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ACOUSTICS OF STRUCTURALLY INHOMOGENEOUS SOLID MEDIA. GEOLOGICAL ACOUSTICS

Relation between the Tidal Modulation of Seismic Noise and the Amplitude-Dependent Loss in Rock

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Abstract—Geological materials and many other microinhomogeneous media exhibit pronounced nonlinear properties under very small strains, when one may expect an almost linear behavior of the material. These properties are conventionally described on the basis of elastically nonlinear or hysteretic models. The present paper discusses the amplitude-dependent dissipation that is unrelated to hysteretic nonlinearity but is also a universal property of microinhomogeneous media. This property allows the explanation of the effect of correlation between the tidal strains of the Earth's crust (on the order of 10^{-8}) and the unexpectedly strong (on the order of 10^{-2}–10^{-1}) variations of seismic noise intensity, which has been observed for more than 25 years without being given any adequate interpretation.

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INTRODUCTION

Qualitatively unusual and strongly enhanced nonlinear acoustic properties are inherent in a wide class of contact- and crack-containing media (including almost all types of rock). Laboratory experiments with these media indicate an increase in nonlinear elastic parameters by a factor of several hundred or even several thousand in comparison with homogeneous media [1–3]. Still higher estimates for nonlinear parameters follow from some field observations (for example, data on the effect of tidal strains of the Earth's crust on the velocity of seismoacoustic waves emitted by high-stability sources [4, 5]). It is also found that, apart from purely elastic nonlinearity, cracks and contacts in the structure of a material may lead to a hysteretic nonlinearity [2, 3], "memory" [6], and other "fast" nonlinear effects, as well as effects of slow dynamics [2, 3, 6–9], the understanding of which has considerably progressed in the last few years.

The effect of seismic noise modulation by tidal strains [10, 11] is one of the exclusions. No convincing interpretation was proposed for this effect despite the increasing amount of reliable independent observations (e.g., [12, 13]). Such a long monitoring [12, 13] was conducted in the regions far from human activity (at the Kamchatka and Hokkaido stations) to exclude the influence of anthropogenic factors. Narrow-band seismic receivers were positioned at the depth down to several tens of meters to exclude the effect of surface temperature variations and winds (the storm periods were excluded from the data, if necessary). The records were processed coherently using a running observation window and coherent summation of noise (intensity) envelopes to separate the periodic variations corresponding to the periods of tidal strains that are known to a high accuracy [12, 13]. As the result of this accumulation with a typical length of several months, the intensity variations with the periodicity corresponding, for example, to tidal components [14] O_1 (the diurnal principal lunar wave, period $T = 25.82$ h), M_2 (the semi-diurnal lunar principal wave, period $T = 12.42$ h), and N_2 (the semi-diurnal major elliptic lunar wave, period $T = 12.66$ h) were separated statistically reliably. Thus, the effects of other potentially possible factors with almost 24-h and 12-h periodicity were reliably excluded. The typical observed depth of modulation for noise varied from 2–3 to 6–8% (Fig. 1). A too large accumulation time (e.g., over 2–2.5 years) caused a decrease in the level of the separated periodic components, which was connected with the long-term instability of the modulation character under the conditions of slowly varying background tectonic stresses.

Let us note briefly the possible ways for interpreting these data. The major difficulty is connected with the seemingly too large modulation depth of the noise level $\sim 10^{-2}$–10^{-1} in comparison with the characteristic amplitude of tidal strains 10^{-8}. In the case of such a level, the reliably determined highly increased elastic nonlinearity of rock provides an opportunity to interpret only the observed tidal variations of the velocities of seismoacoustic waves with a level of 10^{-5}–10^{-3} [4, 5, 15, 16], but this does not help to explain the tidal variations of the seismic noise level that are higher by 2–4 orders of magnitude.

