

STRAIN-AMPLITUDE DEPENDENT DISSIPATION IN LINEARLY DISSIPATIVE AND NONLINEAR ELASTIC MICROINHOMOGENEOUS MEDIA

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We discuss a mechanism of nonhysteretic strain-amplitude dependent dissipation of elastic waves in microinhomogeneous media containing linearly dissipative and nonlinear elastic soft defects. The combined effect of these factors can cause well-pronounced essentially dissipative nonlinearity of elastic waves. The mechanism presumably works in rocks and other materials with microstructure where cracks and grain contacts act as soft inclusions. Unlike homogeneous materials where strain dependent variations of dissipation and elasticity are commonly of the same order, dissipation variations in microinhomogeneous materials can be times as great as the respective elasticity variations. A slight (a few percent) change in Young's modulus can correspond to a manifold dissipation change at moderate strain typical of acoustic and seismic applications. Moreover, in the case of variable frequency, this nonlinear mechanism can increase or decrease Young's modulus or reduce its variation to zero at certain frequencies while dissipation variations remain quite large. The nonhysteretic mechanism may act concurrently with hysteretic mechanisms, which are due to friction and adhesion effects and are commonly invoked for materials with microstructure, and influence the relationship between strain dependent dissipation and elasticity observed in experiments on nonlinear dissipative effects.

Microinhomogeneous medium, elastic nonlinearity, linear dissipation, amplitude dependent dissipation

INTRODUCTION

Nonlinear acoustic and seismic effects have received much recent attention, as their use opens up new prospects for testing applications (nondestructive testing, engineering seismics, etc. [1–7]). Discussions of nonlinearity in rocks in terms of its influence on propagation and interaction of elastic waves most often invoke elastic (reactive) nonlinearity. Indeed, this nonlinearity in rocks is normally times as high as in ideal crystals and amorphous homogeneous materials, though the difference in linear elastic moduli is not so high. The difference is due to the structural mechanism [3, 8–11] of nonlinearity increase in rocks and other microinhomogeneous materials containing cracks, grain boundaries, dislocation clusters and other defects which are much softer than the homogeneous matrix material. The qualitative sense of the mechanism is as follows: High compliance of soft defects allows times greater strain in their immediate vicinity and causes locally high departure from the linear behavior of the material, and thus a rapid growth of nonlinearity of the whole microinhomogeneous medium.

Besides elastic (reactive) nonlinearity, these materials (including rocks) demonstrate prominent strain-amplitude dependent dissipation of elastic waves which was noted quite long ago [11]. Ample experimental evidence gained through four recent decades shows that nonlinear elastic effects in rocks and other microinhomogeneous materials are as a rule accompanied by strain dependent energy loss [11–15] unlike the case of classical elastic nonlinearity of solids caused by weak anharmonicity of the interatomic potential. The totality of the nonlinear reactive and dissipative properties is commonly attributed to hysteretic nonlinearity (friction or adhesion effects)

[11, 12, 16–19]. Experimental elastic stress–strain diagrams for rocks become hysteretic at strain above 10^{-5} – 10^{-4} [11, 17–20]. The available instrumental facilities do not allow direct observation of hysteretic curves at strain below 10^{-5} (typical of acoustic and seismic amplitudes). Then, the behavior of hysteresis is inferred implicitly from nonlinear wave effects, namely, from the relationship between hysteretic (strain amplitude dependent) energy loss for a period of oscillation and the accompanying change in Young's modulus $\Delta E/E$ due to hysteretic nonlinearity. (This change is often called “modulus defect” as Young's modulus decreases with oscillation amplitude in most hysteretic materials [21]). The loss can be estimated using the dimensionless logarithmic decrement θ which is conveniently expressed in terms of energy as $\theta = \Delta W/(2W)$, where ΔW is the energy loss of an elastic wave over a period per unit volume and $W = E\varepsilon^2/2$ is the maximum stored elastic energy, likewise per unit volume. It was mentioned long ago that the relationship between the amplitude dependent θ_H due to hysteretic loss and the hysteresis-related modulus defect $(\Delta E/E)_H$ is often practically independent of amplitude. This idea apparently received its first clear-cut formulation in Read's paper [22], and the coefficient r ($\theta_H = r(\Delta E/E)_H$) is therefore called sometimes “Read parameter”. The measured Read parameter (r) was suggested to be used for identifying the shape of hysteresis loops in stress-strain diagrams [21]. Indeed, in the case of a commonly used piecewise-power-law approximation for the shape of hysteresis branches, the power can be estimated experimentally from the observed dependences of harmonics or from strain dependence of decrement and modulus defect. The obtained power, however, does not provide unambiguous constraints on the shape of hysteresis loops and this can be further constrained using r [21]. The r parameter has additional implications for the shape of hysteresis (in fact, for the ratio of the loop area to the slope and curvature of its branches which are not uniquely defined by the power n of the approximating power-law function). The parameter r takes different values in different hysteresis models at the same power. Lebedev [21] reported different r values for the cases of the Granato–Lücke breakaway hysteresis, Davidenkov hysteresis, and friction hysteresis without restoring force (WRF): $r \approx 1.25$ for Davidenkov hysteresis and $r \approx 2.7$ for the WRF hysteresis in the piecewise quadratic case ($n = 2$) with linear amplitude dependences of θ_H and $(\Delta E/E)_H$ ($n - 1 = 1$) [21]. The r difference in different models being not so drastic, its accuracy is of special importance for the hysteretic equation of state. Moreover, one has to be sure that the behavior of the observed strain dependent effects is due to hysteresis and not to some other mechanisms.

