ISSN 1063-7710, Acoustical Physics, 2008, Vol. 54, No. 3, pp. 398–406. © Pleiades Publishing, Ltd., 2008.
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ACOUSTICS OF STRUCTURALLY INHOMOGENEOUS SOLID MEDIA. GEOLOGICAL ACOUSTICS

Cascade Cross Modulation Due to the Nonlinear Interaction of Elastic Waves in Samples with Cracks

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Received March 7, 2007

Abstract—The phenomenon under study consists in that, in an inhomogeneous material with nonlinearity caused by the presence of soft defects (the so-called "nonclassical" nonlinearity), cascade nonlinear effects are fairly strong and may even become comparable to the first-order effects. Similar cascade effects in media with a common nonlinearity of the crystal lattice are much weaker. This difference can be used as an important diagnostic indicator in nondestructive testing. Experimental data obtained for samples with cracks, which exhibit both ordinary modulation and cross-modulation effects, as well as a cascade cross modulation, are presented. The origin of the enhanced level of cascade effects is explained by modeling with the use of a simple model of nonlinearity of an inhomogeneous material containing soft Hertzian contacts.

PACS numbers: 43.25.+y, 62.65.+k, 62.20.Mk

DOI: 10.1134/S1063771008030160

INTRODUCTION

Microinhomogeneous media containing defects that are much "softer" than the surrounding material form a wide class of media (including almost all kinds of rock, many structural materials, primarily, damaged ones, etc.). Owing to the local strain and stress concentration at the soft defects and, hence, a local increase in nonlinearity, the macroscopic acoustic nonlinearity of such media is often strongly increased even when the defect concentration is very small [1]. Therefore, the possibility of using nonlinear acoustic effects for an early detection of cracks and for other diagnostic purposes has become the subject of intensive research in recent years [2, 3].

In the course of these studies, along with the simplest effects of the type of higher harmonic generation [4, 5], various versions of modulation methods were tested [6–10]. In these methods, the higher-frequency wave is usually much weaker than the other one and is called the probe wave. The second (modulating) wave, i.e., the pump wave, is sufficiently strong to affect the state of the defects in the sample and thereby to cause modulation of the probe wave.

In the simplest case of interaction between the harmonic pump wave of frequency Ω and the probe wave of frequency $\omega \gg \Omega$, their combination harmonics (modulation sidelobes) arise. An impact excitation (not only mechanical [7, 10], but also, for example, thermoelastic excitation by laser pulses [9]) can also be used as a pump action. For samples with a sufficiently high Q factor, the impulse action excites a number of quasi-monochromatic oscillations at the eigenmodes, which modulate the probe wave at the corresponding eigenfrequencies. This method was realized, e.g., in experiments [10] on crack detection in axles and disks of railway wheel pairs.

In addition to the aforementioned cases [6–10], where the origin of modulation components is evident, a somewhat unusual modulation effect was described in [9]. In the latter experiments, cylindrical samples (with a diameter of about 1 cm and a thickness of about 1 mm) were cut out from a metal damaged by preliminary loading; for the excitation of the probe wave (at a frequency of about 1.4 MHz), the samples were glued with their generatrix side to a piezoelectric plate. The pumping was produced by laser pulses, which were absorbed in the sample volume and excited a broad spectrum of eigenmodes with characteristic frequencies on the order of hundreds of kilohertz. In the spectrum of the acoustic signal, around the central frequency of ~1.4 MHz of the probe wave, quasi-monochromatic modulation components were observed at frequencies of about ±9–10 kHz and ±18–20 kHz. The origin of these components was unclear, because these frequencies were much lower than the eigenfrequencies of the sample and, hence, should not be effectively excited [9].

In this paper, we discuss the possible mechanism of this modulation. Since it is rather difficult to reproduce the experiments described in [9], below we use additional data obtained from the experiments [10] on the nonlinear modulation effects in the axles and disks of railway wheels to test our explanation by independent results. The conditions of experiments [10] (appropriately scaled to the size of the samples and the frequencies in use) reproduce the experimental conditions of [9] in the part significant for our explanation. An extra analysis of data that was not used in [10] allows us to reveal new, clearly pronounced modulation effects that differ from all the effects mentioned above. We call them cascade effects. In addition to substantiating the hypothesis proposed for explaining the observations reported in [9], we also employ the experimental data reported in [10] in discussing the prospects of using the cascade modulation effects for diagnostic purposes.

THE ORDINARY MODULATION AND THE CASCADE MODULATION EFFECTS

In view of a certain ambiguity of terminology, before analyzing experimental data, we briefly discuss the similarities and the distinctions of the modulation effects used in nonlinear acoustic diagnostics.

