

Nonlinear Relaxational Absorption/Transparency for Acoustic Waves Due to Thermoelastic Effect

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Summary

A thermoelastic mechanism that manifests itself as nonlinear dissipation of acoustic waves without implying either hysteresis or nonlinear viscous-like losses is considered. This idea was earlier considered at rheological level by the present authors for microinhomogeneous solids with defects possessing elastic nonlinearity and linear viscous-like dissipation. In the present paper, this mechanism is discussed first from a more general point of view in terms of relaxators with parameters varied due to purely reactive nonlinearity. Then it is considered in details for a practically important special case of such relaxators in the form of cracks containing inner contacts. It is shown that, due to strong stress concentration at the contacts and due to enhanced temperature gradients, the thermoelastic dissipation at these relaxators can be very efficient. Implications for linear small-amplitude losses as well as nonlinear effects of induced transparency and induced dissipation are discussed. The two latter effects may coexist and be observed at different frequency ranges (below and above the relaxation maximum). The considered amplitude-dependent effects are typical of the same defects that may also exhibit hysteretic properties due to adhesion or friction mechanisms. Thus the discussed non-hysteretic and hysteretic mechanisms may be activated simultaneously, which should be taken into account in theoretical interpretation of observations.

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1. Introduction

There is currently a growing number of experimental indications and theoretical arguments that the hysteretic nonlinearity [1, 2, 3] of micro-inhomogeneous and damaged solids is not the only reason for the nonlinear dissipation of acoustic waves in these materials [4, 5, 6, 7]. For the explanation of multiple experiments on sound damping by sound in polycrystalline metals ([8, 9, 10], and the references therein) and sandstone [11] it was necessary, in addition to nonlinear hysteretic dissipation, to introduce phenomenologically the dependence on wave amplitude for the parameters, which control viscous-like dissipation [8, 9, 10, 11, 12].

It should be mentioned further that experimentally not only induced (or amplitude-dependent) additional absorption is observed, but also induced (or self-induced) transparency, in particular, in sand, water saturated sand, rocks and crack-containing samples [12, 13, 14, 15]. However, this is not only the observation of the induced transparency phenomena, which excludes the explanations based on the mechanism of hysteretic nonlinearity. Actually, it has been recently demonstrated that effects of both self-induced

transparency and the transparency induced by one acoustic wave for the propagation of another one are theoretically possible due to manifestation of hysteretic nonlinearity only [3, 16, 17, 18]. The problem is that in many cases the physical phenomena contributing to hysteretic nonlinear dissipation (such as the stick-slip motion of the contacts, for example [19]) have a threshold in strain amplitude, although estimates [19, 20, 21] and the experiments [15, 22] demonstrate that the efficient and high sensitive to additional loading dissipation in many materials (e.g. rocks) takes place even when these threshold phenomena are forbidden because of far subatomic displacements of interfaces at micro-defects.

Consequently, for the explanation of the low-amplitude acoustic dissipation it is necessary to propose possible physical mechanisms without a threshold. Since all mechanisms of linear sound absorption have no threshold by definition, then it looks reasonable to investigate the possibility how the amplitude-dependent dissipation in its physical origin can be related to linear dissipation processes. Actually quite recently it has been argued [4] that nonlinear absorption naturally results from the linear absorption mechanisms operating in strongly elastically nonlinear defects. The physics of the non-hysteretic nonlinear absorption phenomena in this model is quite clear. The linear absorption by the defects depends on the local values of strain rate and on the features of the defect geometry.

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Both the local strain and geometrical parameters of the defects (e.g. microcontacts) depend in a highly nonlinear manner on the applied average stress in the material due to strongly enhanced stress-sensitivity of highly-compliant defects and due to stress concentration in the vicinity of the defects. As a result, the ratio of energy dissipated by these defects to the energy accumulated in the material becomes pronouncedly amplitude-dependent, although the background mechanism of the dissipation remains linear.

In the present publication, for the theoretical description of nonlinear absorption/transparency phenomena in acoustics of micro-inhomogeneous materials we propose basically the same physical principle as in [4], that is the modification of the local linear absorption due to strong local elastic nonlinearity. However, starting from the relaxational absorption due to viscous-like losses analysed in [4], we use a more general relaxation model for the description of linear losses (which includes the viscous losses as a special case). The application of the relaxational absorption model provides at least three important advantages. First, it gives a clear physical illustration for the possibility of both additional nonlinearly-induced absorption and transparency. Second, by using an appropriately chosen distribution of the linear relaxation times, the model provides the opportunity to fit rather arbitrary dependences of the linear (and the nonlinear) absorption on frequency. And, third, it is possible to associate the relaxation processes with quite definite physical processes in the acoustical systems. In particular, the sound absorption due to irreversible processes of heat conduction is known [23] as an example of relaxational absorption.

We propose here both qualitative and quantitative description of thermoelastic absorption of sound by contacts between the grains and the crack lips, and elucidate how this mechanism can induce nonlinear absorption/transparency due to high compliance of the elastically nonlinear contacts. Our theory is, in particular, relevant to the explanation of the recent experimental observations of the Luxembourg–Gorky effect for elastic waves and induced transparency/attenuation in rod-resonators containing cracks [15, 22].

The paper is organized as follows. In section 2 we remind the mathematical formalism for the description of the relaxational absorption of sound. In section 3 we discuss how a formal assumption of the dependence of a single relaxation time in the system on the wave amplitude leads to a qualitative understanding of the effects of the induced absorption and transparency. In section 4 it is demonstrated how the relaxation process might be identified with the processes of the irreversible heat conduction in the regions of stress concentration and we present the analytical description of the dissipation. In section 5 it is described how processes of the linear absorption become wave amplitude-dependent due to strong elastic nonlinearity of the regions of stress concentration. In Section 6 the estimates for the magnitude of the considered effects in some practical situations are given followed by discussion and the conclusions.

2. Linear relaxational absorption of sound waves

Perhaps the most general phenomenological description of the linear sound absorption in the material can be formulated by assuming that it can be attributed to some relaxation processes in the material. It is commonly accepted that the physical parameters of a system can not immediately follow the external action but rather they attain the equilibrium values, required by the external action, with some delay. This leads to the memory or rheology phenomena in the system: the current value of a parameter depends not only on the current value of the external loading but, in general, on the complete history of action. In many cases the approach of a parameter to a new equilibrium value can be approximated as being exponential in time.