The presence of nonhysteretic, physically linear (e.g., thermoelastic or viscous) energy losses would appear to cause no difficulty in hysteresis identification. It is assumed that one just has to subtract the strain independent component from the total measured decrement to attribute the remaining strain dependent component to hysteresis. However, the case is more complicated than that in microinhomogeneous materials, such as almost all rocks. Indeed, hysteretic properties of materials with microstructure are due to friction or adhesion effects at highly compressible (compliant) defects (cracks or grain boundaries in granular materials). On the other hand, locally high strain can make soft defects prone to very high losses caused by common linear mechanisms, besides the eventual hysteretic effects. These can be thermoelastic losses at crack mating faces [23, 24], and the loss magnitude strongly depends on the geometry of defects (crack opening or precompression of contacts, etc.). The geometry of highly compliant defects (width of contacts, crack opening, etc.) is, in turn, highly sensitive to very moderate average stress. Inasmuch as the same parameters define the magnitude of linear (e.g., thermoelastic) losses at defects, the losses inherently linear by their physics become strain dependent in microinhomogeneous media. We discussed the mechanism in [25] as amplitude dependent (nonlinear) dissipation in microinhomogeneous materials due to the combined effect of elastic nonlinearity and linear losses at defects. In [25] we did not go far into details of various nonlinear properties of defects but focused on possible difference of the losses for compressional and shear strains. The conclusion relevant to the present consideration is that relative amplitude dependent variation in decrement can be times as great as the accompanying relative change in elastic moduli [25].

In this study we address strain amplitude and frequency dependences of simultaneous variations of dissipation and elasticity, in 1D approximation only, for the nonhysteretic mechanism of strain dependent losses suggested in [25]. First we discuss (using the standard rheological model of viscoelastic material) why this mechanism, though remaining workable, fails to show considerable difference in variations of dissipation and elasticity in a homogeneous viscoelastic medium with elastic nonlinearity. Then we investigate the frequency dependent behavior of strain-amplitude dependent variations of dissipation and Young's modulus for several common types of elastic nonlinearity of defects using a modified model from [26, 27] with regard to microheterogeneity. Finally, we compare the predicted changes in elastic and dissipative properties due to concurrent hysteretic and nonhysteretic mechanisms. The comparison shows that the nonhysteretic contribution can be responsible for a manifold change in the observed ratio between the modulus defect and strain dependent losses. Therefore, the idea that this ratio (r) can be used directly as diagnostic of hysteresis type needs a revision.

RHEOLOGICAL MODELS OF A STANDARD VISCOELASTIC BODY AND ONE-DIMENSIONAL MICROINHOMOGENEOUS VISCOELASTIC MEDIUM

Viscoelastic properties of solids are most often described using equivalent rheological models which are combinations of parallel or series elastic and dissipative elements [28]. These models highlight the most essential features of dispersion in elastic and dissipative parameters of viscoelastic materials. In their “classical” form, they are commonly viewed as lumped models, which in fact implies homogeneity of material. A distributed problem for a homogeneous medium can be solved, in 1D approximation, using a series of identical viscoelastic elements.

Consider such a model which simulates a medium as a series of identical masses m coupled by Kelvin-Voigt viscoelastic elements (Fig. 1). In this context we do not care about complication of the structure of each element, for instance, by including an additional elastic element in a series with a viscous element in a standard viscoelastic body. We assume that the elements have a unit cross section area, the elastic element is characterized by Young’s modulus E and the viscous element by the effective viscosity coefficient g . In a fragment of this series much less than the elastic wavelength, Hooke’s law for the whole fragment, as well as for each separate element, is

$$\sigma = E\varepsilon + g d\varepsilon / dt. \quad (1)$$

For a harmonic perturbation $\varepsilon = \varepsilon_a \exp(i\omega t)$ we derive from (1) the equations for the real and imaginary parts of Young’s modulus $E_{\text{eff}} = \sigma/\varepsilon_a$:

$$\text{Re } E_{\text{eff}} = E, \quad \text{Im } E_{\text{eff}} = g\omega. \quad (2)$$

Then the decrement θ is given by the ratio of the imaginary-to-real parts of Young’s modulus [29]:

$$\theta = \pi \text{Im } E_{\text{eff}} / \text{Re } E_{\text{eff}} = \pi\omega g/E \sim E^{-1}. \quad (3)$$

It follows from (3) that if $E(\varepsilon_0)$ experiences a slight change ($\Delta E/E \ll 1$) for some reason (e.g., due to dependence on the applied static strain ε_0), the complementary relative change in decrement is of the same order: $\Delta\theta/\theta \approx -\Delta E/E$. The same holds for a distributed model consisting of a series of identical elements (Fig. 1), and, in a long-wavelength approximation, at $\Lambda \gg 1$, the wave equation for this case is

$$U_{tt} - c^2 U_{xx} = \alpha U_{xxt} \quad (4)$$

where $c = (El/m)^{1/2} = (E/\rho)^{1/2}$ is the velocity of an elastic wave, $\rho = m/l$ is the density, and $\alpha = gp$. Passing to the “fast running” variable $\tau = t - x/c$ and the “slow” variable x gives a reduced equation corresponding to (4):

$$U_x = \frac{\alpha}{2c^3} U_{\tau\tau}. \quad (5)$$

Substituting the solution $U = V \exp(i\omega t)$ into (5) gives

$$V_x = -\beta V, \text{ or } V = V_0 \exp(-\beta x), \quad (6)$$

where the attenuation coefficient has the form $\beta = \alpha\omega^2/(2c^3)$, wherefrom the equation for $\theta = \lambda\beta$ again arrives at (3):

$$\theta = \lambda\beta = \pi\omega g/E \sim E^{-1}. \quad (7)$$

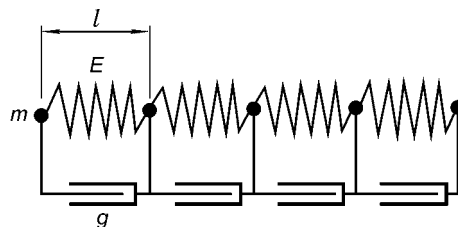


Fig. 1. Medium modeled as identical Kelvin-Voigt viscoelastic elements.

The coincidence of (3) and (7) indicates that the relative changes in elasticity and dissipation are of the same magnitude if the viscous properties of elements remain invariable. This conclusion following from the model (Fig. 1) is at odds with the empirical fact that relative changes in decrement are as a rule much greater than in Young's modulus for many real materials (see above). A better fit to the experiment can be gained using, for instance, a modification of the standard Kelvin-Voigt model (or a standard elastic body) [30] including another dissipative element with its viscosity strongly dependent on stress applied to the whole element (though it remains unclear whether the modified model gives a realistic account of different materials).