Moreover, in the case of rock, the “nonclassical” hysteretic nonlinearity is also typical [2]. It manifests itself as the hysteresis of the quasi-static stress–strain dependence. (Note that here we do not discuss linear viscous losses, which also lead to a phase shift between the current values of stress and strain. It is also called a hysteresis sometimes.) The “genuine” quasi-static hysteretic nonlinearity (for example, caused by adhesion or friction phenomena at microstructure defects of the medium) basically may lead to the variation of dissipation for one oscillation in the presence of another one [17]. However, the induced variation of dissipation for this hysteretic nonlinearity is very small at a large difference between the frequencies of interacting oscillations [17]. Since the frequency ratio for the noise and the tidal strains discussed here is about 10^6–10^7, the tidal variations of loss induced by the hysteretic nonlinearity are negligibly small for high-frequency noise.

An alternative assumption concerning the effect of tides not on the propagation conditions of the noise but rather on its sources also does not seem sufficiently justified, at least as the only important mechanism. Indeed, observations [12, 13] of pronounced tidal modulation of noise were conducted in the regions of increased seismic activity, where several strong earthquakes occurred after the beginning of regular noise monitoring (1992). Pronounced noise modulations were observed at different phases of seismic activity (both before and after earthquakes), when the background stresses of rock should be different. In this connection, an assumption on possible initiation of microdamages and acoustic emission accompanying them (that may be hypothetically assumed for a material exactly at the threshold of fracture) by weak tidal strains clearly cannot be recognized as the major factor for the observed modulation. Therefore, to interpret the entire ensemble of these observations, another, more universal and robust (in the sense of its existence conditions) mechanism is needed.

An explanation of this kind could be based on the existence of one more mechanism of amplitude-dependent loss in the medium, which is sufficiently sensitive to very weak tidal strains. It is desirable that this mechanism would not require a finite threshold in strain (in contrast to hysteresis and adhesion phenomena requiring the value of shifts of contacting medium elements to exceed the atomic size for their activation). The efficiency of the desired mechanism must provide the decrement variations for seismoacoustic perturbations at a level of ~10^{-2}–10^{-1} under the action a quasi-static strain of about ~10^{-8}, which is typical of tides. Note that recent observations of tidal amplitude modulation for seismic waves produced by high-stability sources [15, 16] independently indicate the existence of such a mechanism. For the same reasons that were noted above while discussing the noise modulation, neither purely elastic nor hysteretic nonlinearity can explain the considerable amplitude modulation observed in these experiments. Below, we will explain how well-known and widely discussed microstructure properties of rock and other similar microinhomogeneous media lead not only to a strong increase in nonlinear elasticity (and hysteresis when the friction-adhesion phenomena are taken onto account), but also give rise to a pronounced amplitude-dependent dissipation unrelated to the hysteretic nonlinearity.

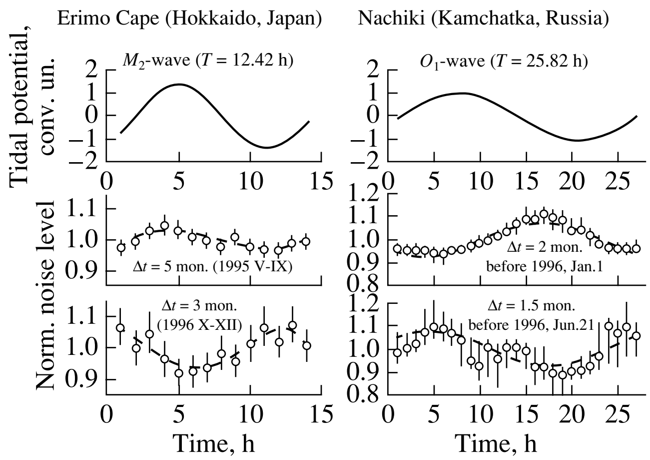

Fig. 1. Examples of time variability of seismic noise intensity that corresponds to the periodicity of tidal strains. Noise with the strain amplitude on the order of 10^{-12}–10^{-14} (i.e., much smaller that the typical amplitude of tidal strains 10^{-8}) was detected in the experiments. The reception was performed by a narrowband seismic receiver with a central frequency of 30 Hz and a quality factor Q = 100 [12, 13].