In mechanics [24] and in nonlinear acoustics [25] this phenomenological approach is attributed to Mandelshtam and Leontovich. Here we briefly recall how it can be mathematically formalized. It is assumed that the acoustic stress σ (that is the deviation of stress from its value in the absence of acoustic loading) depends not only on the acoustic strain s but also on some additional parameters related to some additional internal degrees of freedoms in the material. Assuming the existence of a single internal parameter, its deviation from the equilibrium value in the absence of acoustic loading is denoted by p . Further we call p the acoustical relaxation parameter. Thus the stress is described by dependence $\sigma = \sigma(s, p)$. With all acoustic variables being small (when appropriately normalized if necessary) the latter relation can be approximated by the first two terms of its Taylor expansion $\sigma = \sigma(s, p) \approx s \partial_s \sigma(0, 0) + p \partial_p \sigma(0, 0)$. It is the parameter p that has a tendency to approach its equilibrium value $p_0(s(t))$, which should correspond to static strain coinciding with the current variable strain value $s(t)$. This approaching is not instantaneous, but proceeds with a delay, and the dynamics of this phenomenon is approximated by the equation $\partial_t p = -[p - p_0(s(t))]/\tau$, where τ is the relaxation time. The solution of this equation leads to $p = 1/\tau \int_{-\infty}^t p_0(s(t')) \exp((t-t')/\tau) dt'$. In accordance with this result, as expected, the current value p of the internal parameter does not coincide with the equilibrium value p_0 but has a memory on the history of strain loading. This memory, however, exponentially “weakens” with time.

In linear acoustics it is additionally assumed that the equilibrium (static) value of the unspecified parameter p_0 is itself proportional to acoustic strain. In other words it is assumed that the dependence $p_0 = p_0(s)$ is analytical and can be approximated by the first term of its Taylor expansion $p_0(s) \simeq s \partial_s p_0$. Then the stress/strain relation takes the final form

$$\sigma = Es - \Delta \bar{E} \int_{-\infty}^t \partial_r s e^{-(t-t')/\tau} dt', \quad (1)$$

where $E = \partial_s \sigma(0, 0) + \partial_p \sigma(0, 0) \partial_s p_0(0)$ is the static (equilibrium) Young modulus of the material, and $\Delta \bar{E} =$

$\partial_p \sigma(0, 0) \partial_s p_0(0)$ is the contribution to this modulus from an internal parameter (the latter corresponds to the existence of an additional degree of freedom in the system). Commonly the second term in equation (1) describes a small deviation from the static Hooke's law.

When the rheological model described by equation (1) is substituted into the acoustic wave equation, and the plane acoustic wave of a wave number k and of a frequency ω is assumed (with mechanical displacement $u \propto \exp[i(kx - \omega t)]$, so that strain $s = \partial_x u$), the dispersion relation $k = k(\omega)$ for the acoustic wave in the material is obtained. The real part of the wave number describes the phase velocity of sound

$$c \equiv \frac{\omega}{\Re\{k\}} \simeq \sqrt{\frac{E}{\rho}} \left[1 + \left(\frac{-\Delta \bar{E}}{2E} \right) \frac{\omega^2}{\omega^2 + \tau^{-2}} \right], \quad (2)$$

where ρ is the material density. The imaginary part of the wave number provides the sound absorption coefficient

$$\beta \equiv \Im\{k\} \simeq \sqrt{\frac{\rho}{E}} \left(\frac{-\Delta \bar{E}}{2E} \right) \frac{\omega^2/\tau}{\omega^2 + \tau^{-2}}. \quad (3)$$

Note that the absorption of acoustic energy requires that $\beta > 0$ in equation (3), which corresponds to condition $\Delta \bar{E} < 0$. The latter in accordance with equation (2) implies that the sound velocity at high frequencies $c_\infty \equiv c(\omega \ll 1/\tau) \simeq \sqrt{E/\rho} [1 + |\Delta \bar{E}|/2E]$ exceeds quasi-equilibrium sound velocity at low frequencies $c(\omega \gg 1/\tau) \simeq \sqrt{E/\rho} \equiv c_0$. This is in agreement with the physical intuition indicating that at higher frequencies the material has not enough time to rearrange its own structure in order to accommodate to the external action (what it should do in accordance with Le Chatelier principle), which results in hardening of the non-relaxed material at higher frequencies. In the derivation of equation (2) and equation (3) the assumption $|\Delta \bar{E}|/E \ll 1$, essential for the weak deviation of the material's rheology from the Hooke's law, has been used. Consequently the strength of the relaxation contribution to stress/strain relationship can be characterized by the parameter $|\Delta \bar{E}|/(2E) \simeq c_\infty/c_0 - 1$.

The presentation in equation (3) describes, for the frequencies lower than the inverse relaxation time ($\omega \ll 1/\tau$), a viscous type absorption with a coefficient $\beta \propto \omega^2$. Equation (3) describing the growth of the absorption coefficient with the increase in the acoustic frequency, also indicates that β saturates at high frequencies ($\omega \gg 1/\tau$). It is important that the decrement θ of the acoustic wave in a medium with a single τ exhibits a pronounced relaxation peak of the form:

$$\theta \equiv 2\pi \frac{\Im\{k\}}{\Re\{k\}} \simeq \pi \left(\frac{|\Delta \bar{E}|}{E} \right) \frac{\omega/\tau}{\omega^2 + \tau^{-2}}. \quad (4)$$

It follows from equation (4) that the maximum decrement is expected for $\omega = 1/\tau$. The decrement increases at lower frequencies and diminishes at higher frequencies

$$\theta \simeq \pi \left(\frac{|\Delta \bar{E}|}{E} \right) \begin{cases} \omega\tau & \text{if } \omega \ll 1/\tau, \\ (\omega\tau)^{-1} & \text{if } \omega \gg 1/\tau. \end{cases} \quad (5)$$

Importance of the described relaxation model is related to the opportunity of modelling rather arbitrary dependencies of the decrement on frequency by assuming a continuous spectrum of the relaxation times in the system and by appropriately choosing the dependence $\Delta E(\tau)$ of the "density" of relaxation processes with different relaxation times [26, 27, 28]

$$\theta \simeq \frac{\pi}{E} \int_0^\infty |\Delta E(\tau)| \frac{\omega/\tau}{\omega^2 + \tau^{-2}} d\tau. \quad (6)$$

In other words quite arbitrary dependencies $\theta = \theta(\omega)$ can be modelled as a superposition of individual relaxation peaks (4) with weight $\Delta E(\tau)$ varying continuously. In particular, this is one of very few currently accepted ways to model phenomenologically a nearly-constant decrement (corresponding to $\alpha \propto \omega$) commonly observed in rocks and other micro-inhomogeneous materials in a very wide frequency range [26, 27, 28]. This behaviour can be modelled by assuming that $\Delta E(\tau)$ is inverse proportional to τ in a sufficiently wide interval of the relaxation times: $\Delta E(\tau) = (\Delta \bar{E}/\tau)[H(\tau - \tau_1) - H(\tau - \tau_2)]$ for $\tau_1 \ll \tau \ll \tau_2$, where $H(\cdot)$ is the Heaviside function. In this case the result in equation (6) predicts that $\theta \approx \pi^2 |\Delta \bar{E}|/2E$ in a wide frequency interval $1/\tau_2 \ll \omega \ll 1/\tau_1$.