There is an alternative model [8–10, 25–27] which accounts for the structure of rocks and other microinhomogeneous materials being free from the above assumptions. It provides a rheological formalization of the “contrasting soft-stiff mechanism of elastic (reactive) nonlinearity growth in a material with soft defects (see Introduction). Furthermore, the model predicts the existence of prominent amplitude dependent dissipation in these materials if the viscous (even purely linear) properties of defects are taken into account, it [25]. The model differs from conventional viscoelastic chains (in 1D case) like that of Fig. 1 as it implies the presence of elements (defects) with their compressibility times as great as that of the other stiffer elements corresponding to the homogeneous matrix material. It is the high contrast in linear elastic properties which is responsible for strain in soft inclusions times the average strain of the medium. Therefore, the departure from linear Hooke's law is the greatest in soft inclusions because of the locally high strain, which increases the macroscopic nonlinearity (see [8–10] where this effect is discussed in the low-frequency (quasi-static) limit).

Beyond the quasi-static case, it should be kept in mind that the locally high strain in soft defects causes high dissipation of elastic energy, i.e., the macroscopic dissipative properties of materials with microstructure are likewise due to the contribution of soft defects. The elastic elements corresponding to homogeneous matrix material can be assumed to be linearly elastic. Thus, the modified distributed model of a microinhomogeneous medium can be represented by a heterogeneous chain [26, 27] containing soft elements that are the sites of both nonlinearity and dissipation (Fig. 2). Cracks offer a physical example of these defects in which locally high thermoelastic losses act as effective viscosity [23].

In the modified model, elastic elements corresponding to the matrix material follow linear Hooke's law with the modulus E ,

$$\sigma = E\varepsilon, \tag{8}$$

and soft defects are characterized by the modulus $E_1 = \zeta E$ (where the softness $\zeta \ll 1$), by the effective viscosity coefficient g , and the nonlinear function $F(\dots)$ for the elastic nonlinearity of defects. Thus the equation of state of a soft inclusion is

$$\sigma = \zeta E [\varepsilon_1 + F(\varepsilon_1)] + g d\varepsilon_1/dt. \tag{9}$$

Note that by the strain ε_1 in (9) we mean the strain of a soft defect in its own scale rather than the average strain of material. The number of defects is represented by their linear concentration ν (in the 3D case it is the relative volumetric percentage of soft inclusions). Assuming the characteristic elastic wavelength $\Lambda \gg L \gg l$, the macroscopic equation of state, at a small concentration of defects ($\nu \ll 1$, when the nonlinear correction is small), is as follows [26, 27]

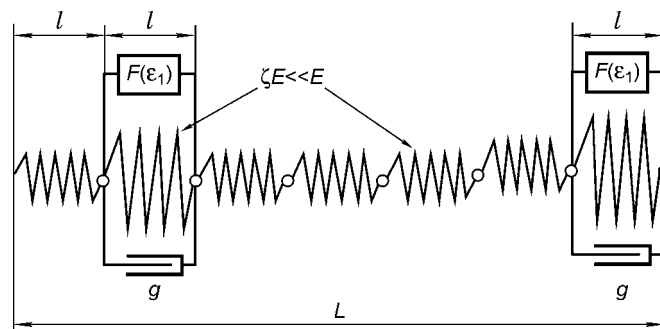


Fig. 2. Distributed rheological model of microinhomogeneous medium with contrasting soft viscoelastic nonlinear inclusions.

$$\sigma(\varepsilon) = E\varepsilon - \nu E\Omega \int_{-\infty}^t \varepsilon(\tau) e^{-\Omega\zeta(t-\tau)} + \nu E\Omega\zeta \int_{-\infty}^t e^{-\Omega\zeta(t-\tau)} f \left\{ \int_{-\infty}^{\tau} \varepsilon(\tau') e^{-\Omega\zeta(t-\tau')} d\tau' \right\} d\tau, \quad (10)$$

where $\Omega = E/(gl)$ has a dimension of frequency so that $\Omega\zeta$ corresponds to the characteristic relaxation frequency of a defect. The first term in the right-hand side of (10) is determined by the homogeneous matrix material that obeys Hooke's law, the second term describes the linear relaxation correction due to soft viscoelastic defects, and the third one corresponds to the nonlinear correction due to the combined effect of elastic nonlinearity and linear relaxation of soft defects. This equation demonstrates that the assumption of additional account for relaxation and nonlinear corrections to the equation of state works in homogeneous but fails in microinhomogeneous media. The physical reason is that both linear relaxation and nonlinear elastic properties in these media occur at the same soft defects. Therefore, relaxational "freezing" of the defects' reaction with frequency in materials with microstructure simultaneously affects its linear and nonlinear properties and is evident in frequency dependence of elasticity and linear dissipation as well as in prominent frequency dependence of nonlinearity [26, 27]. Moreover, below we show for several typical forms of the nonlinear function $F(\dots)$ that, in addition to the elastic nonlinearity growth, structure inhomogeneity causes amplitude dependence of dissipation due to the combined effect of linear losses at defects and their nonlinearity. Unlike the above case of a homogeneous medium (where $\Delta\theta/\theta \approx -\Delta E/E$), relative dissipation changes in materials with microstructure can be times as great as the accompanying relative changes in elasticity.

