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Self-Demodulation of Acoustic Pulses in Partially Water-Saturated River Sand

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Abstract—Laboratory experiments on self-demodulation of high-frequency acoustic pulses in partially water-saturated river sand are reported. Based on experimentally revealed dependences of demodulated pulse parameters (propagation velocity, amplitude, and duration) on the static pressure and on the amplitude of the primary pulses, an equation of state for this medium is proposed. Parameters of this equation are derived from the comparison of theoretical predictions and experimental results.

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INTRODUCTION

River sand is one of the interesting and unusual media that are still poorly studied in nonlinear acoustics. Due to the soft contacts between individual particles of the medium (grains of sand), the propagation of elastic waves in river sand is characterized by high nonlinearity, high attenuation, and low velocity. Taken together, these properties make it possible to perform various experiments on nonlinear acoustic wave propagation and interaction under controlled laboratory conditions by using small amounts of the material. On the other hand, since river sand is a natural medium, the nonlinear wave propagation in it can (in some cases) be studied and modeled under conditions that are rather close to the field conditions.

Many researchers have experimentally studied acoustic wave propagation in river sand (and similar granular media) [1–16]. Nevertheless, no indications of nonlinear acoustic properties in such highly nonlinear media were mentioned in the majority of these works, while the effects observed and apparently caused by the nonlinearity were either ignored or misinterpreted (see, e.g., [9, 15, 16]). It should be noted that the acoustic nonlinearity of granular media is so high that the observation and study of, for example, the effect of self-demodulation of high-frequency (HF) acoustic pulses does not actually require any complex equipment (which is necessary to study nonlinear effects in conventional weakly nonlinear media). Due to the high nonlinearity of the granular medium, HF acoustic pulses excited in it undergo demodulation so that intense secondary low-frequency (LF) video pulses are generated, which propagate with a much lower attenuation than the rapidly decaying (near the pumping radiator) primary HF pulses. When receiving these relatively weak secondary pulses, there is no necessity to suppress the spectral components of the intense HF pumping pulses or to use equipment with a wide dynamic range. It should also be noted that the phase of the demodulated video pulses is independent of the phase of the pumping HF pulses and is determined by the sign of the medium's nonlinearity parameter alone, so that, even if the phase of the primary pulses excited in the medium is random, coherent LF pulses are generated and propagate in it. Therefore, coherent accumulation can be used to detect and extract from noise the demodulated pulses even for a random phase of the pump. Also, the property that the phase of the demodulated pulse is independent of the phase of the primary HF pulse can be used in experiments to check whether the received LF signal is actually a result of demodulation rather than of penetration of low-frequency components of the intense HF pumping pulse.

The effect of self-demodulation of HF acoustic pulses in river sand was first observed and thoroughly (experimentally and theoretically) studied in [5–8]. These experiments used dry sand or almost completely water-saturated sand, which was immersed into water so that its water saturation was close to unity. The analysis of the results obtained showed that, although the waveform of LF pulses demodulated in dry and water-saturated river sand is the same and their amplitude and propagation velocity depend, respectively, on the primary HF pulse amplitude and on the static pressure in the same manner, certain characteristics of LF pulses demodulated in these media, in particular, the dependences of their amplitude and duration on the static pressure, are qualitatively different, which is, in principle, a diagnostic criterion allowing one to distinguish dry sand from water-saturated sand.