3. Nonlinear relaxational absorption / transparency for sound waves

From the physics point of view it looks reasonable in the first approximation to analyze how the presence of one wave in the material could modify the existing mechanisms (channels) for the absorption of another wave, rather than to consider possibilities for creation of some new channels of energy dissipation (although, in principle, the latter opportunity can not be excluded *a priori*).

Using this idea and starting from the framework of the phenomenological models (4) and (6), nonlinear (amplitude-dependent) phenomena in acoustics could be introduced considering how the propagation of an acoustic wave in the material modifies the characteristic times or/and the "strength" ΔE of the relaxation processes. For example, if in the presence of a sufficiently strong acoustic wave the relaxation time in the model (4) increases, then this causes the shift of the decrement relaxation peak to lower frequencies. Consequently, the dissipation in the material has to increase at low frequencies $\omega \ll 1/\tau$, demonstrating induced absorption, whereas at high frequencies $\omega \gg 1/\tau$ the dissipation should diminish, exhibiting induced transparency (Figure 1). This qualitative argumentation suggests a way how the nonlinear absorption (transparency) might be introduced based on the relaxation-type phenomenology. To understand the background physics of the phenomena it is necessary to study different macroscopic and microscopic models of possible relaxation processes. One of these models, relating the relaxation to irreversible heat conduction in the vicinity of the contacts between the crack lips in damaged materials or between the beads in granular materials, will be analysed in the next section.

4. Irreversible heat conduction in the vicinity of contacts as a relaxation process

Thermal conduction equation states that heat propagates in a diffusive manner when the temperature field is not homogeneous in space. Similar to gases, temperature of a solid rises when the latter is compressed. This effect can be taken into account by adding in the equation of heat conduction a term depending on the strain field. In the framework of thermoelasticity, the thermal conduction equation becomes [29]:

$$\partial_t T' - D \Delta T' = -\Gamma T_0 \partial_t s_{ii}, \quad (7)$$

where T' is the temperature deviation from the average temperature T_0 , D is the heat diffusivity, Γ is the Grüneisen parameter characterising the importance of the thermoelastic coupling ($\Gamma = \alpha K / (\rho C)$, where α is the volumetric thermal expansion coefficient, K is the bulk elastic modulus and C is the specific heat capacity per unit volume), s_{ii} is the trace of the strain tensor (local volume variation).

In accordance with equation (7), the correction to the temperature rate (i.e. the rate of the thermoelastic heat production) is proportional to the rate of local volume variation. If the volume variation is uniform in space, the temperature change is also uniform. A non-uniform volume variation leads to non uniform temperature change inducing heat flux between differently strained regions.

When two bodies are pressed together, they are generally non-conforming and they touch each other only at well localized contact regions creating non-uniform deformation field. The entire interaction force between the two bodies passes through the contacts which are therefore much stronger stressed than the surrounding material, so that the contacts are stress and strain concentration zones. Therefore the strain rate is also strongly enhanced when the stress is non-stationary. This non-uniform straining leads to greater temperature changes in the vicinity of contacts than in the rest of the bodies. The temperature field becomes non-uniform and heat flow occurs near the contacts.

An alternating heat flow is not completely reversible, so that a fraction of energy contributes to irreversible entropy increase. Since in the considered case the energy flow is due to acoustical straining, this can be interpreted as a loss of the acoustical energy [29]. The following part of this section is devoted to the evaluation of these losses for the case of point-like and elongated strip-like contacts.

These types of contacts occur respectively when spheres or parallel-oriented cylinders are pressed together. These contacts were named Hertzian after the studies of Heinrich Hertz in the late nineteenth century which opened the field of contact mechanics. In the region near a contact between two bodies, their surfaces can be parametrized by their curvature radii. The Hertz theory provides the characteristics of a contact between two surfaces of arbitrary curvature radii [30, 31]. Provided the strain field is known, the tem-

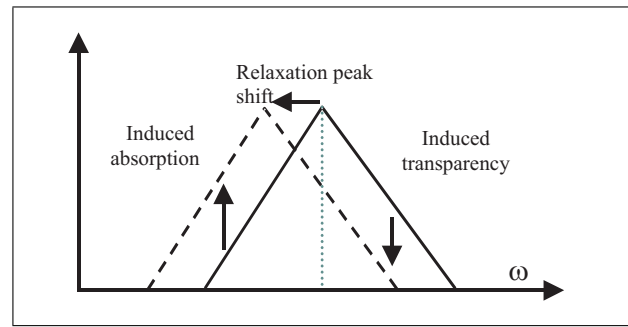


Figure 1. Schematic view of relaxation peak shifting: it can lead to induced transparency or absorption.

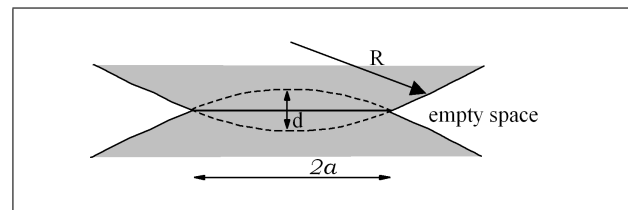


Figure 2. Schematic view of a contact between surfaces of curvature radii R .

perature distribution can be found from (7) by means of the spatial Fourier transform :

$$\tilde{T}' = -\Gamma T_0 \int_{-\infty}^t \partial_{t'} (\tilde{s}_{ii}) e^{-Dk^2(t-t')} dt', \quad (8)$$

where “ \sim ” stands for the spatial Fourier transform of a variable and k is the dual space variable. This temperature field can be used to evaluate the rate of growth of entropy and, correspondingly, the mechanical energy losses [29]:

$$\frac{dW}{dt} = -\frac{\kappa}{T_0} \int (\nabla T')^2 dV, \quad (9)$$

where κ is the thermal conductivity, $\kappa = DC\rho$.