AMPLITUDE DEPENDENT LOSSES FOR DIFFERENT TYPES OF ELASTIC NONLINEARITY OF DEFECTS

Quadratic nonlinear defects. Consider the quadratic elastic nonlinearity of defects $F(\varepsilon_1) = \gamma\varepsilon_1^2$. Note that the coefficient γ for the nonlinearity of soft defects in their own strain scale is of an order of several units and the macroscopic nonlinearity of a microinhomogeneous medium does increase strongly only due to the influence of the locally high strain of defects embedded in a relatively stiff matrix [8–10, 26, 27]. Real materials commonly have $\gamma < 0$, i.e., soften under extension ($\varepsilon > 0$) and stiffen under compression ($\varepsilon < 0$). Then we assume that the static strain ε_0 coexists with the periodic (acoustic) strain $\tilde{\varepsilon}_a = (\varepsilon_a e^{i\omega t} + \varepsilon_a^* e^{-i\omega t})/2$, and $\varepsilon_0 \gg \varepsilon_a$. Substituting into (10) the equation

$$\varepsilon(t) = \varepsilon_0 + \tilde{\varepsilon}_a \quad (11)$$

and holding the periodic terms at $\exp(i\omega t)$ in the elastic stress gives

$$\sigma_a(\varepsilon_a) = Ea - \frac{\nu E}{\zeta} \varepsilon_a \frac{1}{1+\bar{\omega}^2} - \frac{\nu E}{\zeta} \varepsilon_a F' \left(\frac{\varepsilon_0}{\zeta} \right) \frac{1-\bar{\omega}^2}{(1+\bar{\omega}^2)^2} + i \frac{\nu E}{\zeta} \varepsilon_a \frac{\bar{\omega}}{1+\bar{\omega}^2} + i \frac{\nu E}{\zeta} \varepsilon_a F' \left(\frac{\varepsilon_0}{\zeta} \right) \frac{2\bar{\omega}}{(1+\bar{\omega}^2)^2}, \quad (12)$$

where $\bar{\omega} = \omega/(\Omega\zeta)$ is the normalized frequency. Singling out the complex elastic modulus in (11) (as it was done above for (1)–(3)) and separating its real and imaginary parts in view of $F(\varepsilon) = -|\gamma|\varepsilon^2$ gives the equations for the effective elastic modulus and decrement:

$$E_{\text{eff}}/E \approx 1 - \frac{\nu}{\zeta} \frac{1}{1+\bar{\omega}^2} - 2 \frac{|\gamma|\varepsilon_0\nu}{\zeta^2} \frac{1-\bar{\omega}^2}{(1+\bar{\omega}^2)^2}, \quad (13)$$

$$\theta = \theta_{\text{lin}} + \theta_{\text{nl}} \approx \pi \frac{\nu}{\zeta} \frac{\bar{\omega}}{1+\bar{\omega}^2} + 2\pi \frac{\nu\gamma\varepsilon_0}{\zeta^2} \frac{2\bar{\omega}}{(1+\bar{\omega}^2)^2}. \quad (14)$$

The second term in (13) corresponds to the relative correction for defects to Young's modulus E of the matrix. The third term in (13) describes the nonlinear (depending on ε_0) relative correction to E . Unlike Young's modulus defined mostly by the homogeneous matrix material, the contribution of the nonlinear correction to the losses is vanishing relative to the losses at defects. Therefore, both linear and nonlinear contributions in (14) are due to defects and correspond to the absolute losses. The nonlinear-to-linear term ratio ($\theta_{\text{nl}}/\theta_{\text{lin}}$) in (14) is of the same order as $\gamma\varepsilon_0/\zeta$. Therefore, the nonlinear (depending on ε_0) contribution of soft ($\zeta \ll 1$) defects can be of the same order as the linear contribution even at moderate static strain in the medium. Below we discuss in more detail the frequency dependent behavior of the relationship between strain dependent variations in Young's modulus and decrement. We assume that the defect softness is $\zeta = 10^{-4}$ (corresponding to the opening/diameter ratio typical of

thin cracks [31]) and select a moderate value of $\gamma=2$ for the nonlinearity of a crack in its own strain scale. Let the concentration of defects be $\nu = 2 \cdot 10^{-6}$, which corresponds to typically seismic $Q \equiv \pi/\theta \approx 100$ for linear losses (in the vicinity of the relaxation maximum of θ_{lin}). The macroscopic nonlinearity coefficient $2\gamma\nu/\zeta^2 = 800$ in the last term of (13), which is likewise quite common to rocks [1–3, 12, 13]. These parameters were used to obtain frequency dependences (13), (14) for the macroscopic E_{eff} and θ at zero ϵ_0 and moderate $\epsilon_0 = 10^{-5}$ static strain (positive at extension) (Fig. 3).

The plots of Fig. 3 show that the linear contribution to decrement from relaxing defects is the greatest near their relaxation frequency $\omega = \Omega\zeta$, as one could expect. The applied static strain causes an over 1.5 times increase in decrement in the vicinity of the maximum and the accompanying relative change of E_{eff} is less than one percent. The difference is better pronounced in the plots of relative variations in $\Delta\theta/\theta$ and $\Delta E_{eff}/E$ in Fig. 4: The variations in decrement are two orders of magnitude greater in the average than in Young's modulus. Moreover, the strain dependent correction zeroes and changes its sign (the fall of $\Delta E_{eff}/E$ in Fig. 4) in the vicinity of the defects relaxation frequency $\omega = \Omega\zeta$. In this region, decrement can show relative variations arbitrarily much greater than those of Young's modulus. Physically the $\Delta E_{eff}(\epsilon_0)$ sign change is caused by a phase delay in the response of the relaxing defects to oscillation. Note that the maximum phase shift of a single relaxation (in the linear case) is within $\pi/2$ but in our case it can be greater since relaxation occurs repeatedly, as follows from (10) for the nonlinear correction to elastic stress. Indeed, it works in the same way as in the linear case (the relaxator in the argument of the nonlinear function $F(\dots)$) and, besides, the spectrum components that are due to the nonlinearity of defects (see the relaxator outside the nonlinear function $F(\dots)$ in (10)) relax as well. (Note that we took only the fundamental frequency component in this discussion). The additional phase shift due to this cascade relaxation can cause a sign change of the nonlinearity-induced correction to Young's modulus (phase shift over $\pi/2$). The effect is to apparently increase effective stiffness in the range above the relaxation frequency instead of softening observed below the relaxation frequency. Thus, dissipation shows a quite prominent strain-amplitude dependence in the vicinity of the relaxation frequency while elasticity remains almost invariable. This paradoxical effect looks as a purely dissipative nonlinearity though physically it is produced by the combined effect of reactive (elastic) nonlinearity and common relaxation linear losses both localized at soft defects.