Figure 1a schematically illustrates the ordinary modulation of a probe wave of frequency ω_2 by a relatively low-frequency (with a frequency ω_1) pump wave, which leads to the appearance of combination components $\omega_2 \pm \omega_1$ (or, in the more general case, $\omega_2 \pm n\omega_1$, where $n = 1, 2, \ldots$). Figure 1b corresponds to the so-called cross modulation. In this case, the pump wave (with the carrier frequency ω_1) initially has a slow amplitude modulation at a frequency Ω (for example, it is biharmonic, as in Fig. 1b), so that the nonlinear interaction leads to a transfer of the slow modulation to the carrier ω_2 of the probe wave; i.e., components $\omega_2 \pm n\Omega$ arise (in Fig. 1b, only the components with $n = 1$ are shown). The carrier frequency of pumping ω_1 can be higher than the probe wave frequency ω_2 or lower than it (Fig. 1b). In the latter case, an ordinary modulation (of the type of that shown in Fig. 1a) may occur simultaneously with the cross modulation. In this case, because of the splitting of the pump spectrum, the ordinary modulation components also become split (Fig. 1c). It should be stressed that the cross modulation components (shown in Figs. 1b and 1c) appear as a result of the interaction of the modulated pumping with the probe wave at the odd component of nonlinearity. This process differs from the possible similar interaction process at a classical quadratic nonlinearity, where, at first, the demodulation of pumping with the appearance of a low-frequency component of frequency Ω takes place, and then this low-frequency component interacts with the probe wave at the same quadratic nonlinearity, which yields the components $\omega_2 \pm \Omega$. Because the efficiency of the initial generation of the low-frequency Ω component is usually very low, this cascade process is of no practical interest and is not discussed below.

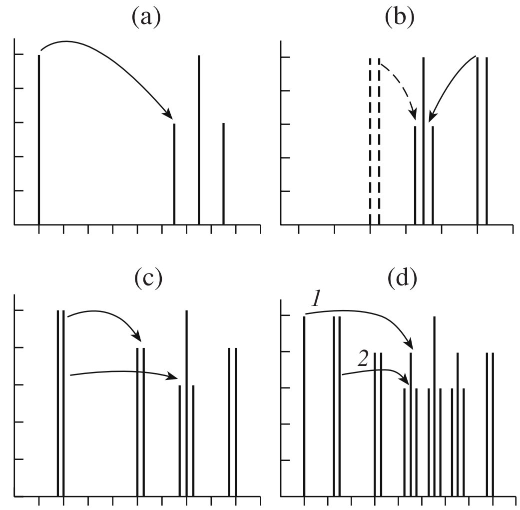

Fig. 1. Schematic illustration of the difference in the character of modulation effects in terms of spectral diagrams (the horizontal axis represents the frequency, and the vertical axis, the level of the corresponding spectral components of the signal): (a) ordinary modulation of the high-frequency (probing) wave by low-frequency pumping; (b) cross modulation, i.e., modulation transfer from the initially modulated "pumping" to another nonmodulated wave; (c) combined modulation action (combining versions (a) and (b)), when a relatively low-frequency and slowly modulated pump wave interacts with the probe wave; (d) cascade modulation, at which the slowly modulated pumping (*2*) modulates the modulation component of the probe wave, this component independently arising due to the interaction of the probe wave with another pump wave (*1*). The combination harmonics that are unimportant for the effects under consideration are omitted.

However, other cascade processes are possible, whose efficiency in media with microstructural nonlinearity can be comparable to that of lower-order modulation effects. Let us consider the case where the pump wave spectrum contains several components that widely differ in frequency and one of them is slowly modulated (for example, split into close components). In this case, the split component of the pump wave may cause the cross modulation of not only the probe wave itself (as in Figs. 1b and 1c), but also its modulation sidelobes produced by other spectral components of pumping, as it is schematically illustrated in Fig. 1d.

Based on the intuition acquired in the course of analyzing nonlinear interactions in media with weak atomic nonlinearity, one may expect that such a cascade process will be virtually unobservable because of its low efficiency. Here, it is implied that the efficiency of modulation should be approximately the same for both the probe wave itself and its modulation sidelobe induced earlier by another pumping component. Indeed, for both ordinary and cross modulations, the typical relative level of modulation sidelobes makes –30 to –40 dB [10–13] (even for samples whose nonlinearity is considerably increased by the defects). There-

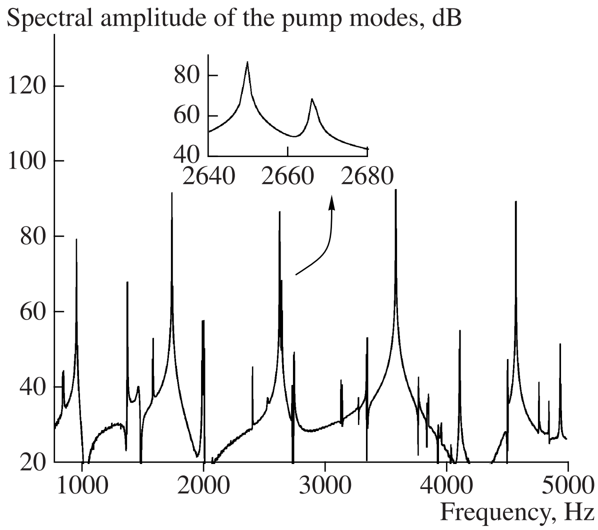

Fig. 2. Example of the spectrum of impact-excited low-frequency modes of pumping. The inset shows the spectral structure of one of the split modes at a frequency of about 2650 Hz with a higher resolution.

fore, one would expect that the cascade effect under discussion should lead to the appearance of secondary modulation sidelobes (marked with index *2* in Fig. 1d), whose level should not exceed –(30–40) × 2 = –(60–80) dB of the probe wave level. For typical acoustic experimental conditions, such a low level of the cascade-induced sidelobes should lie at the noise level or below it. However, these arguments are only valid for a weak atomic nonlinearity.