To carry out the evaluation of the energy dissipation, we use a model strain field which keeps the major features of the Hertzian strain field [30] (i.e. characteristic dimension of the stress-localization near the contact and the stress vanishing outside this region), but allows for easier mathematical treatment than the exact Hertzian strain field. The model strain field used $s(r, t) = s_c e^{-r/a} \cos(\omega t)$ is cyclic in time of frequency ω and Gaussian in space with characteristic width a and maximum strain value s_c (indice c denotes the value at the center of the contact). This approximation by the order of magnitude correctly keeps the vanishing scale of the near-contacts stress-concentration roughly equal to the contact width as predicted by rigorous solutions [30, 31], and thus yields the correct order of magnitude of the predicted thermoelastic losses. Note also that the Gaussian strain distribution effectively cuts contributions to the integral (9) of the regions, where the empty volume in Figure 2 is indicated. Thus the integration of (9) can be carried over the whole space. Then the following solutions are derived for the energy ΔW_p dissipated

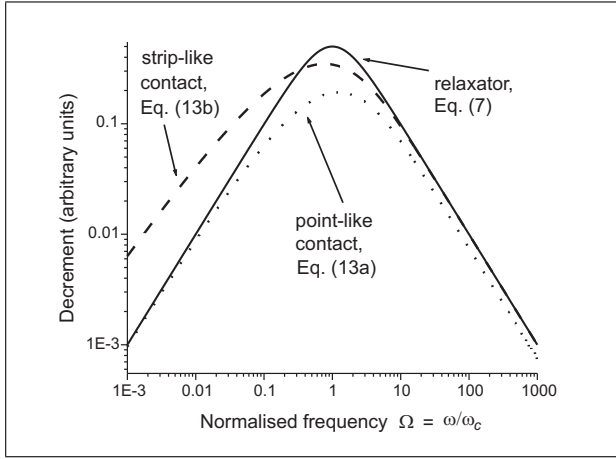


Figure 3. Frequency dependence of the decrement of a simple relaxator and of point- and strip-like contacts.

over a period at a circular (point-like) contact and for the amount ΔW_l dissipated at a cylindrical (strip-like) contact of length L_c :

$$\Delta W_p = \frac{\pi^{5/2} \kappa T_0 \Gamma^2}{\sqrt{2D}} s_c^2 a^3 f_p(\Omega),$$

$$f_p(\Omega) = \Omega \left[1 - \sqrt{2\pi\Omega} F(\sqrt{2\Omega/\pi}) \right], \quad (10a)$$

$$\Delta W_l = \frac{\pi^2 \kappa T_0 \Gamma^2}{2D} s_c^2 L_c a^2 f_l(\Omega),$$

$$f_l(\Omega) = -\Omega \left[Ci(\Omega) \cos \Omega + (Si(\Omega) - \pi/2) \sin \Omega \right], \quad (10b)$$

where $\Omega = \omega\tau/2$ is the normalised frequency, $\tau = a^2/D$ is the characteristic time, f_p and f_l are the functions describing the frequency dependence of the dissipated energy for point-like and strip-like contacts. The special functions F , Ci and Si are the Fresnel auxiliary function and the cosine and sine integrals, respectively [32]. The resultant frequency dependences are shown in Figure 3 together with the frequency dependence (4) for the relaxation peak rewritten in the normalized form $f_{\text{relax}}(\Omega) = \Omega/(1 + \Omega^2)$. The figure shows these dependences in double-logarithmic scale over 6 decades. The shape of this curve is similar to the experimental curves for the internal friction due to thermal conduction for waves in polycrystalline metals or for bending waves in bars [33]. In both cases, the characteristic dissipation frequency arise from the thermal conduction over a characteristic scale of the sample (size of crystallites and width of the bar, respectively).

The main common feature for all three curves in Figure 3 is that either relaxator (4) or the contacts are characterized by some frequency $\omega_c = 1/\tau$, near which the energy dissipated over a period is maximal, whereas below and above this frequency the functional behaviour of the thermoelastic energy losses is close to ω and $1/\omega$ dependencies, respectively, typical of the relaxational absorption. More exactly, the asymptotic behaviour of the

absorption at the cylinder-like contact at low frequencies $\omega \ll \omega_c = D/a^2$ is given by expression $\omega \ln \omega$, however a weak logarithmic dependence is not essential in the latter formula. Consequently the asymptotic behaviour of thermoelastic losses at the contacts is rather well approximated by a generalised linear relaxator considered in section 2. The amount of energy dissipated over a period is proportional to a combination of material properties $\kappa\Gamma^2/D$, to the average temperature T_0 , to the characteristic volumes of the contacts (a^3 or $L_c a^2$, respectively) and to the square of the characteristic material strain near the contact.

Asymptotic analysis of the thermal-conduction equation (7) provides an additional physical insight in the phenomena discussed. The spatial distribution of the temperature field in the considered problem is determined by the contact geometry: compression of the contact results in a local temperature rise T'_c in the vicinity of the contact. For a characteristic contact size a , the Laplacian term $\Delta T'$ in equation (7) can be then estimated as $\Delta T' \sim T'_c/a^2$ [25]. Similarly, since the temporal behaviour of the temperature distribution is controlled by the temporal evolution of the strain, for a sinusoidal excitation with cyclic frequency ω , the time derivative of temperature can be estimated as $\partial_t T' \sim T'_c \omega$. The relative weight of the two terms in the left-hand side of equation (7) essentially depends on the ratio between frequency ω and the characteristic frequency $\omega_c \sim D/a^2$. The local temperature rise T'_c at high and low frequencies can be then evaluated in the following way.

If $\omega \ll \omega_c$, then the spatial derivative term dominates, $\partial_t T' \ll D\Delta T'$, so that the approximation of equation (7) is given by $\Gamma T_0 \partial_t s_{ii} \approx D\Delta T' \sim DT'_c/a^2$, which yields the estimate for the temperature variation

$$T'_c = \Gamma T_0 s_c \omega a^2 / D \quad (11a)$$

$$= \Gamma T_0 s_c (\omega / \omega_c), \quad \text{at } \omega \ll \omega_c,$$

taking into account that $s_{ii} \approx s_c$ at the contact.

If $\omega \gg \omega_c$ then the term with the time-derivative dominates, $\partial_t T' \gg D\Delta T'$, and the thermal conduction equation can be approximated by $\Gamma T_0 \partial_t s_{ii} \approx \partial_t T' \sim T'_c \omega$, which yields the estimate

$$T'_c = \Gamma T_0 s_c \quad \text{at } \omega \gg \omega_c. \quad (11b)$$

Thus the local temperature variation T'_c increases with the increase in frequency from zero at low frequencies and reaches the saturation value $T'_c \sim \Gamma T_0 s_c$ when the excitation frequency approaches and then exceeds the characteristic value $\omega \sim \omega_c \sim 1/\tau$.

