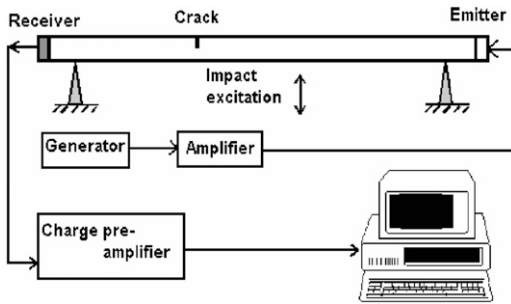
High interest to the nonlinear-modulation acoustic method of crack detection is significantly motivated by expectations to achieve superior detection sensitivity compared to other methods of nondestructive testing [1–10]. To ensure this goal, the main attention in the development of the nonlinear-modulation technique is usually paid to the reduction of various technical nonlinearities, whereas the masking effect of the atomic nonlinearity of the intact material is neglected. Apparently, this view is supported by the notion that the dimensionless quadratic nonlinear parameter  $\beta$  for homogeneous solids is on the order of  $10^0$  [11,12]. Therefore, for typical acoustic strains  $\varepsilon \leq 10^{-5}$ , the corresponding nonlinear correction  $\beta\varepsilon$  should have the relative level about  $-100$  to  $-80$  dB, which is indeed practically negligible. Furthermore, contributions of terms higher than quadratic seem to be beyond reasonable measurable values.

It has recently been argued [13] that under resonant conditions, such simplest quasistatic arguments can drastically underestimate the modulation level due to the atomic nonlinearity. More accurate resonant estimates of conventionally measured modulation components  $\omega \pm \Omega$  (where  $\omega$  and  $\Omega$  are the frequencies of the interacting weak probe and the intense pump waves, respectively) indicate that the contribution of technical nonlinearities of the modern equipment can be equal to or even less than the contribution of the quadratic atomic nonlinearity. Therefore, the atomic nonlinearity becomes the main factor, which limits the ultimate sensitivity of the conventional nonlinear-modulation approach. If the difference between the levels of the  $\omega \pm \Omega$  sidelobes for the studied samples is comparable with the natural variability of the background modulation (typically, 10–15 dB and even more [13]), it can be attributed to many factors other than the sample damage, which complicates the early detection of cracks.

As a natural way to improve the sensitivity and reliability of the nonlinear-modulation approach to crack detection, the possibility to intentionally use higher-order nonlinear interactions was mentioned in Ref. [13]. Indeed, for higher-order

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**Fig. 1.** Schematically shown configuration of nonlinear-modulation experiments [13–15] with sinusoidal probe wave at frequency  $\omega$  tuned in the range 50–70 kHz and impact-excited intense eigenmodes with frequencies  $\Omega_i$  typically ranged from a few hundreds Hz to a few kHz.

interactions, one can expect a drastic decrease in the masking signal caused by the atomic nonlinearity for which the power-series law is typical, such that each additional order in strain ensures  $\sim 80$ – $100$  dB additional reduction of the masking components due to the atomic nonlinearity. In contrast, the nonlinearity of cracks can significantly differ from the power-law type even for moderate acoustic amplitudes, so that the level of higher-order and lower-order modulational components induced by crack-like defects can be comparable. Therefore, one may expect that the observation of higher-order modulational interactions can significantly enhance the contrast between intact and crack-containing samples.