Above we used the simplest version of the model with identical defect parameters (same relaxation frequency), which never occurs in real rocks and other microinhomogeneous materials. In this respect, it appears instructive to see whether our conclusions hold true in the case of defects distributed over their relaxation frequencies. This distribution should better agree with numerous experiments that demonstrate a very weak frequency dependence of decrement in a broad frequency range, which is more typical of rocks and other materials with a similar structure. As shown in [32, 33] within the limits of the same microinhomogeneous model, a frequency independent decrement

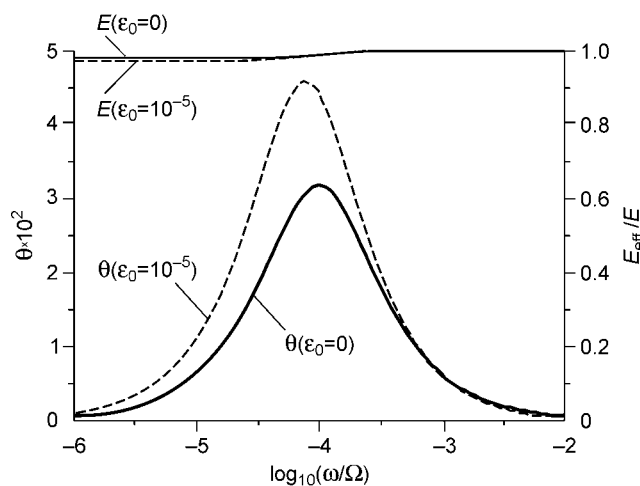


Fig. 3. Frequency dependence of effective Young's modulus (13) and decrement (14) at zero static strain ($\epsilon_0 = 0$) and moderate background strain ($\epsilon_0 = 10^{-5}$) of medium with identical quadratic nonlinear defects with softness $\zeta = 10^{-4}$.

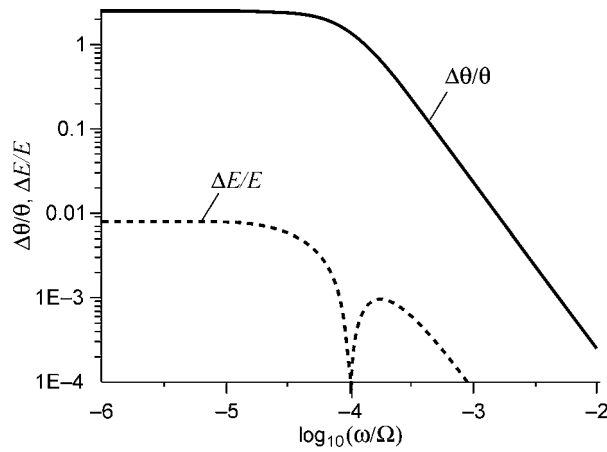


Fig. 4. Frequency dependence of simultaneous relative strain-amplitude-dependent changes in $\Delta\theta/\theta$ and $\Delta E/E$ at defect parameters same as in Fig. 3.

in a linear approximation corresponds to a uniform softness distribution of defects in a broad range, when $v(\zeta) \approx v_0 = \text{const}$ at $a \leq \zeta \leq b$ (where $a \ll b$) and $v(\zeta = 0)$ outside this range.

Summing the contributions from different defects over the distribution $v(\zeta)$ in nonlinear equation of state (10), as in the linear case [32, 33], gives the equations for effective Young's modulus and decrement similar to (13) and (14) derived for the case of identical quadratically nonlinear defects:

$$E_{\text{eff}} \approx 1 - \frac{1}{2} v_0 \ln \frac{\Omega^2 b^2 + \omega^2}{\Omega^2 a^2 + \omega^2} + 2v_0 \gamma \varepsilon_0 \Omega^2 \left\{ \frac{b}{\Omega^2 b^2 + \omega^2} - \frac{a}{\Omega^2 a^2 + \omega^2} \right\}, \quad (15)$$

$$\theta = \theta_{\text{lin}} + \theta_{\text{nl}} \approx \pi v_0 \left\{ \arctan \frac{\Omega b}{\omega} - \arctan \frac{\Omega a}{\omega} \right\} - 2\pi v_0 \gamma \varepsilon_0 \Omega \left\{ \frac{\omega}{\Omega^2 b^2 + \omega^2} - \frac{\omega}{\Omega^2 a^2 + \omega^2} \right\}. \quad (16)$$

The linear (independent of ε_0) terms in (15) and (16) coincide with the results of [32, 33]. The frequency dependent (ω/Ω) behavior of θ and E at the static strain $\varepsilon_0 = 0$ and $\varepsilon_0 = 10^{-5}$ for a uniform softness distribution $10^{-5} \leq \zeta \leq 10^{-2}$ of quadratically nonlinear defects (with the nonlinearity parameter $\gamma = 2$) is plotted in Fig. 5 (similar to Fig. 3). This softness range appears to be a quite reasonable approximation for real cracks [34]. The assumed concentration of defects is $v_0 = 6 \cdot 10^{-3}$, which corresponds to linear ($\varepsilon_0 = 0$) $Q \sim 100$ (i.e., $\theta = \pi/Q \sim 0.03$) in the frequency domain where decrement is roughly constant.

The plot demonstrates that the relative variations in decrement are times greater than in Young's modulus over a broad range of frequencies, as in the case of identical defects (Fig. 3). This difference is highlighted in Fig. 6 where the sign change in $\Delta E_{\text{eff}}/E$ caused by the relaxation phase shift leads to a fall, as in Fig. 4, near which the relative decrement change can be arbitrarily larger than that of Young's modulus.

Consider again the relaxation phase shift effect in a reactive (elastic) response. Both the ideal (identical defects) and more realistic (softness distribution of defects) models predict the paradoxical effect of a Young's modulus increase for a probe wave excited at frequencies above the frequency of the sign change of $\Delta E_{\text{eff}}(\varepsilon_0)$, under extensional static strain. In the case of compression, the modulus, on the contrary, decreases. Positive and negative nonlinear changes in the dynamic modulus were observed in dynamic loading experiments where the self-action of a quite intense wave caused either a decrease or an increase in Young's modulus at increasing amplitude (e.g., [24]). Unlike dynamic loading, quasi-static compression most often increases the elastic modulus. At least to our knowledge, observations of a modulus decrease in these conditions have not been reported.