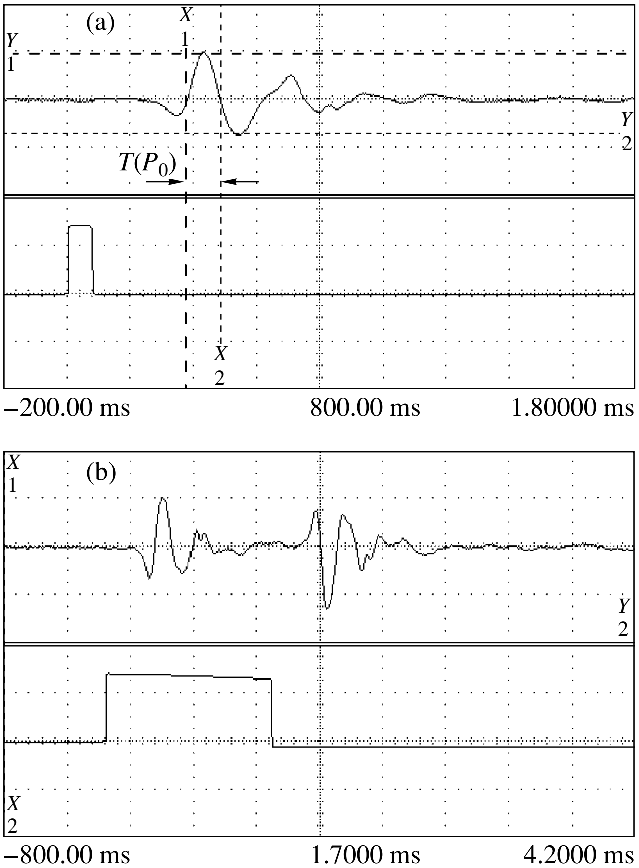

Fig. 1. Oscillograms of (a) short and (b) long demodulated LF pulses received by the accelerometer.

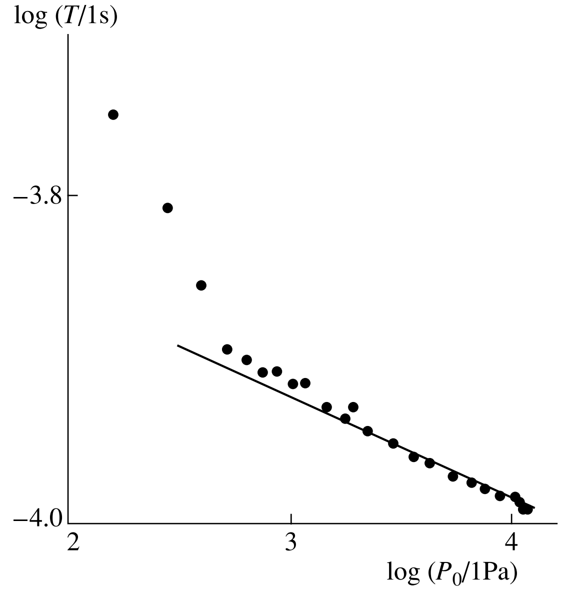

Fig. 2. Duration of LF pulses versus the static pressure. The straight line represents the function $T(P_0) \sim P_0^{-1/16}$.

This paper describes similar (to those reported in [5–8]) experimental and theoretical studies of self-demodulation of HF acoustic pulses and propagation of the LF pulses in partially water-saturated river sand. Results of these studies will be used to show that HF pulse demodulation in this medium is qualitatively different in its character from that observed in dry and in almost completely water-saturated sand.

IDEA OF THE EXPERIMENT

The measurements used the experimental setup were described in [5–7] with the only difference that partially saturated river sand was produced by pouring water into dry sand (rather than by putting sand into water); as a result, air bubbles could remain in the sand (in voids between the sand grains), so that water saturation of sand was less than unity. Due to thin liquid layers ("bridges"), the presence of gas in this three-phase medium apparently causes surface tension forces, which, on the one hand, weaken the Hertz-type clapping nonlinearity (which breaks contacts between loosely pressed grains) and, on the other hand, favor the capillary nonlinearity of thin liquid layers [17]. As a result, the nonlinear acoustic properties of partially water-saturated river sand qualitatively differ from the nonlinear properties of both dry sand and completely water-saturated sand, in which the clapping nonlinearity predominates.