In the case of the microstructural nonlinearity under study, i.e., nonlinearity caused by the presence of soft defects (such as microcontacts or cracks) in the material, the state of the defects can be considerably perturbed by acoustic pumping (for example, a crack may noticeably change its opening or the contacts may begin to clap). Such a perturbed state of a defect may lead to an up to 100% change in the nonlinearity of the medium. It should also be taken into account that the level of lower-order modulation components observed in a sample with defects is mainly determined by precisely this microstructure-induced nonlinearity. Hence, an additional perturbation of the state of the defect by one more action (in the case under study, by a second pump wave) may substantially affect the level of ordinary lower-order modulation components (i.e., cause their modulation). Thus, in a medium with microstructural nonlinearity, the efficiency of such a cascade process (see Fig. 1d) may be comparable to the efficiency of the ordinary first-order nonlinear modulation interaction.

The situation where, along with the probe wave, several intense pump waves of different frequencies are excited in the sample is rather common for experiments with an impulse pump excitation [7–10]. In addition, for samples with a pronounced symmetry (e.g., almost cylindrical samples), almost degenerate modes with close frequencies arise in the natural way. Under an impact excitation, these modes may give rise to low-frequency intermode beatings. Such beatings of the pump modes can lead to the appearance of a low-frequency modulation of the probe wave owing to their cross-modulation interaction at the odd component of the microstructural nonlinearity (as was mentioned above). This effect can explain the experimental observation [9] of relatively slow modulation components that appeared in the kilohertz frequency range around the probing ultrasonic wave (~1.4 MHz) and were found to lie much lower than the characteristic eigenfrequencies of the samples (300–400 kHz).

EXPERIMENTAL DATA

In the experiments, whose results are partially presented in [10], the probe wave was an ultrasonic wave generated by a piezoelectric transducer and tuned in a stepwise manner in the frequency range from 45 to 70 kHz. A special shock device provided an impulse excitation of pump oscillations at the lowest-order eigenmodes (usually, within the frequency range 1–4 kHz) of the samples under study. The Q factors of the eigenmodes were on the order of several thousands, so that the characteristic decay time of low-frequency oscillations was on the order of a second. The typical length of a signal record at a sampling frequency of 200 kHz also was about one second.

Figure 2 shows an example of the experimental spectrum of low-frequency oscillations, in which one can clearly distinguish pronounced discrete components corresponding to the low-frequency eigenmodes of the high-Q sample. They give rise to discrete modulation components at sum and difference frequencies around the carrier frequency of the probe wave, and it is these components that were studied in [10]. On average, the relative level of modulation components was 20–30 dB higher for samples with defects as compared to the reference samples without defects. This difference shows that, even in the case of isolated cracks, their contribution to the nonlinearity of the sample dominates over the contribution of the volume nonlinearity of the defect-free material.

Note that the violation of cylindricity of the samples or the presence of the points of support or suspension naturally lifts the degeneracy of some of the modes and causes a splitting of their frequencies, as is shown in the inset in Fig. 2. Such a splitting of modes in many cases resulted in the appearance of clearly audible slow beatings (with frequencies from several to several tens of hertz) under the impulse excitation of the sample. Figures 3a and 3b show examples of records of such natural oscillations with beatings for different samples. In Fig. 3a, the beatings are clearly distinguished against the whole set of low-frequency modes in the frequency range 0.8–5 kHz. In Fig. 3b, for the sake of illustration, by filtering the whole signal in a frequency band of ±100 Hz, we separated the contribution of the mode (with a frequency of about 1.8 kHz) that exhibited the most pronounced splitting and an almost 50% depth of beatings. Such beatings of the pump modes were observed to one or another extent for all of the samples. The occurrence and stability of this effect confirms that similar (but scaled in frequency) beatings should also be present in geometrically similar samples in the experiments described in [9], and, therefore, could be the origin of the low-frequency cross-modulation components observed in these experiments.

When performing the comparison with the data of [9], it should be taken into account that, in [9], the frequency ratio of the probe wave to the low-frequency modulation observed by the authors was about 150, whereas, in the experiments described in [10] with a probe wave frequency of 45–70 kHz and a beat frequency of several hertz, this ratio was on the order of 10^4. At such a high ratio between frequencies, the observation of the cross modulation in the probing signal spectrum is close to the spectral resolution limit (the resolution is limited by a record time of ~1 s). Therefore, to reveal the possible modulation, it was more convenient first to separate the time variations (beatings) of the pump envelope and the envelope of the high-frequency probe wave component under analysis. Then, it was possible to compare either the shapes of these envelopes or their spectra. In addition, to simplify the comparison of the quasi-harmonic pump oscillations decaying with time and the corresponding modulation sidelobe, their amplitude decay with time was compensated for by introducing a correcting factor. For the same sample as that of Fig. 3b, the left-hand plots in Fig. 4 show the results of separating the aforementioned envelopes with the use of the Hilbert transform. The mean value was subtracted, and the residual variation was normalized to the mean level. The right-hand plots in Fig. 4 show the spectra of the variations of these envelopes. In both temporal and spectral representations, one can clearly see the modulation components at a frequency of about 6 Hz. The resolution did not exceed 1 Hz, because it was limited not only by the duration of the observation window of about 1 s, but also by the noticeable decay of the modulating oscillations themselves and the modulation sidelobes. Thus, the cross-modulation effect shown in Fig. 4 is analogous to the probe wave modulation observed in [9] at frequencies much lower than the frequencies of the sample eigenmodes.