MECHANISM OF THE NONHYSTERETIC AMPLITUDE-DEPENDENT LOSS DUE TO THE PRESENCE OF “SOFT” DEFECTS IN MATERIAL’S STRUCTURE

Today, it is conventionally believed [1–5] that the greatest strains (and strain rates), which lead to both a strong increase in the elastic nonlinearity of a medium and a growth of dissipation in comparison with a homogeneous material, are localized at these defects because of the local compressibility increase. However, the amplitude-dependent dissipation in these media, which is not connected with the conventionally discussed amplitude-dependent loss due to the quasi-static hysteresis, is usually ignored.

To explain the nature of this dissipation not connected with the hysteretic nonlinearity, let us use an instructive rheological model of a microinhomogeneous medium (Fig. 2). Here, we restrict ourselves to the one-dimensional form of the model [18], which is sufficient for our consideration. In the three-dimensional case, it is possible to introduce analogous differently oriented soft defects (as is done, for example, in [19]). To reveal the amplitude-dependent dissipation, it

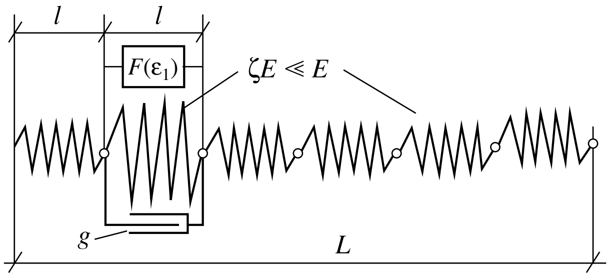

Fig. 2. Rheological model for a microinhomogeneous elastic medium with soft defects. The function $F(\ldots)$ and the parameter g describe their elastic nonlinearity and effective viscosity (for example, on account of thermoelastic and real viscous losses in the presence of a filling liquid). The number of defects is characterized by their linear concentration $\nu = l/L$ in the one-dimensional case or relative volume content in the case of generalization to the three-dimensional situation.

is sufficient to take into account the nonlinear character of strain and the presence of losses only at soft defects, while ignoring the nonlinearity and losses in the matrix medium, as is shown in Fig. 2. In this case, the equation of state for defects can be written in the form $\sigma = \varsigma E[\varepsilon_1 + F(\varepsilon_1)] + g d\varepsilon_1/dt$, where the elastic stress σ in the medium is connected with the local strain ε_1 of a soft defect, the value of ε_1 being considerably greater than the average strain ε of the material because of high softness of the defect characterized by the small parameter $\varsigma \ll 1$. The elasticity modulus E characterizes a homogeneous matrix medium. To discuss the effects observed at very small strains, it is sufficient to take into account the lower order of the elastic nonlinearity of defects that is quadratic in strain ε_1: $F(\varepsilon_1) = \gamma\varepsilon_1^2$. We consider the nonlinearity parameter γ to be negative, since, under the effect of the tensile stress $\varepsilon > 0$, a material usually becomes softer. It is necessary to indicate that, in terms of the local strain ε_1 of a defect, its nonlinearity has a quite common low level; i.e., the local nonlinearity parameter γ has a value of about several units. Assuming that a large number of defects is contained in a microinhomogeneous medium within the length of an elastic wave (though, the correction introduced by them to the elastic modulus E of the matrix medium is still small) and summing the contributions of soft and relatively rigid elements, it is easy to obtain the following expression for the relation between the macroscopic strain and the elastic stress in the medium [18]:

$$\sigma(\varepsilon) = E\varepsilon - E\Omega\int d\varsigma\nu(\varsigma)\int_{-\infty}^{t}\varepsilon(\tau)e^{-\varsigma\Omega(t-\tau)}d\tau$$
$$+ E\Omega\int d\varsigma\nu(\varsigma)\varsigma\int_{-\infty}^{t}e^{-\varsigma\Omega(t-\tau)}F\left[\Omega\int_{-\infty}^{\tau}\varepsilon(\tau')e^{-\varsigma\Omega(\tau-\tau')}d\tau'\right]d\tau, \quad (1)$$

where $\Omega = E/g$, so that $\zeta\Omega$ corresponds to the characteristic relaxation frequency of defects and the function $\nu(\varsigma)$ describes their distribution in the softness parameter ς The term $E\varepsilon$ in Eq. (1) corresponds to the contribution of the homogeneous matrix medium, the second relaxation term describes the linear contribution made by defects to elasticity and dissipation, and the last term corresponds to the nonlinear contribution of defects. In the low-frequency limit, where the characteristic frequencies ω of strain lie much lower than $\zeta\Omega$, the last nonlinear term describes a common "instant" nonlinear elastic response of the medium. The presence of a relaxation operator in the nonlinear term describes the relaxation "freezing" of the defect's response to fast varying actions with the frequencies $\omega \gg \zeta\Omega$. This means that, in the case of microinhomogeneous media, a pronounced dispersion of their nonlinear properties must appear [18].

In addition, the last nonlinear term describes a pronounced nonlinear dissipation as well [20]. Let us compare the contributions of the last nonlinear-relaxation term in Eq. (1) to the variation in the elastic modulus and the complementary variation of the damping rate in the microinhomogeneous medium. We assume that a static strain ε_0 and an oscillating strain ε_ω at the frequency ω are produced in the medium (a quadratic approximation is sufficient to evaluate the role of small quasi-static tidal strains). For clarity of the results, we take identical values of ς for the softness parameters of defects. Separating the real and imaginary parts of the oscillating stress under the above assumptions, we obtain from Eq. (1) the following expressions for the effective elastic modulus E_{eff} of the medium and the decrement θ for the oscillating component of strain in the presence of a quasi-static action ε_0:

$$E_{\text{eff}}/E \approx 1 - \frac{\nu}{\zeta}\frac{1}{1+\bar{\omega}^2} - 2\frac{\nu|\gamma|\varepsilon_0}{\zeta^2}\frac{1-\bar{\omega}^2}{(1+\bar{\omega}^2)^2}, \quad (2)$$

$$\theta = \theta_{\text{lin}} + \theta_{\text{nl}} \approx \pi\frac{\nu}{\zeta}\frac{\bar{\omega}}{1+\bar{\omega}^2} + 2\pi\frac{\nu|\gamma|\varepsilon_0}{\zeta^2}\frac{2\bar{\omega}}{(1+\bar{\omega}^2)^2}, \quad (3)$$

where the normalized frequency $\bar{\omega} = \omega/(\varsigma\Omega)$ is introduced and a reasonable assumption already taken into account in Eq. (1) is used that one may ignore the contribution made by the homogeneous matrix medium to absorption. For identical defects, their linear contribution to the decrement θ_{lin} demonstrates a well-known relaxation peak corresponding to $\omega = \varsigma\Omega$. Taking into account a more realistic wide and smooth distribution of defects $\nu(\varsigma)$ in their softness (and relaxation frequencies) provides an opportunity to obtain [19, 20] the decrement value that is approximately constant in a wide frequency range, which, as is known, is typical of rock and many other microinhomogeneous media. However, simple equations (2) and (3) with a single relaxation time give an especially clear understanding of interconnections between the linear and nonlinear (in our case, depending on the applied strain ε_0) elastic and dissipative properties of the material, which are usually excluded from consideration in much more complex models of the medium that are used in acoustics and geophysics.

