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EFFECT OF HYSTERESIS SATURATION ON NONLINEAR INTERACTION OF ELASTIC WAVES IN MATERIAL WITH HYSTERETIC NONLINEARITY

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Numerous experimental data on nonlinear-acoustic effects consistently indicate the important role of hysteretic nonlinearity in many microinhomogeneous media (e.g. rocks, granular material, etc.) The most well studied manifestations of the hysteretic nonlinearity relate to the effects of self-action of acoustic perturbations (such as the generation of harmonics, self-induced modulus defect and hysteretic losses). At the same time, in the recent years, experimental methods based on interaction of waves having different frequencies have been attracting increasing attention. In this context, development of models for adequate description of such interactions in hysteretic materials is required. Even for a significant difference in the amplitude of the interacting perturbations having comparable strain rates, a significant qualitative transformation of the hysteresis loop with creation of embedded inner loops may occur (the case of non-simplex deformation), which prevents the use of perturbation methods. Besides, the hysteretic nonlinearity may exhibit significant qualitative changes even within the amplitude range typical of acoustic perturbations (for example, the hysteresis saturation may manifest itself), which can strongly influence the character of the self-action and interaction of elastic perturbations. This communication presents results of a numerical simulation that allows one to correctly take into account both the non-simplex character of the material deformation and the hysteresis saturation effects. It is shown that for the hysteretic corrections to the elastic moduli and for the nonlinear dissipation, the combined action of these factors results in complex non-monotonic dependences, which should be taken into account in the interpretation of experimental results. Some experimental data indicating the hysteresis saturation are discussed.

Numerous experimental data on nonlinear acoustic effects in a wide class of microinhomogeneous solids indicate hysteretic character of the “stress-strain” relationship for these materials. However, direct observations of quasistatic hysteretic loops are available only for fairly large amplitudes (about 10^{-4} and higher), that is beyond typical amplitudes of acoustic excitations. In the acoustic range of amplitudes, hysteresis is normally studied via observations of self-action of acoustic excitations. The main difficulty in the description of interactions between excitations having different frequencies is related to the regimes when complex hysteresis loops with embedded inner loops can be created even if the oscillations have strongly different strain amplitudes, but comparable strain rates. Besides, experimental data indicate that even in the acoustic range of amplitudes, the hysteresis loops can have not self-similar shapes in different amplitude ranges, which introduces additional complexity in the description of self-action and interaction between different excitations. One of the popular approaches to the description of hysteretic nonlinearities is the Preisach-Krasnoselskii-Mayergoyz (PKM) approach [1,2]. This approach is based on the representation of the macroscopic response of the material as a superposition of individual responses of a large ensemble of elementary hysteretic units. Such a representation has proven its usefulness, although these hysteretic units do not directly correspond to the ensemble of real physical defects (such as crack, contacts, etc.), at which the hysteretic nonlinearity is physically localized. The PKM approach allows for a correct description of complex hysteretic loops taking into account the history of the previous loading of the material, which is difficult to make using conventional piece-wise power or other approximations of hysteretic loops (see, for example, [3]). For hysteretic PKM-units, it is usually supposed that their shape is rectangular and properties of particular materials are accounted via different distributions $f(s_c, s_o)$ of the hysteretic units on the plane of their control parameters (for example, strains s_c and s_o), at which the switching of each element into another state occurs (see the “opening” and “closing” of the hysteretic unit shown in Fig. 1).

For sufficiently small amplitudes, the distribution $f(s_c, s_o)$ can be considered as uniform, so that the individual contributions of the hysteretic elements can be summed analytically, which predicts a piece-wise quadratic antisymmetric shape of the macroscopic loop, which remains self-similar at different amplitudes [3,4]. There are experimental data indicating clear signs of the hysteresis saturation (which is qualitatively similar to the saturation of hysteresis in magnetic materials) with increasing amplitude of the deformation. On the PKM-plane such saturation corresponds to an inhomogeneous distribution $f(s_c, s_o)$ of the hysteretic units. As it was elucidated in [5], the inhomogeneity of the dis-