EXPERIMENTAL RESULTS

As in [5–8], the excitation of short (T_1 = 80 μs) and long (T_2 = 1300 μs) HF acoustic pulses at a frequency of f = 180 kHz was accompanied by the self-demodulation effect. Oscillograms of envelopes of the radiated HF pulses and of the waveforms of the demodulated LF pulses received by an accelerometer are shown in Figs. 1a and 1b; their shape (as in [5–8]) is close to the third time derivative of the envelope of the HF pumping pulse and, therefore, the parametric radiator operated in the Westerwelt mode [18, 19]. When the static pressure P_0 in the sand increases, the duration $T = T(P_0)$ (see notations in Fig. 1a) of the demodulated video pulses decreases (Fig. 2), because the attenuation in sand decreases, whereas their propagation velocity $C = C(P_0)$ and amplitude $A = A(P_0)$ increase (Figs. 3 and 4). The following dependences are valid in the range $P_0 \geq 2 \times 10^3$ Pa (when the pressure in sand is primarily determined by a system of weights with a mass of $M \geq 5.6$ kg):

$$T(P_0) \sim P_0^{-1/16}, \quad (1)$$

$$C(P_0) \sim P_0^{1/6}, \quad (2)$$

$$A(P_0) \sim P_0^{1/3}. \quad (3)$$

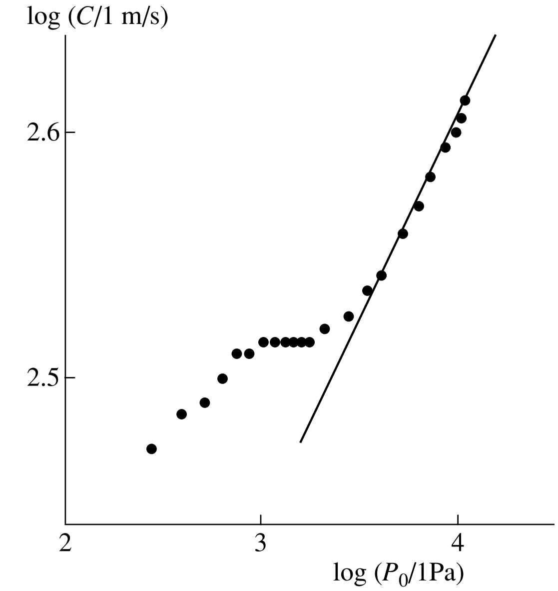

Fig. 3. Propagation velocity of LF pulses versus the static pressure. The straight line represents the function $C(P_0) \sim P_0^{1/6}$.

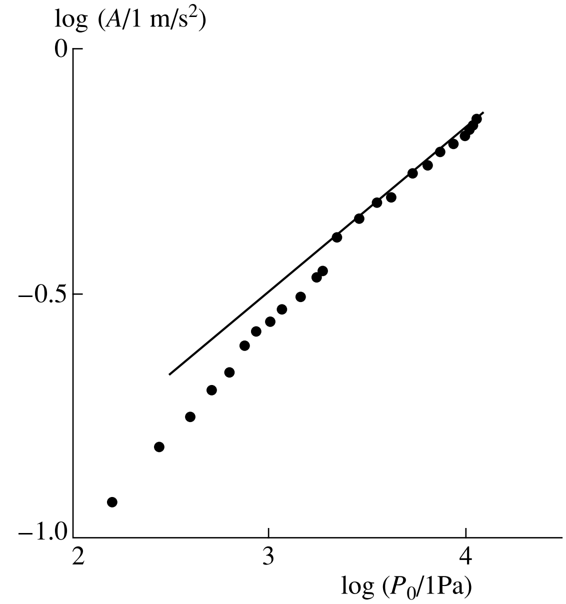

Fig. 4. Amplitude of LF pulses versus the static pressure. The straight line represents the function $A(P_0) \sim P_0^{1/3}$.