In addition to analyzing the ordinary modulation and cross-modulation (arising due to the pump beatings) components of the probe wave, we analyzed the data of [10] from the viewpoint of revealing the cascade modulation of the first-order modulation sidelobes. We stress once again that this effect is not an exotic example of modulation interaction, but is of particular interest for diagnostic applications, because the cascade modulation should be the specific feature of the "nonclassical" nonlinearity caused by the "soft" defects (cracks). Of special interest is the situation where the presence of a small number of defects increases the nonlinearity of the sample not strongly enough, so that the slight increase in the first-order nonlinear effects cannot be uniquely attributed to the presence of defects. In the case of such an insufficient contrast in the ordinary modulation level, it is possible to use other indica-

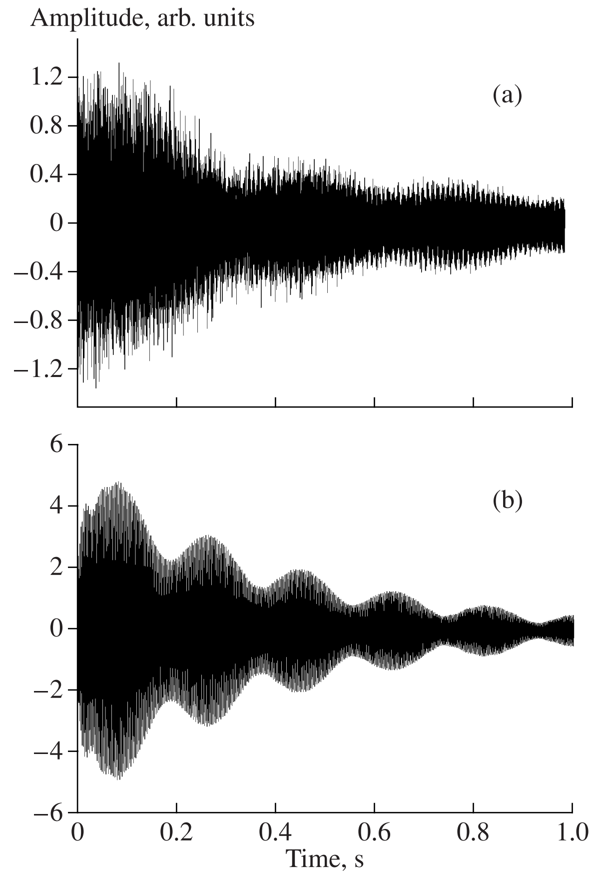

Fig. 3. Examples of a slow modulation of pumping, which naturally arises because of the low-frequency beatings of the split modes at an impulse excitation of almost cylindrical samples. (a) Modulation with a characteristic frequency of about 3 Hz, which was observed for the axle of a railway wheel pair. The signal is formed by all the excited modes in the frequency band from 800 Hz to 5 kHz. (b) Similar modulation with a frequency of about 6 Hz, which was observed for another axle. The modulation looks clearer, because, in this plot, the signal is only formed by the split mode with a frequency of about 1800 Hz in the chosen frequency band of ±100 Hz. The characteristic values of the mode's Q factors are 3000–6000.