Further, for a contact with characteristic volume of strain-concentration equal to V_c we estimate the loss of the energy over a period using equation (9), which yields for the cylinder-like contact

$$\Delta W = \frac{2\pi}{\omega} \left\langle \frac{dW}{dt} \right\rangle \sim \frac{2\pi}{\omega} \frac{\kappa}{T_0} \left(\frac{T'_c}{a} \right)^2 V_c$$

$$= 2\pi \frac{\kappa T_c'^2 a^2 L_c}{T_0 D \omega \tau}, \quad (12)$$

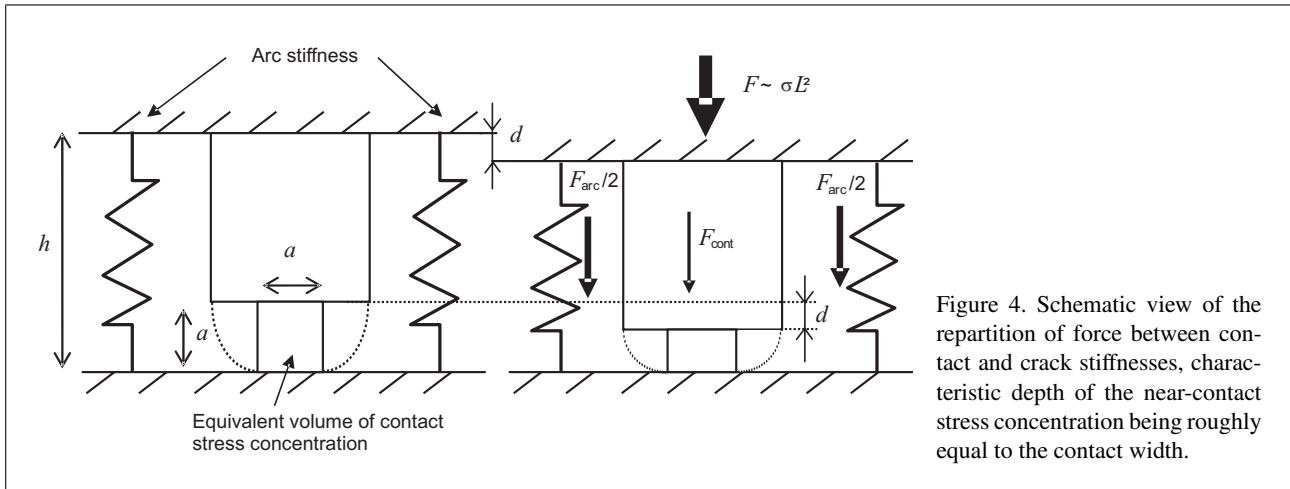


Figure 4. Schematic view of the repartition of force between contact and crack stiffnesses, characteristic depth of the near-contact stress concentration being roughly equal to the contact width.

where notation $\langle \dots \rangle$ means the time-averaging. Relation (12) for the strip-like (cylinder) contact is readily transformed into the expression for a point-like contact putting $L_c = a$, which corresponds to the difference between characteristic volume of the strain concentration regions, $L_c a^2$ for the strip-like contact and a^3 for the point-like one, respectively. Then one may substitute in equation (12) asymptotic values (11a) and (11b) for T'_c in order to get the low- and high-frequency approximate asymptotic solutions for the energy dissipated during a period

$$\Delta W_{LF} \sim 2\pi \frac{\kappa \Gamma^2 T_0 a^2 L_c}{D} \omega \tau s_c^2 \quad \text{at } \omega \ll \omega_c, \quad (13a)$$

$$\Delta W_{LF} \sim 2\pi \frac{\kappa \Gamma^2 T_0 a^2 L_c}{D} \frac{s_c^2}{\omega \tau} \quad \text{at } \omega \gg \omega_c. \quad (13b)$$

Note that from the physics point of view one may argue that, at $\omega \ll 1/\tau = \omega_c$, the energy dissipated increases with frequency rise because of sufficiently fast increase of the local temperature rise (described by equation 11a). Further, when $\omega \gg \omega_c$, the dissipated energy decreases with frequency rise due to combined effect of the diminishing of the period $2\pi/\omega$ with simultaneous saturation of the temperature variation equation (11b) due to enhanced heat diffusion. Expressions (13) indicate the same dependence on temperature, material parameters, width and strain at the contact, and similar asymptotic frequency behaviour as equation (10). Even the dissipation magnitude at the relaxation maximum can be reasonably well evaluated by matching the asymptotes (13a), (13b) at $\omega \approx \omega_c$.

The above considered asymptotic derivation (in which the ratio of the thermal wave length to the characteristic stress-localization scale is the key parameter) corroborates additionally the possibility to use the Gaussian approximation for the stress distribution if the characteristic vanishing scale is correctly kept.

Note further that the energy losses at the contact are still expressed via the local strain at the contact, whereas for comparison with experiments it is desirable to get expressions via average macroscopic acoustic strain in the sample. Let us consider first the important case of a material containing cracks with inner contacts. In the most part of

practically interesting cases (e.g. for damaged engineering materials or rocks) the crack density is sufficiently small, so that the averaged elastic modulus weakly differs from that for the intact matrix material. Let us suppose existence of a soft contact inside the crack. As it was argued in [15, 22] based on literature data and our own experimental indications, the contacts in cracks are mostly strip-like rather than point-like because of the 2D-character of crack initiation. In order to correctly estimate stress (strain) concentration at a contact one should take into account that the applied external loading is distributed between the arc-stiffness of the crack and the stiffness of the contact, the two stiffnesses acting in parallel. Besides, in comparison with the arc-stiffness of the cracks and, moreover, with the rest of the matrix material, the inner contacts can be much more compliant, so that the global displacement field remains nearly the same with and without the contact. Therefore, for such a contact, its deformation is controlled by the mean acoustic strain.