The last term in Eq. (3) shows that the well-known properties taken into account in the model lead to a pronounced amplitude-dependent dissipation. As one can see from Eqs. (2) and (3), linear and nonlinear contributions of defects (both reactive elastic and dissipative) are determined by the same parameters in similar combinations. However, in this case, there is the following essential difference: defects make the nonlinear contributions equal in the order of magnitude to the relative variation of the elastic modulus E_{eff}/E and to the absolute (and not relative) variation of the decrement θ. Therefore, since the decrement is almost always much smaller than unity ($\theta \ll 1$) even in media with defects (such as rock), it turns out that the amplitude-dependent (depending on ε_0 in our case) relative variations of the decrement $\Delta\theta/\theta \sim \theta_{\text{nl}}/\theta_{\text{lin}}$ are much greater that the variation of the elastic modulus accompanying them: $\Delta\theta/\theta \gg \Delta\theta \sim \Delta E/E$.

COMPARISON OF THE MODEL WITH THE DATA ON TIDAL MODULATION OF WAVES EMITTED BY HIGH-STABILITY SEISMIC SOURCES

Before turning directly to the modulation of seismic noise, let us verify the validity of the choice of the parameters for the model medium by comparing it with the data on the tidal modulation of radiation from artificial seismic sources [5, 6, 15, 16]. To make necessary estimates, the parameters introduced into the model can be readily compared with the properties of real defects in geological materials. For example, as is well known [5], different models of cracks predict that cracks with the aspect ratio α (i.e., the ratio of the crack opening to its characteristic length) can be closed if a macroscopic strain $\varepsilon_0 \sim \alpha$ arises in the medium. This means that, in our terms, the softness index of such a defect is $\varsigma \sim \alpha$. In this case, the values $\alpha \sim 10^{-4}$–10^{-5} [5] are quite typical of cracks in real rock. Moreover, it is necessary to take into account that real cracks are not just smooth cuts but have rough surfaces. This wavy roughness creates elongated regions in cracks, which produce (or can produce under a small additional compression) elongated strip-like contacts. The fact important for our discussion is that, in the region of this roughness, a contact between crack surfaces may arise (or, on the contrary, disappear) even in the case when the medium's strain much smaller (e.g., by a factor of 10–100) than the strain necessary for a complete closing (opening) of a crack [8, 9]. It is essential in this case that the rigidity introduced by this contact with an extended strip may be already compared to the arch rigidity of the whole crack [21]. This means that, for a crack with the aspect ratio $\alpha \sim 10^{-4}$–10^{-5} and extended contacts inside (their presence is rather a rule for real cracks) the effective rigidity can be changed by ~50% in the case of producing a strain $\varepsilon_0 \sim 10^{-6}$–$10^{-7}$ in the medium; i.e., 10–100 times smaller than the strain $\varepsilon_0 \sim \alpha \sim 10^{-4}$–$10^{-5}$ needed for a complete closing (opening) of a crack. Therefore, in terms of the discussed model, the effective softness parameter ς for such a crack with a contact, which characterizes its sensitivity to the strain produced in the medium, is already not on the order of magnitude of the aspect ratio $\alpha \sim 10^{-4}$–10^{-5}, but may reach the values $\varsigma \sim 10^{-6}$–10^{-7}. It is necessary to stress that, within the framework of the conventionally used understanding about a crack in the form of a cut, these values of the softness parameters seem to be unrealistic, since they require the corresponding unreal small values of the aspect ratio, at which cracks will be completely closed by minor external stresses.

We note that linear thermoelastic losses (which are orders of magnitude higher at soft contacts and crack-like defects because of locally higher strains and gradients of temperature variations produced by them [8, 9]) can be compared to the linear relaxation absorption introduced into the model at the rheological level. These can be also common viscous losses produced by the presence of a liquid in a defect. However, to obtain further estimates, a detailed knowledge of the nature of a corresponding defect is not needed, since these losses are taken into account in the model via characteristic relaxation frequencies of relaxing defects and their concentration ν.