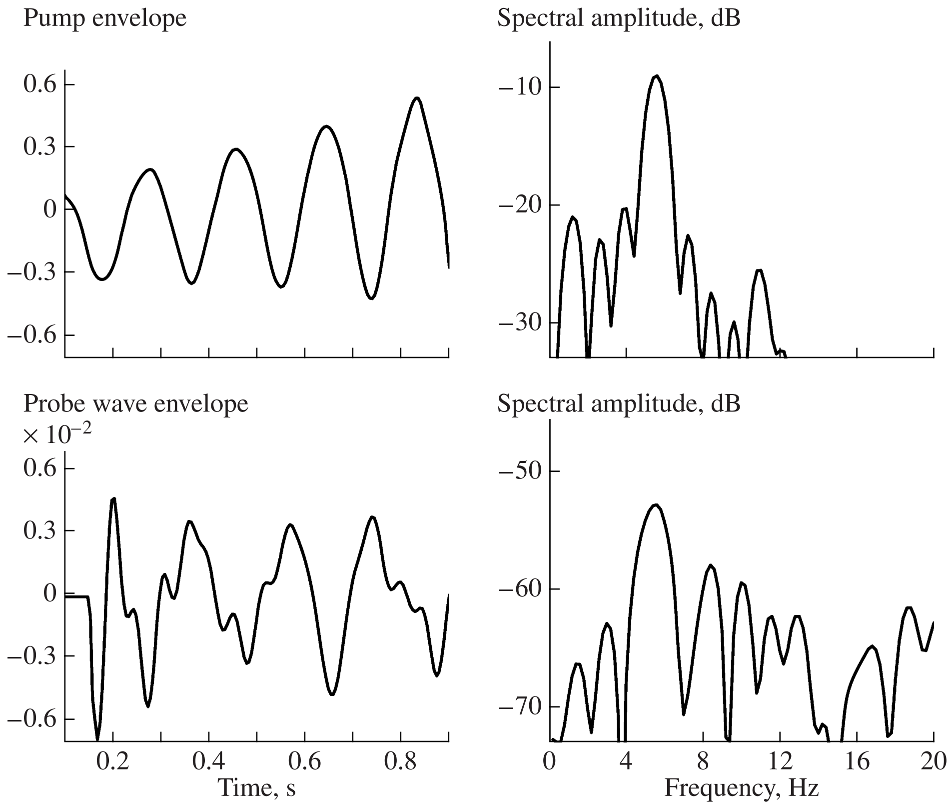

Fig. 4. Example of slow beatings of the low-frequency pumping due to the natural splitting of an intense mode at a frequency of about 1800 Hz and the cross-modulation-caused amplitude variations of the ultrasonic probe wave (with a carrier frequency of 51 kHz). The upper plots represent the normalized envelope of pumping (with correction for its decay with time), which is reduced to zero average value (the left-hand plot), and the spectrum of its variations with the main peak at a frequency of about 6 Hz (the right-hand plot). The lower plots represent the pump-beat-induced slow modulation of the envelope of the probing ultrasonic wave and the corresponding spectrum of variations. The sample is a wheel axle with a single crack.

tors of the defect, specifically, the observation of nonlinear effects that do not manifest themselves in materials with only the volume atomic (lattice) nonlinearity. Precisely this kind of "nonclassical" effect is the cascade modulation under discussion.

For its detection, from the signal records we separated the examples where the pump oscillations caused by impulse excitation exhibited beatings due to the splitting of the frequencies of some of the low-frequency modes. We performed the correction of the time-decaying pump signals and modulation components, as described above. The spectral components under analysis corresponded to the mode frequencies that were not naturally split. Figure 5 shows the plots obtained in this way for the envelope of a split pump mode (a central frequency of about 2325 Hz) and the spectrum of the variation of this envelope; it also shows a similar envelope and the spectrum of its variations for the modulation component caused by a nonsplit mode (900 Hz). In the spectra, one can clearly distinguish the components at a splitting frequency of about 6.5 Hz. The scale on the vertical axis is normalized so that the numbers (in decibels) correspond to the variations of the envelopes with respect to their mean levels. Despite the fact that the first-order modulation components themselves have a level of –30 to –40 dB with respect to the carrier component of the probe wave [10], the level of the cascade amplitude variations of the modulation component proves to be rather high (about –10 dB or higher). Such a high level of the cascade effect is characteristic of samples with defects (even samples that exhibit pronounced pump beatings), whereas for defect-free samples, the cascade effect was practically invisible against the noise.

MODEL ARGUMENTATION

Let us reveal the origin of the high level of the cascade effect by considering the elastic nonlinearity of Hertzian contacts as an example. As is known, the influence of both cracks and contacts present in them can manifest itself not only as changes in the elastic properties of the medium, but also as a considerable change in absorption [11, 12, 15]. However, in the case under study, we ignore this small difference in the manifestations of the defect properties and concentrate on the dif-