Further, as shown schematically in Figure 4 force $F \sim \sigma L^2$ [34] applied to the crack with characteristic area $\sim L^2$ (where σ is the stress in the material) is divided into a part F_{arc} supported by the arc-stiffness of the crack and a part F_{cont} supported by the stiffness of the contact with characteristic length $L_c \leq L$ and width a .

$$F = \sigma L^2 = F_{\text{arc}} + F_{\text{cont}}. \quad (14)$$

As argued above, due to its high compliance, the displacement d (see Figure 4) is the same for the contact region and for the crack interfaces. Compressibility of narrow cracks can be characterized by non-dimensional compliance parameter ζ , whose value is roughly equal to the crack aspect ratio $\zeta \approx h/L \ll 1$, not depending on a particular crack model [21]. Thus the force supported by the arc-stiffness of the crack can be written as:

$$F_{\text{arc}} \approx E \zeta \frac{d}{h} L^2 \approx EdL, \quad (15)$$

where E is elastic modulus of the matrix material and ζE the effective “elastic modulus” of the crack, d/h is the crack strain.

For expressing the force F_{cont} supported by the contact one should take into account that the characteristic depth of the near-contact stress concentration is roughly equal to the contact width a [30], as we have already used in the above estimates (see equations 10). Thus the force F_{cont} acting on the contact with area $S_c = aL_c$ experiencing strain d/a can be expressed as follows:

$$F_{\text{cont}} \approx E \frac{d}{a} a L_c = E d L_c. \quad (16)$$

Substitution of equations (15), (16) into equation (14) yields the following expressions relating the contact force F_{cont} and local contact stress $\sigma_c = F_{\text{cont}}/S_c$ with the average acoustic stress σ :

$$F_{\text{cont}} \approx \frac{L_c}{L_c + L} \sigma L^2, \quad (17)$$

$$\sigma_c \approx \frac{L^2}{(L_c + L)a} \sigma \sim \frac{L}{a} \sigma \gg \sigma. \quad (18)$$

The same relation $s_c \sim (L/a)s$ can be written for the average macroscopic strain and the local enhanced strain s_c at the contact, since even the increased strain is still much smaller than unity and we may use locally linear relation between the stress and strain in the material. It is important to underscore that the concentration factor in (18) weakly depends on the contact length $L_c \leq L$, so that at strip-like contacts strain concentration is roughly the same as at point like contacts.

Now we may substitute the concentration factor given by equation (18) into equation (13) and to recover result [22]

$$\Delta W_{LF} \approx 2\pi\omega T_0 \frac{\alpha^2 K^2}{\kappa} a^2 L_c L^2 s^2, \quad \omega \ll \omega_c \approx \kappa/(\rho C a^2), \quad (19a)$$

$$\Delta W_{HF} \approx \frac{2\pi}{\omega} \kappa T_0 \left(\frac{\alpha K}{\rho C} \right)^2 L_c \frac{L^2}{a^2} s^2, \quad \omega \gg \omega_c, \quad (19b)$$

$$\Delta W_{\text{cont}}^{\text{max}} \approx 2\pi T_0 \frac{\alpha^2 K^2}{\rho C} L_c L^2 s^2, \quad \omega \approx \omega_c. \quad (19c)$$

The decrement $\theta = \Delta W_{\text{diss}}/(Es^2V)$ is proportional to the ratio of the energy dissipated and stored in a volume V . The stored energy is proportional to the square of the acoustic wave strain s and the dissipated energy (19) is also proportional to the square of the strain s . The asymptotic forms of the contribution to the decrement from n contacts in cracks contained in a unit volume is then of the same form as the relaxational decrement (5):

$$\theta_{LF} \approx 2\pi\omega T_0 \frac{\alpha^2 K^2}{\kappa} a^2 L_c L^2 \frac{n}{E}, \quad \omega \ll \omega_c \approx \kappa/(\rho C a^2), \quad (20a)$$

$$\theta_{HF} \approx \frac{2\pi}{\omega} \kappa T_0 \left(\frac{\alpha K}{\rho C} \right)^2 L_c \frac{L^2}{a^2} \frac{n}{E}, \quad \omega \gg \omega_c, \quad (20b)$$

$$\theta_{\text{cont}}^{\text{max}} \approx 2\pi T_0 \frac{\alpha^2 K^2}{\rho C} L_c L^2 \frac{n}{E}, \quad \omega \approx \omega_c. \quad (20c)$$

Thermoelastic dissipation by a contact can then be identified to a relaxational process with a characteristic time $\tau \sim a^2/D$ which depends on the contact width. However, it is important to point out that the maximal value of energy dissipated by contacts in cracks does not depend on the contact width (see equations 19c, 20c). In a medium with distribution of contacts of different sizes, each contact contributes to the global decrement which can be presented in a form similar to equation (6) for relaxation processes with a wide spectrum.

In order to show that thermoelastic effect is not negligible with respect to other channels of internal dissipation, we will evaluate the relative change caused by the presence of contacts in initially defect-free steel and glass samples. Knopoff [35] reports typical values for the quality factor $Q = \pi/\theta$ to be of the order of 3000 and 300 for steel and glass respectively, leading to values of the decrement in the range of 10^{-3} and 10^{-2} . Let us consider the estimation for a rod of 1 cm diameter and 25 cm long. The elastic energy stored in the rod is of the form $Es^2V/2$ where V is the volume of the rod. If we suppose that this rod contain several cracks with similar characteristics, the maximum contribution to the decrement from the considered thermoelastic mechanism is given by:

$$\theta_{\text{max}} = \frac{\Delta W_{\text{max}}}{Es^2V} = \left(\frac{2\pi\kappa\Gamma^2 T_0}{DEV} \right) L^2 L_c N, \quad (21)$$

where N is the number of contacts in the sample. The factor in the bracket in the right-hand side of formula (21) depends on the material parameters and at $T_0 = 300^\circ\text{K}$ roughly equals to 500–2000 for steel and 300–3000 for glass. Assuming $N = 6$ contacts of length $L_c = 3$ mm in cracks of size $L = 4$ mm one obtains for the thermoelastic contribution to the decrement: $1.4\text{--}5.8 \cdot 10^{-4}$ for steel and $0.9\text{--}9 \cdot 10^{-4}$ for glass. In the cracked sample, the thermoelastic contribution from cracks and contacts to the decrement is added to its initial value. Expressing the dissipation variation in terms of the quality factor leads to a decrease of the order of 15–60% for the steel sample and of 1–10% for the glass-like materials. The estimated magnitudes of the quality factor decrease are expected at the frequency of maximum absorption. Let us evaluate this frequency in order to show that it could be sufficiently low to allow for the observation of both induced absorption and transparency (if the absorption frequency is higher than any measurable frequency, the effect of a downward shift of the absorption peak would only lead to induced absorption in the whole measuring range). Assuming that thermal diffusivity in glass and steel is of the order of $5 \cdot 10^{-7}$ and 10^{-5} m²/s respectively, and considering contact size in the range 1 to 10 μm lead to characteristic absorption frequency in the range from 800 Hz to 80 kHz in glass and from 16 kHz to 1.6 MHz in steel. This evaluation shows that the presence of contact in a medium is likely to produce noticeable change in the dissipation and may give rise to both induced absorption and transparency.