Here, as in [5–8], the function $C = C(P_0)$ corresponds to the Hertzian dependence of the static pressure P_0 on static compressive strain ε_0 [20–22]:

$$P_0 = B(\varepsilon_0)^{3/2}, \quad B = \text{const}. \tag{4}$$

This expression, in the linear approximation, yields the equation of state (i.e., the relation between acoustic stress σ and strain ε)

$$\sigma(\varepsilon, P_0) = (3/2)B^{2/3}P_0^{1/3}\varepsilon. \tag{5}$$

Figure 5 shows the amplitude A_0 of the demodulated LF pulses as a function of the amplitude ε_0 of short HF pumping pulses (at $P_0 = 10^4$ Pa). As can be seen from this figure, in the range $5 \times 10^{-6} \le \varepsilon_0 \le 1.2 \times 10^{-5}$, the relation $A_0 \sim \varepsilon_0^n$, where $n \cong 2$, is valid and, therefore, acoustic nonlinearity in the range $5 \times 10^{-6} \le \varepsilon_0 \le 1.2 \times 10^{-5}$ must be quadratic rather than Hertzian ($n \cong 3/2$) for both dry and completely water-saturated sand. The reason for this may be the effect of surface tension of thin liquid layers that lie between the sand grains and border on gas, which first of all affects the properties of a considerable part of weak contacts, for which static strain is much smaller than the average strain and which are responsible for the acoustic nonlinearity in sand with the Hertzian exponent $n = 3/2$ [5–8, 23]. Thus, to analytically describe the quadratic demodulation of HF pulses and the propagation of LF video pulses in partially water-saturated sand, equation of state (5) should be supplemented with the quadratic nonlinearity term $\alpha(P_0)\varepsilon^2$ and a dissipation term, which is responsible for the attenuation of acoustic waves:

$$\sigma(\varepsilon, P_0) = (3/2)B^{2/3}P_0^{1/3}\varepsilon - \alpha(P_0)\varepsilon^2 + L(P_0, \dot{\varepsilon}), \tag{6}$$

where $L(P_0, \dot{\varepsilon})$ is the linear operator, which determines the frequency dependence of the damping constant of acoustic waves in sand.

Now, let us use equation of state (6) to describe the processes observed in the experiments (in the pressure

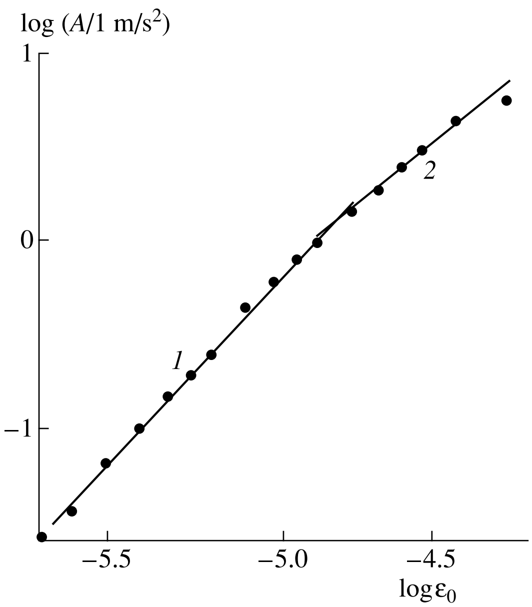

Fig. 5. Amplitude of the demodulated pulse versus the strain amplitude of the HF pulse ($P_0 = 10^4$ Pa). The straight lines represent the functions (*1*) $A_0 \sim \varepsilon_0^2$ and (*2*) $A_0 \sim \varepsilon_0^{3/2}$.

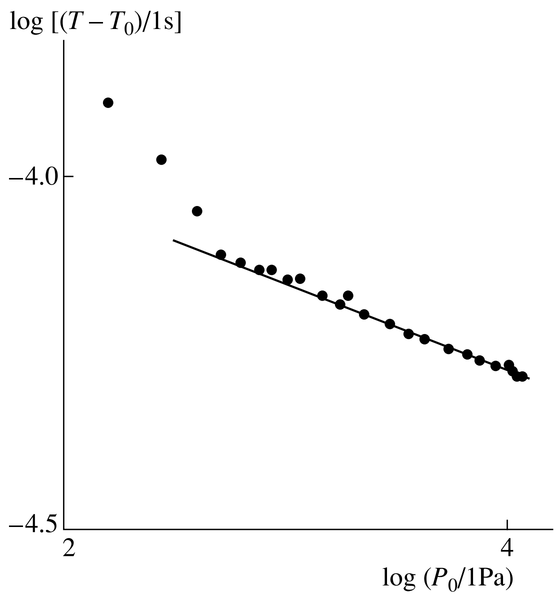

Fig. 6. Function $T(P_0) - T_0 = \beta(P_0)z$ versus the static pressure. The straight line represents the function $\beta(P_0) \sim P_0^{-1/8}$.