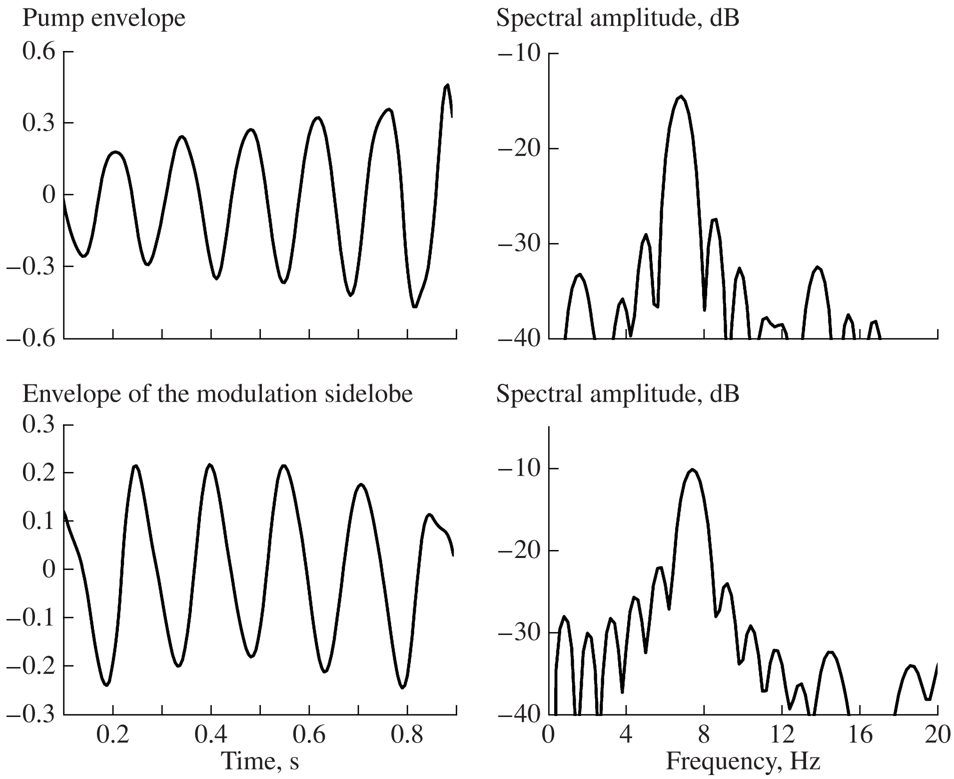

Fig. 5. Example of the cascade modulation of the probe wave modulation sidelobe at an excitation frequency of 49 kHz. The upper plots represent the variations of the split pump mode envelope and the corresponding spectrum. The lower plots represent the variations and the corresponding spectrum for one of the first-order modulation components. The chosen component arises as a result of the interaction of the probe wave with the nonsplit pump mode of 0.9 kHz and corresponds to the difference combination frequency of (49–0.9) kHz. The frequency of the modulating split pump mode is 2325 Hz, and the frequency of slow beatings is about 6.5 Hz. The sample is a wheel axle with a single crack.

ference in the cascade effect levels for the cases of the conventional nonlinearity and the "nonclassical" nonlinearity associated with the influence of weak contacts. It is important to demonstrate that their state may be strongly perturbed by an additional acoustic action, which leads to considerable changes in the effective nonlinearity parameters responsible for changes in either reactive or dissipative properties.

As in our previous publication [13], we use a simple model of an elastic sample that contains a defect represented by a weak Hertzian contact. In this case, the stress–strain relation has the form

$$\begin{aligned}\sigma_0 + \tilde{\sigma} &= A(\varepsilon_0 + \tilde{\varepsilon})^{3/2} H(\varepsilon_0 + \tilde{\varepsilon}) \\ &+ B(\mu\varepsilon_0 + \tilde{\varepsilon})^{3/2} H(\mu\varepsilon_0 + \tilde{\varepsilon}).\end{aligned} \quad (1)$$

Here, σ_0 is some static value (of no significance for our consideration) of elastic stress in the sample; $\tilde{\sigma}$ is the oscillating (acoustic) component of elastic stress; ε_0 is the static strain of the material that corresponds to the stress σ_0; $\mu\varepsilon_0$ is the initial strain of the weakly compressed contact, where the small parameter $\mu \ll 1$ characterizes the smallness of its initial compression; H is the Heaviside function, which shows that the contact does not "hold" a tensile stress; and $\tilde{\varepsilon}$ is the oscillating (acoustic) strain component. The parameters A and B in Eq. (1), which have the dimension of stress (elastic modulus), are determined by the elastic properties of the material and, in addition, depend on the geometric size of the contacts and on their number. Assuming that the acoustic perturbation amplitude is small, i.e., $\tilde{\varepsilon} \ll \mu\varepsilon_0$, from Eq. (1) relating the acoustic stress and strain, we approximately obtain

$$\begin{aligned}\tilde{\sigma} \approx \left.\frac{d\tilde{\sigma}}{d\tilde{\varepsilon}}\right|_{\varepsilon_0} \tilde{\varepsilon} + \frac{1}{2}\left.\frac{d\tilde{\sigma}^2}{d\tilde{\varepsilon}^2}\right|_{\varepsilon_0} \tilde{\varepsilon}^2 &= \left(\frac{3}{2}A\varepsilon_0^{1/2} + \frac{3}{2}B(\mu\varepsilon_0)^{1/2}\right)\tilde{\varepsilon} \\ &+ \left(\frac{3}{4}A\varepsilon_0^{-1/2} + \frac{3}{4}B(\mu\varepsilon_0)^{-1/2}\right)\tilde{\varepsilon}^2.\end{aligned} \quad (2)$$

From this expression it follows that, when A and B are comparable, the contribution of weak contacts to the value of the elastic modulus is small (which is determined by the smallness of the parameter $\mu^{1/2} \ll 1$ at $\mu \ll 1$). By contrast, the contribution of weak contacts to the nonlinear (quadratic in the approximation under study) components is greater, because $\mu^{-1/2} \gg 1$ when $\mu \ll 1$. In

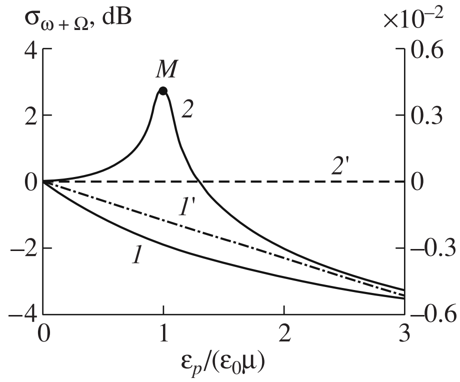

Fig. 6. Numerically calculated amplitude variation of the modulation component $\sigma_{\omega+\Omega}$ as a function of the normalized amplitude of the perturbing action ε_p. The left-hand ordinate axis corresponds to the calculation in the presence of a soft defect, and the right-hand ordinate axis corresponds to the calculation in the absence of the defect. Curves *1* (a static compressional action) and *2* (a dynamic action ε_p) are obtained in the presence of the defect, and curves *1'* and *2'* are obtained in the absence of the defect (when in expression (2) the parameter B is $B = 0$).