5. Amplitude dependent relaxation process due to high stress-sensitivity of the contacts

In this section we consider the nonlinear variation of the contact parameters under the action of a sufficiently strong average stress. Indeed, due to the high compliance of the contacts (comparing with the surrounding bulk material), quite moderate macroscopic stress in the material may cause rather significant deformation of the nearest contact vicinity. Since width a of a contact depends on the mutual approach d of the contacting surfaces (see Figure 2): when the contact is compressed, its area increases leading to its widening. This variation results in the consequent change of the thermoelastic relaxation at this contact and thus leads to the amplitude dependence of the relaxational dissipation as it was qualitatively discussed in section 3.

Let us consider a contact with the curvature radius R , which has initial characteristic width a_0 and initial mutual approach $d_0 = a_0^2/R$. An acoustic wave of strain amplitude s_a will produce a strain oscillation s_{ca} at the contact enhanced by the concentration factor (see equation 21): $s_{ca} = (L/a)s_a = d_a/a$. The oscillation amplitude d_a of the mutual approach caused by the acoustic wave then leads to the variation of the contacts width a according to the following expression:

$$a = \sqrt{Rd} = \sqrt{R(d_0 + d_a)} = a_0 \sqrt{1 + \frac{LR}{a_0^2} s_a}. \quad (22)$$

Formula (22) demonstrates the fact that the acoustic wave has an influence on the contact size. As is discussed in section 4, the contact size controls the characteristic frequency of the thermoelastic absorption peak. Consider now how this acoustically-induced variation can modify characteristics of the dissipation.

We will first discuss the effect of a large amplitude wave (pump) of frequency ω_{pm} on a weak probe wave with a higher frequency ω_{pr} . The total acoustic strain has the following form: $s_a = s_{pm} \cos(\omega_{pm}t) + s_{pr} \cos(\omega_{pr}t)$, where s_{pm} and s_{pr} are the pump and probe wave strain amplitudes, respectively. We suppose additionally that $\omega_{pm} \ll \omega_{pr}$. For the relatively high-frequency probe wave, the value of the contact radius is then slowly varying under the action of the low-frequency pump wave:

$$a = a_0 \sqrt{1 + \frac{LR}{a_0^2} s_{pm} \cos(\omega_{pm}t) H(\bullet)}$$

(the dot in the Heaviside function represents the same argument as in the square root). Due to the pump-controlled time-dependence of the contact width, the characteristic frequency of the absorption peak also becomes time-dependent. A strong enough pump wave can induce considerable variation in the contact size. For a sufficiently strong pump, the argument under the radical becomes negative during a part on the pump wave period, so that the contact gets into clapping regime (for pump strain $s_{pm} >$

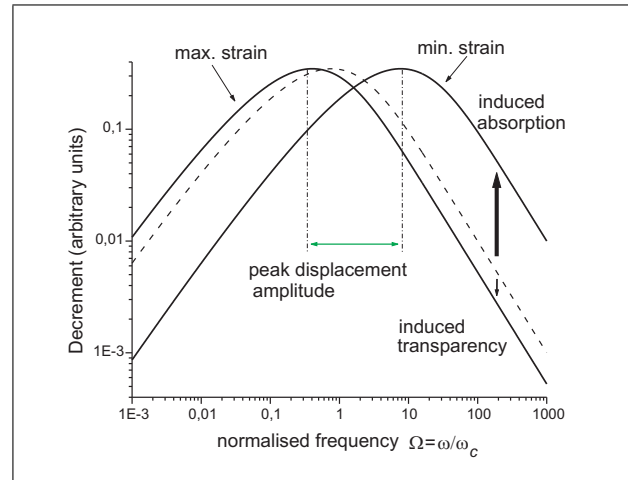


Figure 5. Frequency dependence of the decrement in different phases of the pump-oscillation period. The dashed curve is the unperturbed absorption peak (zero strain).

$a_0^2/(LR) = s_{th}$). In terms of the characteristic frequency of the absorption peak, this implies that during a pump period, it can be shifted from its initial value (without pump) to formally infinite frequency during the dilatation phase, when the contact size tends to zero. In contrast, during the compression phase, the relaxation peak is shifted to lower frequency.

Figure 5 shows the frequency dependences for the probe-wave decrement in the absence of the pump wave (middle dashed curve), during the maximum of the compression phase (left curve) and for the dilatation phase (right curve), the pump wave amplitude being at 90% of the clapping threshold. This modification of the absorption will manifest itself by a modulation of the amplitude of the probe wave with the period of modulation that is controlled by the pump wave. As Figure 5 indicates, a probe wave whose frequency is above the initial characteristic absorption frequency, will manifest induced absorption when the strain is minimal (see the bold arrow of Figure 5) and induced transparency when the strain is maximal (thin arrow). As the peak shift is more pronounced during minimum strain than during maximum, the average effect will be induced absorption. In contrast, if the probe wave frequency is below the initial relaxation peak frequency, then the effects will be opposite and the wave will experience induced transparency in average.

The average amplitude of the probe wave is related to the average value of the dissipation. Due to asymmetrical reaction of the contact to the external loading, its averaged state can be affected by an oscillating pump action, which may lead to the time-averaged variation of the probe-wave dissipation.

As previously, the contact size is controlled by the pump wave, but is not a constant at the timescale of the probe. The time-average contact size depends of the pump-oscillation amplitude in a different manner before and after transition to the clapping regime. Figure 6 shows how the average size $\langle a \rangle$ behaves as a function of the pump ampli-

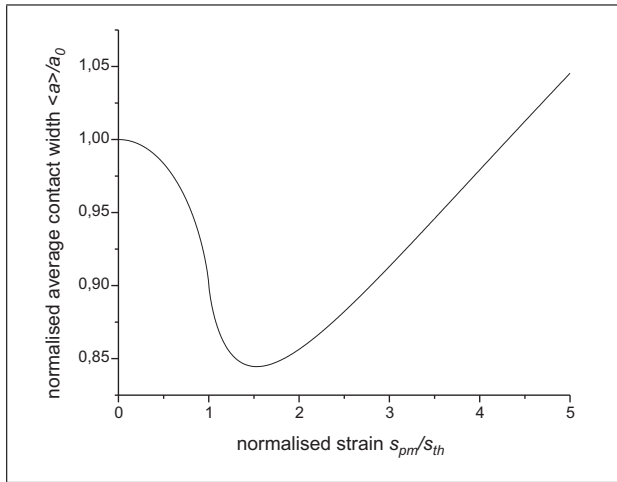


Figure 6. Evolution of the average contact width as a function of the pump strain.

tude normalised to the clapping threshold s_{pm}/s_{th} . It is important to underline the non-monotonous behaviour of the averaged contact width: after initial decreasing, the contact size increases in the essentially clapping regime. This is due to the fact that for the pump wave amplitude far higher than the clapping threshold, the contact size is zero during approximately half a period, but increases essentially during the other half period, resulting in the average contact-size growth.