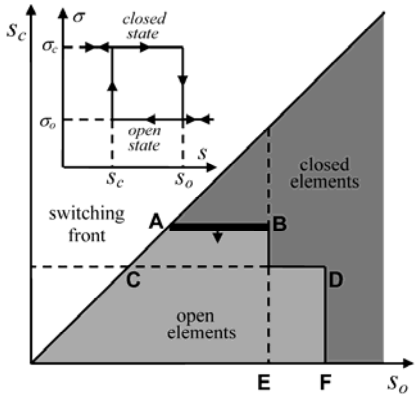

Fig. 1. Model of an individual hysteretic unit (in the inset) and PKM-plane (s_o,s_c) with the distribution of the hysteretic elements $f(s_c, s_o)$ localized below the diagonal s_o>s_c. Each element is described by threshold deformations s_o, s_c of switching between two states ("closed" and "open"). The corresponding elementary jump in the strain is equal to $\Delta\sigma = \sigma_c - \sigma_o$.

tribution $f(s_c, s_o)$ along the diagonal direction in the PKM-plane results in the appearance of nonlinear-elastic terms in the stress-strain relationship (in the lowest order this corresponds to the appearance of quadratic terms which break the antisymmetrical form of the hysteresis loop). The inhomogeneous distribution in the direction orthogonal to the diagonal keeps the antisymmetry, but results in the hysteresis saturation in a quite pure form without introduction of additional non-hysteretic nonlinear-elastic terms.

Below we will use such a model of antisymmetric hysteresis with saturation due to inhomogeneous distribution of the hysteretic element density in the direction orthogonal to the diagonal. The inhomogeneity is supposed to be exponential. The distribution law $f(s_c, s_o)$ will be convenient to reformulate in terms of new variables $s_{\|} = (s_o + s_c)$ and $s_{\perp} = (s_o - s_c)$ corresponding to the along-diagonal and off-diagonal directions in the PKM-plane, so that

$$f(s_o, s_c) = f_0 \exp(-(s_o - s_c)/s_{\perp}^0) \equiv f_0 \exp(-s_{\perp}/s_{\perp}^0) = f_0 \exp(-S_{\perp}) \qquad (1)$$

Here, the strain variable $S_{\perp} = s_{\perp}/s_{\perp}^0$ is normalized by the characteristic scale $s_{\perp}^0$ of the inhomogeneous distribution of the hysteretic units in the direction orthogonal to the diagonal. We assume that during the material deformation, the global maximum $S_{\max}$ and minimum $S_{\min}$ are achieved along with intermediate extrema S_k. Then, following the procedure described in ref. [6], for all descending (with negative derivative $S_t < 0$) branches starting from the respective maxima $S_{\max}^{(k)}$ one obtains the following expression for the hysteretic contribution $\sigma_H(S)$ to the total stress in the material.

$$\sigma_H = \sigma_H(S_{\max}^{(k)}) - \left[S - S_{\max}^{(k)} + 1 - \exp(S - S_{\max}^{(k)})\right], \qquad (2)$$

Analogously, for all ascending ($S_t > 0$) branches starting from the respective minima $S_{\min}^{(k)}$ we get

$$\sigma_H = \sigma_H(S_{\min}^{k}) - \left[S - S_{\min}^{(k)} + 1 - \exp(S - S_{\min}^{(k)})\right]. \qquad (3)$$

Note that in expressions (2) and (3) the stress is normalized to the elastic modulus E of the matrix material in which the hysteretic units-defects are embedded: $\tilde{\sigma}_H = \sigma_H/(h_H E)$, where $h_H = f(0,0)\Delta\sigma/E$ is the dimensionless nonlinear parameter of the hysteretic nonlinearity. When the minor embedded branch during the return movement reaches the previous larger branch starting from the previous stronger extremum, the expressions (2) and (3) should be correspondingly "switched" to this stronger extremum as is illustrated in Fig. 2. This figure shows complex hysteretic loops with embedded branches in the case of the material deformation by the superposition of two excitations with 7 times different frequencies. Note that for small amplitudes ($S << 1$), far below the characteristic amplitude of the hysteresis saturation, expressions (2) and (3) reduce to the conventional expression corresponding to the piece-wise quadratic shape of the hysteresis loop [4] like in the case of a uniform distribution $f(s_c, s_o) = const.$. The inhomogeneity of the distribution $f(s_c, s_o)$ (decrease at higher amplitudes of the deformation) breaks the self-similarity of the hysteresis loops. This is illustrated in Fig. 3 for different amplitudes A (from A<<1 to A =5) of simplex-type deformation with one minimum and one maximum over the period.