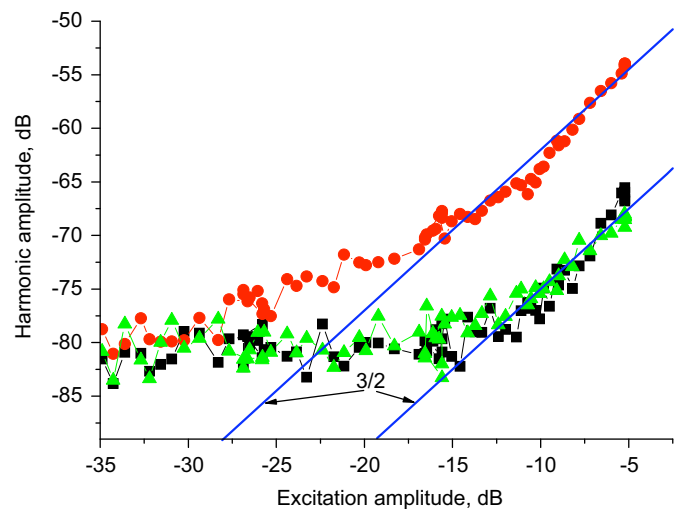
In what follows, we discuss in more detail this idea and make comparison with the same series of experiments as discussed in Refs. [13–15]. We recall that in those measurements (for which the experimental configuration is schematically shown in Fig. 1), favorable conditions for multi-wave interactions were ensured by the impact excitation of a significant number of intense (“pump”) low-number resonant eigenmodes of the sample at frequencies  $\Omega_i$ . Initially, those measurements were processed according to the conventional nonlinear-modulation approach by singling out the quadratic-type modulational components  $\omega \pm \Omega_i$  in the vicinity of the fundamental frequency  $\omega \gg \Omega_i$  of a sinusoidal probe ultrasound wave. By re-processing the experimental records, we demonstrate that more complex components of the  $\omega \pm \Omega_i \pm \Omega_j$  type can also be well observed. In what follows, we call them “cascade components” (instead of “higher-order”) to avoid the incorrect impression that their amplitude is of the next order of smallness in comparison with the conventionally discussed  $\omega \pm \Omega_i$  modulation sidelobes. We show that the cascade components can ensure noticeably higher contrast between the reference and crack-containing samples. We also show that the amplitude dependences for the cascade components  $\omega \pm \Omega_i \pm \Omega_j$  (i.e., formally cubic type) significantly differ from the scaling law expected for classical cubic nonlinearity, which can be used as an additional signature of the presence of crack-like defects. We demonstrate that the actual amplitude behavior of the cascade components can be fairly well modeled by clapping nonlinearity of inner Hertzian contacts in cracks.

**2. Preliminary note on indications of clapping Hertzian nonlinearity in acoustically driven crack-like defects**

For the further discussion, it is essential to recall that high softness of crack-like defects results in high increase of the local strain, which is the physical reason of the strongly increased nonlinearity of damaged samples. It can readily be shown that in the expansion of the defect’s equation of state in a power-series in

strain, the nonlinear terms of  $n$ th order increase as  $1/\zeta^{n-1}$ , where  $\zeta \ll 1$  is the small parameter describing the defect softness compared to the surrounding intact solid [16,17]. Formally this means that for strains  $\varepsilon \sim \zeta$ , the higher-order nonlinear terms should be of the same order as the lower-order terms. Actually, however, the power-series expansion is no more valid for such oscillation amplitudes, and the regime of the defect deformation becomes essentially non-analytical, so that nonlinear components of formally different orders can have amplitudes of the same order of smallness. For example, if Hertzian contacts at the crack interface begin to clap, then their nonlinearity can be approximated as  $\varepsilon^{3/2}H(\varepsilon)$  (where strain  $\varepsilon$  is considered positive for compression and  $H(\varepsilon)$  is a Heaviside function). Consequently, amplitudes of all higher harmonics produced by such nonlinearity under sinusoidal excitation  $\varepsilon = A \cos(t)$  exhibit the same functional dependence  $\propto A^{3/2}$ , which is easily seen by taking the Fourier transform of  $\varepsilon^{3/2}H(\varepsilon)$ .

Due to high softness of crack-like defects, such a non-analytical clapping regime for inner contacts can occur for fairly moderate average acoustic strains, e.g., on the order of  $\sim 10^{-5}$  and even less. Fig. 2 shows experimental examples of amplitude dependences of the 2nd, 3rd, and 4th harmonics in a sinusoidally excited resonant sample containing an artificial crack-like defect. The same power-law behavior with the exponent 3/2 for all harmonics (see the slopes of the solid lines) corresponds to the clapping Hertzian nonlinearity. The sample excited near its first resonance represented a glass rod (1 cm in diameter and about 20 cm in length) with one free and another acoustically rigid (due to cemented massive backload) boundary. Like in Ref. [18], the defect was modeled by a transversal diamond-saw-cut (1 mm in width and 4–5 mm in depth) in which a small metal plate was inserted. By slightly changing the plate position it was possible to observe different nonlinear regimes of the defect oscillations, including the contact-clapping regime, which could be attained at fairly moderate strains of  $10^{-6}$ – $10^{-5}$ . For the classical power-law nonlinearity, the higher harmonics should exhibit the quadratic, cubic, fourth-power, etc. dependences with pronouncedly different slopes in log-log scale. Certainly, the relative levels of the harmonics in Fig. 2 are affected by resonant properties of the sample, so that the resonant odd-type 3rd harmonic is higher than the non-resonant 2nd and 4th ones. When the inset was removed (which corresponded to the disappearance of the



**Fig. 2.** Amplitudes of the 2nd (squares), 3rd (circles), and 4th (triangles) harmonics in a rod-shape resonant sample with an artificial crack-like defect as functions of the fundamental-harmonic amplitude. The maximal strain of the harmonic excitation in the defect vicinity was  $5 \times 10^{-6}$ .



