range $P_0 \geq 2 \times 10^3$ Pa) and determine the parameters of this equation. Let the boundary condition at the radiator be

$$\epsilon_1(r, z = 0, t) = \varepsilon_0 \Pi(r/a)\Phi(t/T_0)\sin\omega t, \qquad (7)$$

where $\Pi(r/a) = 1$ for $r/a \leq 1$ and $\Pi(r/a) = 0$ for $r/a > 1$, a is the radius of the radiator, and $\Phi(t/T_0)$ and T_0 are the envelope and duration of the HF pumping pulse. After some algebraic transformations similar to those performed in [5–8], we obtain an expression for the demodulated pulse received by an accelerometer located in the far-field region on the radiator axis:

$$A(\tau, z) = -\frac{a^2\alpha(P_0)\varepsilon_0^2}{8\pi\rho C^3(P_0)z}\frac{\partial}{\partial\tau}$$

$$\times \int_0^z \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Omega^2\Phi^2(\tau'/T_0)\exp[-2\beta_1(P_0)\omega z'] \qquad (8)$$

$$\times \exp[-\beta_2(P_0)|\Omega|z - j\Omega(\tau' - \tau)]dz'd\tau'd\Omega,$$

where $\tau = t - z/C(P_0)$, ρ is the density, and $\beta_{1,2}(P_0) = \theta_{1,2}(P_0)/2\pi C(P_0)$ with $\beta_{1,2}(P_0)$ and $\theta_{1,2}(P_0)$ being the HF and LF coefficients and damping constants, respectively. (We assume here that the damping constant $\theta_2(P_0)$ of river sand in the LF range is independent of frequency (see review [24]).)

For the pumping pulse envelope in the form $\Phi(\tau/T_0) = [1 + (\tau/T_0)^2]^{-1/2}$, the integral in Eq. (8) can be calculated explicitly. In this case, the waveform $A(\tau, z)$, amplitude $A(P_0)$, and duration $T(P_0)$ of the demodulated LF pulse are determined by the expressions (at $2\beta_1(P_0)\omega z \gg 1$)

$$A(\tau, z) = -\frac{\pi a^2\gamma(P_0)T_0\varepsilon_0^2}{4\omega\theta_1(P_0)T(P_0)z}\frac{\partial^3}{\partial\tau^3}\left\{\frac{1}{1 + [\tau/T(P_0)]^2}\right\}, (9)$$

$$A(P_0) \cong \frac{3.64a^2\gamma(P_0)T_0\varepsilon_0^2}{\omega\theta_1(P_0)T^4(P_0)z}, \qquad (10)$$

$$T(P_0) = T_0 + \beta_2(P_0)z, \qquad (11)$$

where $\gamma(P_0) = \alpha(P_0)/\rho C^2(P_0)$ is the quadratic nonlinearity parameter.

Certain acoustic parameters of partially water-saturated sand can be estimated by comparing experimental (1)–(3) and theoretical (9)–(11) dependences. First, we set $T_0 \cong 50$ μs and use Fig. 6 and Eq. (11) to determine the LF coefficient $\beta_2 = \beta_2(P_0)$ and damping constant $\theta_2(P_0)$: $\beta_2(P_0) \sim (P_0/P_m)^{-1/8}$, $\theta_2(P_0) \approx \theta_m(P_0/P_m)^q$, where $P_m = 10^4$ Pa, $q = 1/24$, and $\theta_m = 1.3$. Note that, here, the damping constant slowly but noticeably grows with pressure P_0, unlike the behavior observed in almost completely water-saturated sand, for which the damping constant decreases as a power function with the exponent $q = -1/3$ as the pressure increases [5–8].