Further, in the analysis of nonlinear interactions between oscillations with different frequencies as well as for the self-action of one oscillation we will use the approach developed in [6]. For arbitrarily complex shape of the hysteresis "stress-strain loop" corresponding to a given protocol of the strain, this approach allows one to calculate the variations in the elastic modulus and in the dissipation, which are locally introduced in the material by one oscillation for another one. To this end, it is sufficient to

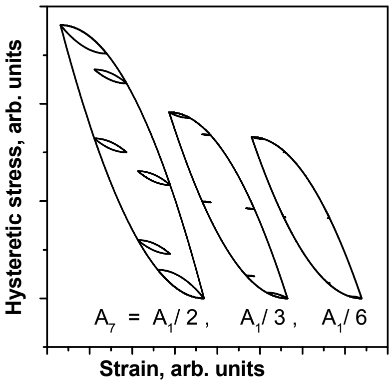

Fig. 2. Evolution of complex hysteretic loops $\sigma_H(S)$ for oscillations with 7 times different frequencies for different amplitudes of the faster oscillation

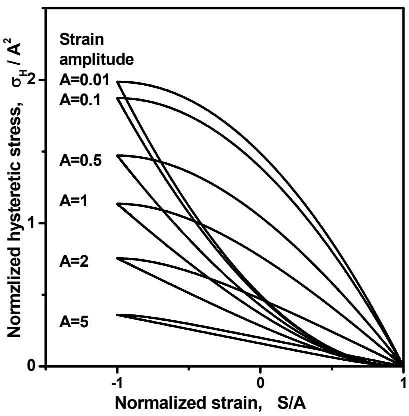

Fig. 3. Illustration of the hysteresis saturation $\sigma_H(S)$ with increased amplitude. At amplitudes A<<1, the loops are almost self-similar and peace-wise quadratic, but are strongly saturated for A >1.

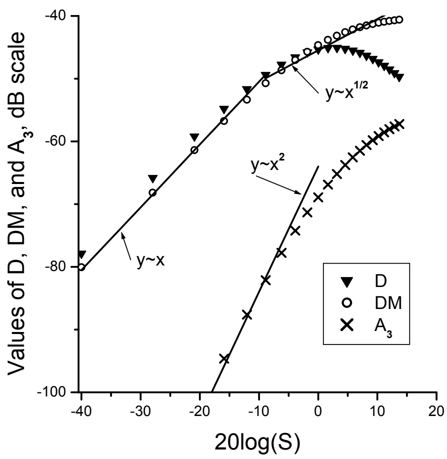

Fig. 4. Theoretically calculated amplitude dependences for the self-induced decrement (D), modulus defect (DM) and the 3rd harmonic amplitude in a hysteretic material with saturation.

find the integrals of the respective combinations of the hysteretic stress and strain [6] over the total period of the complex hysteretic loop (which in certain cases can be done analytically or numerically in the general case). In the simplest case of finding the self-induced hysteretic losses, such integration is reduced to the known procedure of determining the area of the hysteretic loop. In a similar way, amplitudes of higher harmonics generated due to the hysteretic nonlinearity can be found. Unlike the case of a uniform distribution $f(s_c, s_o) = const.$ (and, correspondingly, for the piece-wise quadratic self-similar shape of the hysteresis), the hysteresis saturation due to inhomogeneous distribution $f(s_c, s_o)$ can lead to qualitatively different character of nonlinear effects in hysteretic materials for different amplitudes ranges. In such a case, in contrast to the self-similar loops, which were considered in details in ref. [6], an important role plays not only relative amplitudes of the interacting oscillations, but also the ratio of each oscillation amplitude to the characteristic amplitude of the hysteresis saturation. The latter amplitude, in terms of the normalized strain S introduced in Eqs.(2) and (3), corresponds to $S = 1$. For the simple case of self-action of one harmonic oscillation in a material exhibiting such saturation of the hysteresis, the hysteretic losses (expressed via the hysteretic contribution $D(S)$ to the decrement), the nonlinear variation of the effective modulus $DM(S)$, and the amplitude of the 3rd harmonic of the stress are shown in Fig. 4 as functions of the normalized strain S. The curves clearly indicate the change in the functional character of the dependences for different amplitude ranges. In is worth to underscore that clear experimental indications of the hysteresis saturation are known in the acoustic range of amplitudes. Examples of experimental curves similar to those shown in Fig.4 are presented in Fig. 5. The comparison between these figures indicates that the characteristic deformation of the hysteresis saturation corresponds to the deformations of the order of 10^{-6}.