The conclusion on the amplitude dependent variations in dissipation much greater than in elasticity agrees well with most of experimental evidence (see for instance the known laboratory experiments reported in [11] and [34]). For field conditions, recently reported [35] modulation of a seismoacoustic wave by diurnal tidal strain likewise agrees with (13)–(16). Namely, the amplitude-phase variations of the probe wave correspond to the absolute

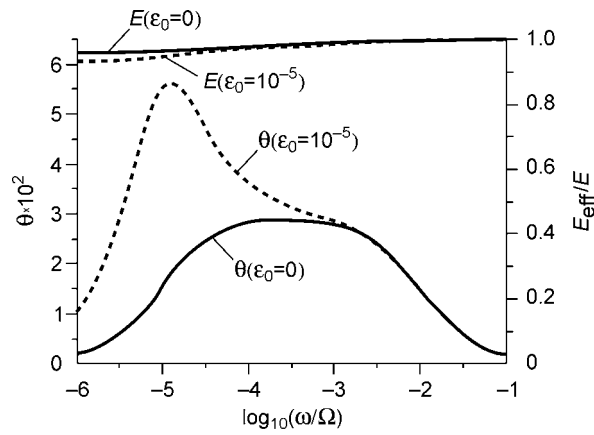


Fig. 5. Frequency dependence of decrement (15) and Young’s modulus (16) at static strain $\epsilon_0 = 0$ and $\epsilon_0 = 10^{-5}$ of medium with quadratic nonlinear defects with softness distribution in a broad range.

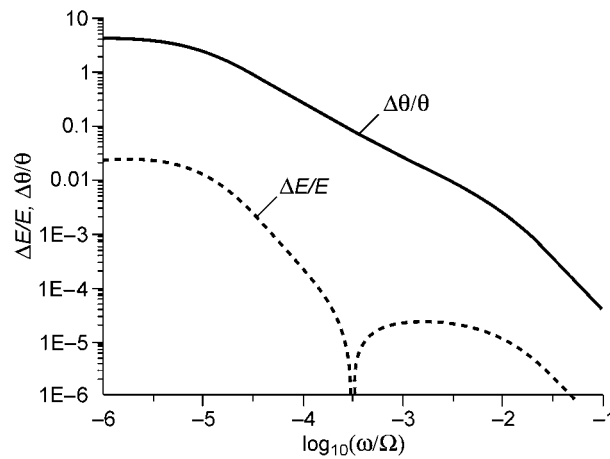


Fig. 6. Frequency dependence of simultaneous relative strain-amplitude dependent changes in $\Delta\theta/\theta$ and $\Delta E/E$ at defect parameters same as in Fig. 5.

changes in decrement and relative changes in Young’s modulus of the same order, namely, $\Delta\theta \sim (3-4) \cdot 10^{-3}$ and $\Delta E/E \sim (7-9) \cdot 10^{-3}$, as the model predicts. Relative variations in decrement are thus times as great as in Young’s modulus.

A qualitatively similar excess of relative dissipation variations over velocity variations under strain from a powerful low-frequency vibrator was reported in [15]. Geza et al. [15] noted that the average effect of the vibrator field cannot be interpreted as mere averaging of quasi-static instantaneous variations, i.e., the effect was essentially dynamic. Below we discuss the possibility of obtaining a nonzero average of a periodic symmetrical impact if the defects nonlinearity contains an odd component.

Cubic nonlinear defects. The quadratic (even-type) nonlinearity of defects causes nonlinear variations in a probe wave proportional to the value ϵ_0 of the additional static strain (see above). This even-type nonlinearity in the case of sine-like variations in ϵ_0 (we call it “pumping” following the terminology of nonlinear optics) does not lead to a period-averaged change. In this section we consider a simple example of odd nonlinear defects with a cubic nonlinearity which yield nonzero average changes for a periodic dynamic pumping. By analogy with the quadratic case, cubic nonlinear defects are characterized by dimensionless nonlinearity coefficient β as

$$F(\varepsilon) = \beta\varepsilon^3. \quad (17)$$

Note that periodic loading normally causes average softening, which corresponds to $\beta < 0$. Then we assume that a medium experiences two perturbations, a weaker probe wave ε_{pr} at the frequency ω with the strain amplitude a and a stronger periodic loading (pumping) ε_{pm} at the frequency w and the strain amplitude $|A| \gg |a|$:

$$\varepsilon = \varepsilon_{pr} + \varepsilon_{pm} = \frac{1}{2}(ae^{i\omega t} + ae^{-i\omega t}) + \frac{1}{2}(Ae^{iwt} + Ae^{-iwt}). \quad (18)$$

Substituting (18) into (10) derived for identical defects and holding the terms $\sim \exp(i\omega t)$ gives

$$E_{eff}/E \approx 1 - \frac{\nu}{\zeta(1+\bar{\omega}^2)} - \frac{3}{2}\nu\beta|A|^2 \frac{1-\bar{\omega}^2}{\zeta^3(1+\bar{\omega}^2)(1+\bar{\omega}^2)^2}, \quad (19)$$

$$\theta \approx \pi \frac{\nu\bar{\omega}}{\zeta(1+\bar{\omega}^2)} + 3\pi|A|^2 \frac{\nu\beta}{\zeta^3} \frac{\bar{\omega}}{(1+\bar{\omega}^2)(1+\bar{\omega}^2)^2}, \quad (20)$$

where $\bar{\omega} = \omega/(\Omega\zeta)$ and $\bar{w} = w/(\Omega\zeta)$ are the normalized frequencies of the probe wave and the pumping, respectively. These equations show that, at sine-like pumping, odd cubic nonlinearity (unlike even quadratic nonlinearity) leads, at sine-shaped pumping, to nonzero average corrections for the Young's modulus and decrement of the probe wave, the sign of the corrections being dependent on the sign of the nonlinearity coefficient. Then we set $\zeta = 10^{-4}$ for the softness and $\nu = 2 \cdot 10^{-6}$ for the concentration of defects and assume their own cubic nonlinearity to be $\beta = 8$, proceeding from the same ideas as above for (13) and (14) of the quadratic case. Let the pumping amplitude be moderate (bearing in mind acoustic and seismic applications) ($A = 10^{-5}$) at the frequency $\bar{w} = 1$ (the \bar{w} dependence, as follows from (19) and (20) is the same for nonlinear corrections to Young's modulus and decrement and thus does not influence their relationship). The resulting frequency dependences of $\Delta\theta/\theta$ and $\Delta E/E$ for the probe wave are plotted in Fig. 7, and Fig. 8 shows the respective frequency dependences for their pumping-related variations, for the same parameters.