The ratio ν/ς necessary for further consideration, which is contained in both linear and nonlinear terms connected with the presence of defects, can be estimated using the values of linear absorption typical of rock. In fact, assuming the typical value of the quality factor $Q \equiv \pi/\theta \approx 100$ for the relaxation peak described by the linear term in Eq. (3), we obtain $\nu/\varsigma = 2 \times 10^{-2}$. For example, $\nu = 2 \times 10^{-6}$ for $\varsigma = 10^{-4}$ and $\nu = 2 \times 10^{-7}$ for $\varsigma = 10^{-5}$. Further, to estimate the values of nonlinear terms, we take the values of the local (intrinsic) nonlinearity parameter γ for defects at the "common" nonlinearity level $\gamma = 3$–8 that is typical of a homogeneous medium [22]. This is sufficient to obtain the estimates for the value of the macroscopic nonlinearity parameter on the basis of Eqs. (2) and (3), $\gamma_{\text{macro}} = 2\gamma\nu/\varsigma^2 = 800$–$2400$ for $\varsigma = 10^{-4}$ and $\gamma_{\text{macro}} = (8$–$24) \times 10^3$ for $\varsigma = 10^{-5}$. Assuming (as is explained above) the presence of defects with even greater effective softness, for example, $\varsigma = 10^{-6}$, we obtain even higher values for the macroscopic parameter of quadratic nonlinearity $\gamma_{\text{macro}} \sim 10^5$–$10^6$ for the same value of ν/ς.

It is necessary to indicate at this stage that the initial Eq. (1) was obtained [18] for a rather small concentration of defects $\nu \ll \varsigma$, the increase in their concentration producing only a limited growth of the nonlinearity parameter. The growth of nonlinearity at an arbitrary concentration ν was analyzed in [23] using a simpler quasi-static approximation of the model (i.e., ignoring relaxation, $\bar{\omega} \longrightarrow 0$). It was demonstrated that, at $\nu \sim \varsigma$, the quadratic macroscopic parameter passes through the maximum $\gamma_{macro}^{max} = \gamma\varsigma/4$ and decreases again with the growth of the defect concentration. Moreover, one has to understand that the "giant" [3] values of the nonlinearity parameter (for example, $\gamma_{macro} \sim 10^6$) are meaningful only for sufficiently small strain amplitudes until $\varepsilon < \varsigma$, since stronger strains just close soft defects and the nonlinearity parameter of the material turns out to be lower than that in the case of smaller strains (see analogous discussion for the role of soft cracks in [5]). In this sense, the tidal strains of about 10^{-8} that are of interest to us are still sufficiently small even for the effective softness of defects $\varsigma \sim 10^{-6}$–10^{-7}.

Thus, the above estimates for the nonlinearity of a microinhomogeneous medium agree well not only with numerous laboratory data indicating the values of the effective nonlinear parameters on the order of 10^3–10^4 (see [2, 3] and the literature cited there), but also with even higher values following from in situ seismic observations [4, 5]. In particular, in terms of the macroscopic nonlinearity parameter γ_{macro}, the data [4, 5] obtained for rock with a strongly damaged structure correspond to the values $\gamma_{macro} \geq 10^5$. The value of γ_{macro} that is close in order of magnitude (but still 4–5 times smaller) can be basically obtained under the assumption that all existing defects have the softness index $\varsigma = 10^{-5}$ (compare with the discussion on the necessity in the presence of cracks with the aspect ratio $\alpha = 10^{-5}$ in [5]). However, in this case, one more unrealistic assumption is required that the concentration of these identical and very thin crack defects must correspond to the value optimal for the growth of γ_{macro} [23]. In reality, these assumptions on the optimal concentration and equal and very small aspect ratio $\alpha = 10^{-5}$ for cracks are impracticable. Nevertheless, the observed high values of $\gamma_{macro} \geq 10^5$ are adequately explained by the presence of a certain part of very soft defects with the efficient softness $\varsigma \sim 10^{-6}$–10^{-7}. As was already noted above, this value can be provided on account of cracks containing contacts with quite real values of the aspect ratio $\alpha \sim 10^{-5}$–10^{-3}, which, at the effective softness of defects $\varsigma \sim 10^{-6}$–10^{-7}, allow them not to close even at average strains of the medium that are much greater than $\varepsilon \sim \varsigma \sim 10^{-6}$–$10^{-7}$.