view of this fact, Eq. (2) can be represented in the form $\tilde{\sigma} \approx E\tilde{\varepsilon}(1+\gamma\tilde{\varepsilon}+\ldots)$, where the elastic modulus is

$$E \approx \frac{3}{2}A\varepsilon_0^{1/2} \quad (3)$$

and the dimensionless nonlinear parameter is

$$\gamma \approx \frac{1}{2}\varepsilon_0^{-1} + \frac{1}{2}(B/A)\varepsilon_0^{-1}\mu^{-1/2}. \quad (4)$$

In fact, in Eqs. (1)–(4), we could simply introduce some unperturbed (determined by the homogeneous medium, i.e., matrix) values of the elastic modulus and the nonlinearity coefficient, to which the “defect,” i.e., contact, would make an additional contribution. However, the expressions in the form of Eqs. (1)–(4) show a considerable difference in the nature of the contributions of weak and strongly compressed contacts to the linear and nonlinear characteristics.

The amplitudes of the combination (modulation) components $\omega \pm \Omega$, which result from the interaction of the initial components with frequencies ω and Ω, are proportional to the nonlinear parameter γ. If we impose an additional perturbation $\tilde{\varepsilon}_p$ on the medium, so that this perturbation is much smaller in amplitude than ε_0 but may be comparable to $\mu\varepsilon_0$, we find that, in the absence of weak contacts, nonlinear parameter (4) will remain virtually invariable. Hence, the level of the combination components $\omega \pm \Omega$ will also be virtually unchanged, and only in the next order of interaction ($\sim\gamma^2\varepsilon_p\varepsilon_\omega\varepsilon_\Omega \ll \gamma\varepsilon_\omega\varepsilon_\Omega$) will a weak modulating effect of the additional perturbation $\tilde{\varepsilon}_p$ manifest itself. However, if the material of the sample contains weak contacts, so that $\tilde{\varepsilon}_p \sim \mu\varepsilon_0$, then, according to Eq. (4), the effect of $\tilde{\varepsilon}_p$ can change the value of the nonlinear parameter γ by 100%. Correspondingly, an equally large (but not in the next order of smallness) change may occur in the level of the combination components $\omega \pm \Omega$. In other words, the result of the cascade effect may turn out to be comparable to the result of the first-order modulation interaction. Strictly speaking, in the case of comparable values of $\tilde{\varepsilon}_p \sim \mu\varepsilon_0$ (and the more so, when $\tilde{\varepsilon}_p > \mu\varepsilon_0$), the power series expansions (2) and (4) become inapplicable, so that it is necessary to use the full expression (1), which allows for the clapping nonlinear mode of contact oscillations.

Now, let us consider typical numerical examples. We assume that, in a sample with a defect whose nonlinearity is modeled by Eq. (1), we introduce a high-frequency probing oscillation ε_ω of frequency ω, a low-frequency pump oscillation ε_Ω of frequency Ω, and an additional action ε_p, so that we have $\tilde{\varepsilon} = \varepsilon_\omega + \varepsilon_\Omega + \varepsilon_p$. As a result of the nonlinear interaction of ε_ω and ε_Ω, modulation components $\omega \pm \Omega$ will appear in the spectrum around the carrier component of the high-frequency probing oscillation.

Let us trace the dependence of these components on the amplitude of the additional action ε_p of both quasi-static and dynamic (oscillatory) types with a frequency different from Ω. As an example, in our calculations we set $\varepsilon_0 = 10^{-1}$, so that, in the absence of the soft defect, the quadratic nonlinearity parameter takes a value typical of homogeneous solids: $\gamma = 5$ [16]. For the softness parameter involved in Eq. (1), we choose the value $\mu = 4 \times 10^{-4}$ and then set $B = 0.04A$. According to Eq. (4), this means that the presence of defects causes an increase in the nonlinearity coefficient γ by a factor of 3 (and, hence, leads to an increase in the level of the first-order modulation components by about 10 dB). Such a difference in the mean levels of modulation components between the samples is usually insufficient to make reliable conclusions concerning the presence or absence of defects. On the other hand, according to the arguments presented above, one should expect a much stronger cascade effect, i.e., a multiple increase in the effect of the additional action ε_p on the level of the modulation component with frequency $\omega \pm \Omega$, which arises as a result of interaction of the ε_ω and ε_Ω components. Indeed, at the chosen parameters, a soft contact can change to the clapping mode when the strain amplitude of acoustic action reaches the value $\varepsilon_p = \mu\varepsilon_0 = 4 \times 10^{-5}$, which seems to be a typical value for internal contacts in cracks [11, 12]. Figure 6 shows the numerically calculated relative amplitude variations of the modulation component at the frequency $\omega + \Omega$ as a function of the normalized amplitude $\varepsilon_p/(\mu\varepsilon_0)$ of the additional action for the cases of the presence and absence of the soft defect (in the latter case, in Eqs. (1)–

(4), we take $B = 0$). The initial level of the modulation component $\sigma_{\omega+\Omega}$ corresponds to zero decibels in the plot. In the absence of the defect, the additional dynamic action leads to variations of $\sigma_{\omega+\Omega}$ on the order of 10^{-5}, which are practically unavailable to experimental observation, and, in the quasi-static case, to variations on the order of 10^{-3} (see the right-hand ordinate axis in Fig. 6). The appearance of a soft defect leads to an increase in the cascade effect of modulation component variation by 2 to 3 orders of magnitude. For example, at $\varepsilon_p \approx \mu\varepsilon_0$, for both quasi-static and dynamic actions, the level of the modulation component varies approximately by a factor of 1.5 compared to its level in the absence of the ε_p action. Remember for comparison that the experimentally observed variations of the modulation component were about 20% at a 20–30% depth of naturally arising split mode beatings.