Along with clear differences in the character of self-action of harmonic excitations in materials with hysteresis saturation, significant new features appear in the case of interaction of oscillations with different frequencies when at least for one of the oscillations its amplitude is comparable with the characteristic amplitude of the hysteresis saturation. Indeed, qualitatively is clear that instead of the induced dissipation considered in details in [6] (where the interaction regime below the saturation was studied), for an oscillation with sufficiently high amplitude, an additional intensive action (pump) will result in the induced transparency, because for stronger amplitudes, the hysteretic nonlinearity with

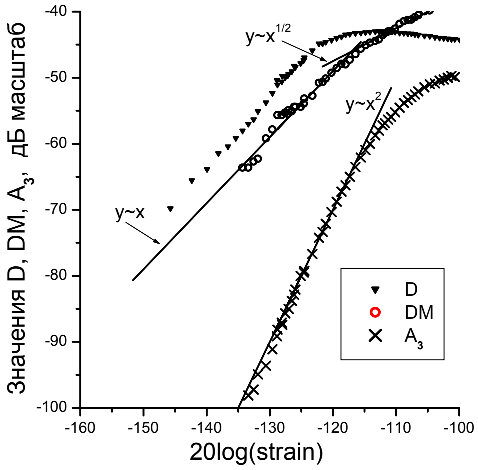

Fig. 5. Experimentally observed amplitude dependences of the self-induced decrement (D), modulus defect (DM) and the 3rd harmonic level for a resonator prepared of annealed-copper rod.

saturation becomes weaker (that is on the PRM-plane the system moves to regions distant from the diagonal, where the density of the hysteretic units is smaller). An example of the calculated induced decrement for a weaker probe wave and the self-induced decrement for the stronger wave is shown in Fig. 6. In the figure, initial growth of the dissipation and then saturation and the subsequent decrease in dissipation is visible. This saturation occurs earlier for the probe wave. Qualitatively similar experimental example demonstrating clear saturation of the probe-wave dissipation and approaching the saturation for the pump-wave dissipation is shown in Fig. 7. For sufficiently intensive probe waves, the initial region of the dissipation increase was not observed at all so that the additional excitation of the pump wave lead to the induced transparency for the probe wave. Therefore, the examples shown in the figures indicate that even in the acoustic range of amplitudes the influence of the hysteresis saturation can be very significant. Such saturation effects should be thus taken into account in the interpretation of experimental observations of self-action and interactions of acoustic oscillations in hysteretic materials. In particular, the proposed approach can be used for this purpose.

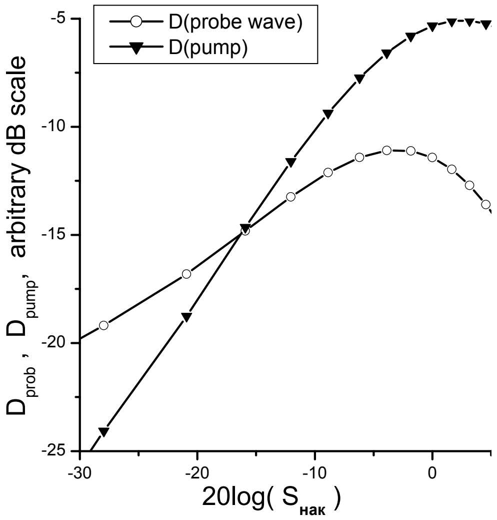

Fig. 6. Calculated variations in the induced decrements of a weak probe-wave and the pump wave for a material with the hysteresis saturation.

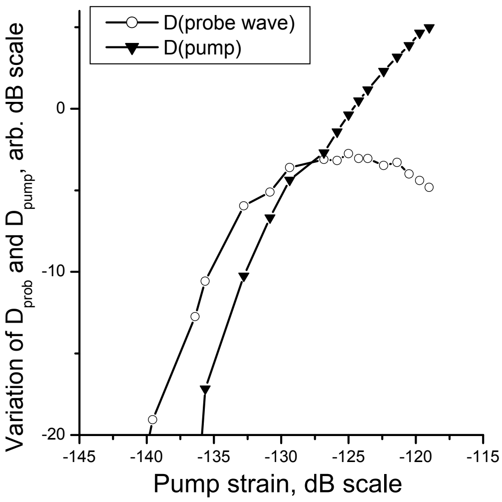

Fig. 7. Experimentally observed nonlinearity-induced variations in the decrements of a weak probe wave and the pump wave in a resonator made of annealed-copper rod.

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R E F E R E N C E S

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