Dynamic pumping causes an effect qualitatively similar to that of static loading in the case of quadratic nonlinear defects. Again, $\Delta\theta/\theta > \Delta E/E$, and the nonlinear correction to Young's modulus changes its sign in the vicinity of the defects relaxation frequency because of the relaxational phase shift.

Then, as in the previous section, we consider the case of defects with a uniform softness distribution $\nu(\zeta)$ in a broad range of $10^{-5} \leq \zeta \leq 10^{-2}$ at $\nu_0 = 6 \cdot 10^{-3}$. This density is selected such to provide $Q \sim 100$ in the region of frequency independence of the linear decrement. Below we give only the frequency dependences of absolute (Fig. 9) and relative (Fig. 10) variations in decrement and normalized Young's modulus and omit the cumbersome equations given by the integration of (19) and (20) over the distribution $\nu(\zeta)$.

The results are qualitatively similar to those of the static case. Namely, there is likewise a characteristic frequency at which the nonlinear correction to Young's modulus changes its sign and can be arbitrarily small, whereas the accompanying changes in dissipation, both absolute and relative, are rather high. Therefore, the combined effect of linear relaxation and elastic nonlinearity in the vicinity of this characteristic frequency looks as a purely dissipative nonlinearity. When a probe wave travels long distances (in the scale of the elastic wavelength), this difference in the amplitude dependent variations in Young's modulus (wave velocity) and dissipation becomes more prominent. Indeed, the phase (group) delay is proportional to distance and dissipation-related damping accumulates exponentially with distance.

Moreover, (18)–(20) also indicate that nonlinear variations in elasticity and dissipation occur even in the absence of pumping for a single sufficiently intense periodic perturbation due to its self-action. In the case of self-action there is also some characteristic frequency at which the nonlinear correction to Young's modulus changes its sign. Thus, the effective Young's modulus can either increase or decrease depending on wave frequency (below or above the characteristic frequency), at the same sign of the defects' nonlinearity. Note that this mechanism may be related to amplitude dependent velocity increase of elastic waves observed in some experiments, though rocks and other materials with microstructure more often exhibit an odd nonlinearity of a "soft" type, when wave velocity decreases with amplitude. Interpretation of experimental results complicates if several nonlinearity mechanisms act jointly (see Introduction). Namely, the elastic nonlinear and relaxation effects can coexist with hysteresis (related to adhesion or friction) at the same defects. In the following section we show that manifestations of both the hysteresis and the nonhysteretic mechanism can result in nonlinear variations of Young's modulus and dissipation that are quantitatively of the same order and qualitatively similar.

Odd quadratic defects: comparison with quadratic hysteresis. There is ample experimental evidence that the nonlinear response of many rocks (prevalence of odd harmonics of quadratic amplitude dependence, nonlinear

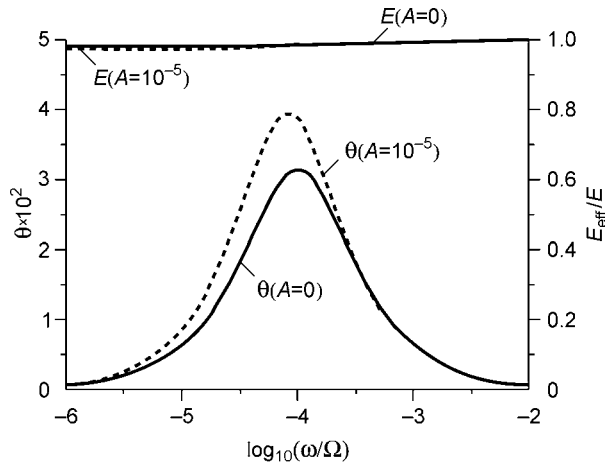


Fig. 7. Frequency dependence of Young's modulus (19) and decrement (20) of probe wave for case of equal-softness cubic nonlinear defects at strain amplitudes of pumping wave $A = 0$ and $A = 10^{-5}$.

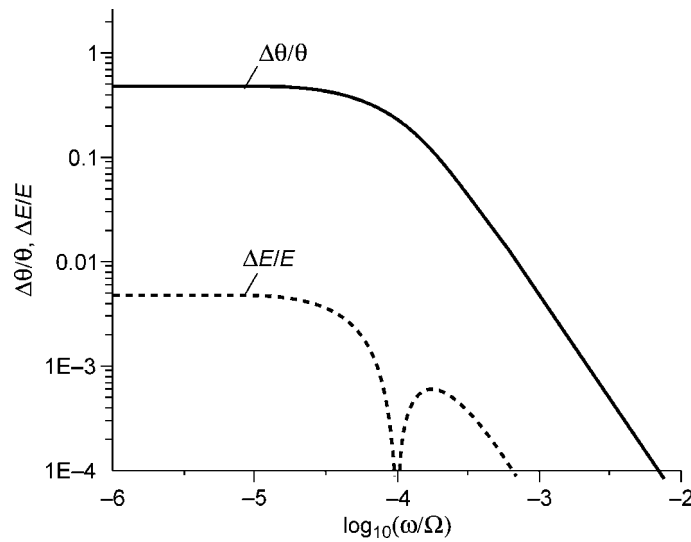


Fig. 8. Frequency dependence of simultaneous relative strain-amplitude dependent changes in Young's modulus and decrement for probe wave at defect parameters same as in Fig. 7.