Now, let us discuss the variations of elasticity and variations of absorption accompanying them in the medium, which were revealed recently in observations [15, 16] of tidal amplitude-phase modulation of radiation from stable sources. For example, the data of the interwell observations [16] obtained on a path with a length of 360 m at a wave frequency of 167 Hz and a wave propagation velocity of ~3000 m/s demonstrated a tidal modulation of the wave phase ~0.05 rad and, similar to [5, 6], showed the values $\gamma_{macro} \sim (1–2) \times 10^5$ for the parameter of elastic nonlinearity involved in Eq. (2). For these values of γ_{macro}, Eq. (3) predicts the value for the accompanying variations of the decrement $\theta_{nl} \sim (2–5) \times 10^{-3}$ at the amplitude of tidal strain equal to 2×10^{-8}, which corresponds to a 10–20% variation of the amplitude of the received wave under the experimental conditions [16] and agrees well with the observed value of ~10%.

In other independent studies [15] of long-range (356 and 430 km) propagation of waves with frequencies of 5–8 Hz, somewhat smaller values of variations were observed (1°–2° in phase and 2–4% in wave amplitude). These data at a distance of about 600–800 wavelengths correspond to the relative variation of the elastic modulus $\sim 10^{-5}$ and decrement variations at the level of $\theta_{nl} \sim 3 \times 10^{-5}$. Both values indicate the value of $\gamma_{macro} \sim 500–700$ averaged over the track. Although this value still considerably exceeds (by two orders of magnitude) the value typical of homogeneous media, it is much smaller than the estimates based on the data of [5, 6, 16]. This difference is not surprising, since, in the experiments [15], the propagation depth of waves reached several tens of kilometers, where soft defects responsible for the growth of nonlinearity are already closed because of the pressure of upper layers.

COMPARISON WITH THE DATA ON TIDAL MODULATION OF ENDOGENOUS SEISMIC NOISE

After evaluating the tidal effect on the propagation of stable signals excited artificially, let us evaluate the expected depth of tidal modulation of endogenous noise. For this purpose, we take into account the fact that, for a receiver positioned at the origin of coordinates and tuned to the spectral component at the frequency ω, which is radiated by a noise source located in the ith elementary volume with the center at the point $\mathbf{r}_i$, the corresponding amplitude $A_i(\omega)$ at the point of reception has the form

$$A_i = \frac{A_0^i f_i(\phi)}{r_i}\exp\left[-\theta(\omega)\frac{r_i}{\lambda}\right], \quad (4)$$

where λ is the elastic wavelength corresponding to the frequency ω. Equation (4) takes into account the spherical divergence and the exponential attenuation of a signal, and the function $f_i(\phi)$ of the spatial angle ϕ describes the possible angular directivity of the source. The resulting spectral intensity $I(\omega)$ of noise is determined by summing (integrating) the contributions of all ambient sources. In this case, averaging over directions produces a certain factor independent of the properties of the medium, and the summation over the volume of sources is represented in the form of an integral over the radial coordinate:

$$I(\omega) \propto \sum_i A_i^2 \propto \int_0^\infty \left(\frac{\exp\left[-\theta(\omega)\frac{r}{\lambda}\right]}{r}\right)^2 r^2 dr \propto \frac{\lambda}{\theta(\omega)}. \quad (5)$$