Further, Eq. (10) yields the expression for the dimensionless ratio $\Gamma(P_0) = \gamma(P_0)/\theta_1(P_0)$ of the quadratic nonlinearity parameter to the HF damping constant; this important characteristic of the medium determines the efficiency of the parametric acoustic radiator [18, 19]. Figure 7 shows the function $\Gamma = \Gamma(P_0)$ obtained for the dependences $A = A(P_0)$ and $T = T(P_0)$ determined above at $a = 4$ cm, $\varepsilon_0 = 10^{-5}$, $\omega = 2\pi \times 1.8 \times 10^5$ s^{-1}, $T_0 = 50$ μs, $C_0 \cong 3.1 \times 10^4$ cm/s, and $z = 11$ cm. It can be seen that, for the partially water-saturated sand, the ratio $\Gamma = \Gamma(P_0)$ noticeably grows ($\Gamma(P_0) \sim P_0^{1/12}$) with increasing static pressure P_0, unlike the behavior observed in [5–8], where this ratio was found to fall for dry sand ($\Gamma(P_0) \sim P_0^{-1/6}$) and for almost completely water-saturated sand. (Recall, however, that the nonlinearity of the medium in the latter cases was Hertzian rather than quadratic.)

Finally, note that, to explain the high amplitude of the demodulated pulse (and the dependence of its amplitude as $\sim\varepsilon_0^{3/2}$) in dry and completely water-saturated sand, we proposed [5–8] to take into account loose intergrain contacts. Their number in the medium can be comparable to or even greater than the number of average-loaded contacts with the static compressive strain $\mathcal{E}_0$, which determine the elastic modulus of the granular medium. As shown in [14, 23], loose contacts with the loading $\mu\mathcal{E}_0$ at a small parameter $\mu \ll 1$ ($\mu \approx 1$ characterizes the average-loaded contacts) give a negligible contribution $\sim\mu^{1/2} \ll 1$ to the elastic modulus; however, they are the very factor that provide the dominant contribution $\sim\mu^{-1/2} \gg 1$ to the quadratic nonlinearity at small pumping wave amplitudes $\varepsilon_0 < \mu\mathcal{E}_0$. At high amplitudes $\varepsilon_0 > \mu\mathcal{E}_0$, the loose contacts start "clapping" and produce the Hertzian nonlinearity. For the dependence of the amplitude of the demodulated pulse on loose contacts, the shape of their distribution function $n = n(\mu)$ at $\mu \ll 1$ is very important. The resulting behavior of the amplitude of the demodulated pulse as $\sim\varepsilon_0^{3/2}$ is observed only when the distribution $n = n(\mu)$ is strongly localized near the zero values (this was the distribution used in [5–8]). When the distribution $n = n(\mu)$ is close to the uniform distribution $n(\mu) = \text{const}$, the proportion of the clapping contacts increases with the pumping amplitude ε_0, so that the amplitude of the demodulated pulse grows faster than prescribed by the Hertzian law ($\sim\varepsilon_0^{3/2}$) and the behavior remains close to $\sim\varepsilon_0^2$, although it is produced by the clapping contacts. Until the strain $\varepsilon_0 \sim 10^{-5}$ exceeds the average static strain $\mathcal{E}_0$, the amplitude of the demodulated pulse appears to be noticeably higher (for example, by almost an order of magnitude when the numbers of loose and average-loaded contacts are comparable [14], which is typical of granular media) than could be expected for the quadratic nonlinearity of average-loaded contacts in the granular skeleton, because $(\varepsilon_0/\mathcal{E}_0)^{3/2} \gg (\varepsilon_0/\mathcal{E}_0)^2$ until $\varepsilon_0 \sim \mathcal{E}_0$. In this situation, the nonlinear term in the equation of state remains smaller than the linear term also until $\varepsilon_0 \sim \mathcal{E}_0$. These arguments show that we should actually expect a similar behavior (and level) of the acoustic nonlinearity of dry and water-saturated sand [5–8], because the total saturation with a liquid must not noticeably change the distribution function of loose contacts, which is strongly localized at $\mu \ll 1$, this property being typical of dry media [14]. Conversely, unlike these extreme cases, the partial water saturation strongly changes the character of acoustic nonlinearity of the granular medium, which may be attributed to capillary forces, whose effect on the weakest (and most nonlinear) contacts is particularly high, connecting them by capillary bridges and preventing them from clapping. This modifies the $n = n(\mu)$ distribution by eliminating its increasing behavior for low pressures ($\mu \ll 1$) and making the distribution function more uniform. The deficiency of data on the contact distribution function in real granular media and the complexity of describing the capillary effects prevent us from predicting the resulting behavior of the nonlinearity versus water content and static pressure in detail. One can, however, expect that, under certain conditions, partially water-saturated sand must exhibit a behavior transition from the quadratic nonlinearity of closed contacts to the clapping Hertzian nonlinearity at high pumping amplitudes. Indeed, this transitional behavior of the demodulated pulse amplitude from $A(\varepsilon_0) \sim \varepsilon_0^2$ to $A(\varepsilon_0) \sim \varepsilon_0^{3/2}$ was observed at $\varepsilon_0 \approx 1.2 \times 10^{-5}$ (see Fig. 5).