This non monotonic variation of the average contact size induces non monotonic shift of the time averaged position of the absorption peak for the probe wave. For an initially closed contact, the average contact size could decrease roughly of 15%, inducing a 30% (quadratic) increase of absorption frequency. Further increase of the pump wave amplitude will cause decrease of the absorption frequency to a value that can be lower than 70% of its initial value if the amplitude is several times greater than the clapping threshold.

Such an effect on the absorption peak will manifest itself in the non monotonic variation of the probe wave amplitude depending on the amplitude of the pump wave. This effect has already been observed experimentally [22].

The considered thermoelastic mechanism of the amplitude-dependent dissipation can relate directly to observations of the Luxemburg-Gorky effect for elastic waves [6, 7]. This effect manifests itself by the transfer of the modulation of strong pump wave on a weak probe wave. Let us consider a pump wave of the form

$$s_{pm}(1 + B \cos(\omega_M t)) \cos(\omega_{pm} t),$$

where B is the modulation ratio and ω_M is the modulation frequency (much lower than the pump frequency). The average value of the contact size is again controlled by the amplitude of the pump wave, but since it is periodic in time, the average contact size becomes periodic too, and the characteristic absorption frequency in its turn becomes a periodic function with the modulation frequency of the pump wave (and its higher harmonics). Thus the probe

wave should acquire slow modulation much like in above considered case of the low-frequency (quasi-static) pump wave.

In the discussion above we have considered first of all the variation of the relaxational thermoelastic absorption of a weak wave under the action of another static or dynamic (acoustic) load on the sample containing cracks with inner contacts. However, in a similar manner, a sufficiently intensive acoustic wave can also cause self-induced variations in the dissipation in function of its own amplitude. By the order of magnitude these self induced variations can be estimated as it has been discussed for the probe wave above. It is important to note that along with the considered mechanism of the thermoelastic (linear) dissipation (and complementary variations in the elasticity), defects of the same type can also exhibit hysteretic properties, for example, due to manifestations of friction or adhesion at crack/contact interfaces. Thus the hysteretic properties and the considered non-hysteretic mechanism are inherent for the same type of the microstructure, so that their very similar manifestations can superimpose in different proportions for different amplitude and frequency ranges. In particular, such a superposition of the two mechanisms can essentially contribute to the ratio between the complementary amplitude-dependent variations in the dissipation and in elasticity observed in microinhomogeneous materials [36, ?]. Traditionally, this ratio (often called the Reed parameter in literature) is discussed in the framework of hysteretic mechanisms only; however, the superposition of the two aforementioned mechanisms should be taken into account in theoretical interpretations.

6. Conclusion

In the previous sections we have considered both general phenomenological features of the relaxational dissipation and the complementary variation in the elasticity. We have discussed in details a special, but a rather important example of such relaxation phenomena due to the enhanced thermoelastic coupling at inner contacts in cracks. The inferences from the considered thermoelastic mechanism can be directly related to various physical problems. In particular, in the low-amplitude range, the performed analysis indicates that the role of thermoelastic losses in rocks is very strongly underestimated, if the estimates are made using conventional equations derived for homogeneous materials [29]. Even in comparison with strongly enhanced thermoelastic losses of the type considered by Savage [23] for solids with cracks, our results modify radically the estimates for higher frequencies (especially in kHz and ultrasound range which is often used in laboratory studies of rocks and in non-destructive testing). Indeed, equations (19) indicate that, for narrow strip-like contacts of length $L_c \sim L$ (comparable with the whole crack diameter), and width a , near its relaxation frequency $\omega_c \approx \kappa/(\rho C a^2)$ corresponding to the contact width, the dissipation magnitude is of the same order as the "global" thermoelastic losses considered for cracks in paper [23]. The difference is that the "global" mechanism corresponds

to the relaxation maximum is determined by the whole crack size $\omega_L \approx \kappa/(\rho CL^2)$. The above considered modified mechanism predicts that, for such a single narrow inner strip-like contact, the amount of the dissipation is proportional to $L_c L^2 \approx L^3$ (see equation 19c). On the other hand, the “global” losses [23] for a small crack of size a correspond to the same position of its relaxation peak $\sim \kappa/(\rho Ca^2)$, but are proportional to a^3 . This indicates the difference in the efficiency of the two mechanisms of the order of $(L/a)^3 \gg 1$ times for absorption at this relaxation frequency. Quantitatively the considered mechanism can increase the estimate of the thermoelastic contribution to the elastic wave dissipation in crack-containing solids by 4–6 orders of magnitude. The total difference in absorption should be especially important for the small amplitude absorption, because threshold-type frictional losses and adhesion mechanisms are not capable to contribute to dissipation in this amplitude range.

Further, for higher amplitudes, as is argued above, the considered mechanism can essentially contribute to amplitude-dependent variation in the dissipation, including both self-induced effects for the strong oscillation and induced dissipation/transparency effects for a weaker probe wave. These manifestations of the thermoelastic mechanism combined with the elastic nonlinearity of the soft contacts have much in common with manifestation of hysteretic nonlinearity of defects of the same type. It is important that the two mechanisms normally can act simultaneously. At the rheological level these conclusions were obtained in paper [4], and the above consideration of thermoelastic effects additionally confirms that the superimposing of the two mechanisms should be taken into account in the interpretations of experimental data on nonlinear effects in microinhomogeneous solids (such as rocks, damaged materials, etc.) A rather peculiar feature of the considered mechanism consists in the fact that the nonlinear shift of the relaxation maximum along the frequency axis is not accompanied by simultaneous noticeable variation in its magnitude (see Figures 1 and 5). This feature results in the possibility of co-existence of induced absorption and induced transparency in different frequency ranges.

Note finally that in fluid saturated solids presence of viscous liquids in high-compliant cracks can also result in similar relaxational manifestations (as discussed, for example, in [?]).

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