One can see from the structure of Eq. (5) that the resulting value of $I(\omega)$ for the given intensity of sources is determined by the contribution from the region with the size limited by the characteristic length of attenuation λ/θ. Hence, at a given level of sources in the medium, where an external influence modulates its elastic-dissipative properties, the relative intensity variation of received noise is determined first of all by the variation of dissipation:

$$\frac{\Delta I(\omega)}{I(\omega)} \propto \Delta\left(\frac{\lambda}{\theta(\omega)}\right) \Big/ \left(\frac{\lambda}{\theta(\omega)}\right) = \frac{\Delta\lambda}{\lambda} - \frac{\Delta\theta}{\theta} \approx \frac{\Delta\theta}{\theta}. \quad (6)$$

In Eq. (6), we take into account the fact that, for microinhomogeneous media, $\Delta\lambda/\lambda \approx \Delta E(2E) \ll \Delta\theta/\theta$, as is explained above.

To estimate the decrement variations $\Delta\theta/\theta$, we turn to Eq. (3), which demonstrates that, at a given value of ε_0 of the quasi-static perturbation of the medium, the relative decrement variations $\Delta\theta/\theta = \theta_{nl}/\theta_{lin} \sim 2|\gamma|\varepsilon/\zeta$ are determined in fact by only the intrinsic nonlinearity of defects γ and their effective softness ζ and do not depend on the concentration of defects ν. For estimation, let us assume a moderate value $|\gamma| = 5$ for the parameter of intrinsic defect nonlinearity and select the value for the softness parameter ζ within the range 10^{-5}–10^{-6}, as was done above while comparing with the data on the tidal modulation of fields from artificial sources [5, 6, 15, 16]. In this case, for the amplitude of tidal strains $\varepsilon_0 \sim 10^{-8}$, the expected value of the induced variations of endogenous noise is $\Delta I(\omega)/I(\omega) \sim 0.01$–$0.1$ for $\zeta = 10^{-5}$–10^{-6}. This estimate agrees well with the characteristic depth of observed variations [12, 13] (Fig. 1).

Remember that, so far, we used simplified equations (2) and (3) obtained for identical defects. More realistic wide distributions $\nu(\varsigma)$ smooth out the frequency dependences of both linear and amplitude-dependent components of the decrement, although the contribution of the softest defects prevails in the variation of the last, while simple estimates obtained on the basis of the assumption on the identical character of defects already give a correct value for the relative variability $\Delta\theta/\theta$ [20].

CONCLUSIONS

The proposed model of a microinhomogeneous medium, which takes into account several common well-known properties of rock and similar microinhomogeneous media, provided an opportunity to obtain estimates for nonlinearly elastic properties that agree well with both laboratory data on their "giant" [3] nonlinearity and in situ experiments on the tidal modulation of the velocity of seismic waves from artificial high-stability sources [5, 6, 15, 16].

In addition, a conclusion on the accompanying variations of not only elastic, but also dissipative properties of the medium, which are caused by the nonlinearity of defects, naturally follows from the model under consideration. These variations of dissipation turn out to be significant even in the acoustic range of strains $\varepsilon \leq 10^{-5}$ [20]. They arise due to the combined action of elastic nonlinearity and linear absorption. Therefore, this model of the medium provides an opportunity to interpret not only the data on the pronounced tidal modulation of velocities, but also the amplitudes of signals from stable seismic sources [15, 16], which could not be explained within the framework of nonlinear elastic models and needed phenomenological introduction of the amplitude-dependent losses of different origin [16].

Finally, the model makes it possible to propose a mechanism and obtain estimates that agree well with observations (at the same parameters "calibrated" according to the data for artificial signals) of the effect of tidal modulation of endogenous seismic noise [10–13], which was known for over 25 years and had no adequate physical explanation. For many acoustic and seismic experimental situations, the aforementioned amplitude-dependent losses must coexist with the manifestations of commonly discussed hysteretic nonlinearity [2, 17]. Identification and separation of contributions from these mechanisms on the basis of the predicted amplitude–frequency differences extends the prospects discussed in [2, 3, 24] for diagnostic applications of nonlinear acoustic and seismic effects.

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