Here, it should be stressed that the aforementioned difference in the levels of the $\sigma_{\omega+\Omega}$ component (approximately by a factor of 1.5) is observed for a single sample and, therefore, can be reliably detected by varying the level of the additional action ε_p, which perturbs the state of the defect. For this purpose, it is possible to use either one of the low-frequency pump modes with a frequency different from Ω (a quasi-static action) or an oscillating but slowly amplitude-modulated oscillation (a dynamic action). If the material of the sample contains no soft defects, i.e., its nonlinearity is of a "classical" nature typical of a crystal lattice, the cascade nonlinear effects will be practically unobservable (see the right-hand ordinate axis in Fig. 6). Thus, the observation of the cascade effect of the type described above can be used for obtaining new diagnostic data in the cases where the direct comparison of the levels of the first-order modulation components for different samples does not allow one to make an unambiguous conclusion concerning the presence of the defect.

Returning to Fig. 6, it should be noted that, for the cases of the dynamic and compressional static actions ε_p, the characters and even signs of the variations of the modulation component levels are different. This difference is related to the fact that (unlike the monotonic effect of a static load), at not too high oscillation amplitudes, the mean rigidity of Hertzian contacts first decreases and then, in the clapping mode, begins to increase. At a moderate variation of the dynamic pump amplitude ε_p on opposite sides of the maximum marked as point M (where $\varepsilon_p \sim \mu\varepsilon_0$) in Fig. 6, the resulting variations of the $\varepsilon_{\omega+\Omega}$ component should have an opposite phase, and, in the immediate vicinity of the maximum M, in the case of a harmonic variation of the ε_p amplitude, the contribution of the second harmonic of modulation should predominate. All this may lead to a considerable difference between the shape of the envelope of the modulating action ε_p and the result of the modulation of the $\varepsilon_{\omega+\Omega}$ component. From the point of view of comparison with the experiment, it is important to stress that, in reality, the levels (and phases) of the nonlinearly excited harmonics (as well as the levels and phases of the primary, linearly excited ones) will be to a considerable extent determined by the resonance properties of the samples, and not only by the level and phase of the sources. When the pump level varies within ~1 s (as in the experiments under analysis), the phase shifts by $\sim\pi$ due to the aforementioned effects may additionally cause smearing and shifts of the characteristic modulation frequency under observation by values on the order of 1 Hz (see examples of modulation envelopes and modulating signals in Fig. 4).

However, the influence of the amplitude- and frequency-dependent factors mentioned above does not cancel the basic conclusions concerning the relative variability of the cascade modulation components in the presence and absence of defects. Precisely for excluding the influence of the largely random positions of resonance peaks in real experiments, the modulation levels were averaged over the probe wave frequency, which could be varied over wide limits [10]. Thus, simple estimates obtained above provide adequate values of the expected level of cascade modulation. Note that, in the experiments under discussion, as the additional action, the naturally arising beatings of the split modes were used, although, it is possible to produce a more effective (more deeply modulated) action in the case of a practical application of the cascade effects.

CONCLUSIONS

The analysis of data obtained from the experiment [10] on the acoustic modulation interactions confirmed that, for high-Q samples of a symmetric shape, a typical effect of slow beatings of some of the impulse-excited eigenmodes owing to the their natural splitting takes place. A detailed analysis of modulation spectra showed that, in addition to the ordinary modulation of the probing ultrasonic wave by natural vibrations of the sample, in the presence of the aforementioned slow beatings of the pump modes, a cross-modulation effect occurs (i.e., the transfer of the slow modulation, which is caused by the beatings of intense oscillations, to the carrier frequency of the probing ultrasonic wave). The analysis suggests that, in the qualitatively analogous experiments [9], the slow modulation of the probe wave, which was observed at frequencies much lower than the eigenfrequencies of the samples, was also related to the beatings of split modes that naturally arise in almost symmetric samples.

In addition to this cross-modulation effect, the analysis performed by us demonstrated a high efficiency of cascade modulation effects, whose level for samples with "nonclassical" nonlinearity caused by soft defects can be comparable to the level of ordinary modulation effects. For homogeneous samples with a conventional nonlinearity of the crystal lattice, such cascade effects are practically unobservable. Therefore, a noticeable variability of the ordinary modulation component within ~100% due to the cascade modulation definitely testifies to the relation of the nonlinearity of the sample to the presence of soft defects. Thus, the difference in the levels of cascade effects by several orders of magnitude can be used as an important diagnostic indicator at the initial stage of the sample damage, when the increase in the level of the conventional first-order nonlinear effects is insufficient for making unambiguous conclusions on the presence of defects.

ACKNOWLEDGMENTS

We are grateful to our coauthors in the cited paper [10] for the cooperation in the experimental part of this work. The work was supported by the Russian Foundation for Basic Research (project no. 06-02-72550-NTsNIL) and by the PECO-NEI project no. 19118.

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Translated by E. Golyamina