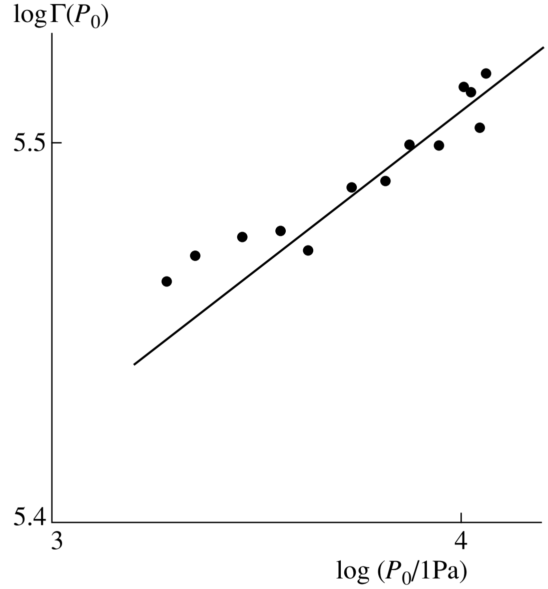

Fig. 7. Parameter $\Gamma = \Gamma(P_0)$ versus the static pressure. The straight line represents the function $\Gamma(P_0) \sim P_0^{1/12}$.

CONCLUSIONS

In this paper, we presented experimental and theoretical results on the demodulation of HF acoustic pulses and on the propagation of the LF pulses in partially water-saturated river sand. It was found that the acoustic nonlinearity of this three-phase medium is quadratic in a substantial part of the pumping amplitude range (up to $\varepsilon_0 \sim \mathcal{E}_0$), unlike the Hertzian clapping contact nonlinearity observed in both dry and almost completely water-saturated river sand [5–8]. This difference is presumably caused by capillary forces in thin liquid "bridges" between the grains, which (i) change the distribution of the weakly compressed grains, i.e., the most nonlinear ones, in the initial pressure, (ii) prevent them from exhibiting their clapping behavior, and (iii) favor the capillary and viscous nonlinearity of thin liquid layers between the grains. It should apparently be expected that the quadratic nonlinearity parameter of partially water-saturated sand depends on the concentration of these bridges, which can be used to determine the water content in the medium. To make this possible, one should, however, formulate not a phenomenological but a macroscopic equation of state whose basic parameters (elasticity, nonlinearity, and attenuation) depend on the structural characteristics of the medium (grain packing, grain size distribution, grain concentration, surface tension and viscosity of the liquid, contact angle, etc.).

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