modulus defect, and nonlinear losses proportional to wave amplitude) are accounted for by an odd piecewise quadratic hysteretic stress-strain relationship. The shape of hysteretic relationships is commonly described using the Preisach-Krasnosel'sky approach [36] in which macroscopic hysteresis is represented as a combined response of a large ensemble of elementary hysteretic units into a linearly elastic matrix. This model, under quite general assumptions, predicts a piecewise quadratic hysteresis loop [37]:

$$\sigma_H = \begin{cases} \sigma_H(\epsilon_{\max}) + \frac{h_H E}{2} (\epsilon - \epsilon_{\max})^2, & \partial E / \partial t < 0; \\ \sigma_H(\epsilon_{\min}) - \frac{h_H E}{2} (\epsilon - \epsilon_{\min})^2 = \sigma_H(\epsilon_{\max}) - \frac{h_H E}{2} [(\epsilon - \epsilon_{\min})^2 - (\epsilon_{\max} - \epsilon_{\min})^2], & \partial E / \partial t > 0, \end{cases} \quad (21)$$

where E is again Young's modulus of the matrix containing hysteretic defect inclusions, h_H is the dimensionless hysteretic nonlinearity parameter, and ϵ_{\max} and ϵ_{\min} are the maximum and minimum strains in the medium. Note that (21) does not include the dominant linear component of the stress-strain relationship but includes only the

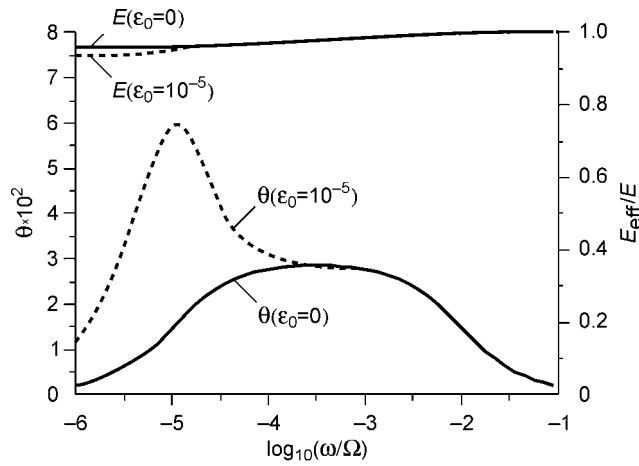


Fig. 9. Frequency dependence of Young’s modulus and decrement of probe wave for case of cubic nonlinear defects with softness distribution over a broad range at strain amplitudes of pumping wave $A = 0$ and $A = 10^{-5}$.

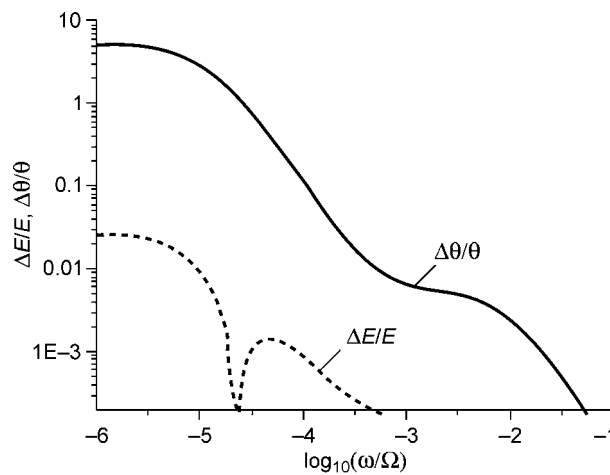


Fig. 10. Frequency dependence of simultaneous relative strain dependent changes in Young’s modulus and dissipation for probe wave, in case of cubic nonlinear defects with softness distribution over a broad range and other parameters same as in Fig. 9.

nonlinear hysteretic component which is essential for this consideration. The constant $\sigma_H(\varepsilon_{\max})$ in (21) is undetermined as it does not influence the area of the hysteresis loop and its curvature corresponding to the losses and the modulus defect. The shape of the hysteresis loop according to (21) is shown in Fig. 11.

At the strain amplitude A it is easy to find strain dependent dissipation and modulus defect that correspond to (21) [37]:

$$\theta_H = (4/3)h_H A, \quad (\Delta E/E)_H = h_H A. \quad (22)$$

Therefrom, for a quadratic hysteresis of the form (21), the Read parameter r is

$$r = \theta_H / (\Delta E/E)_H = 4/3. \quad (23)$$

Then one should bear in mind that microheterogeneities (contacts or other “weak sites” in a material) which exhibit hysteretic behavior (due to adhesion and/or friction) are at the same time the sites of linear (thermoelastic or viscous) losses, i.e., of relaxation effects. Therefore, the nonlinear elastic component in hysteretic nonlinearity which is responsible for the modulus defect should likewise cause strain dependence of losses due to the combined

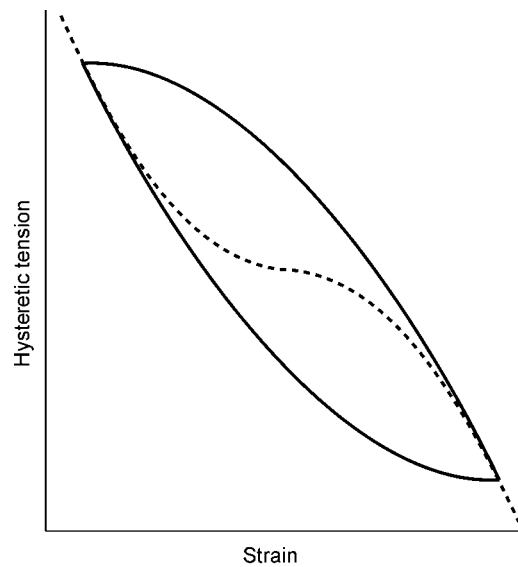


Fig. 11. Odd quadratic hysteresis (21) following from Preisach-Krasnosel'sky approach (solid line) and equivalent nonhysteretic odd quadratic nonlinearity that produces a similar modulus defect (both in functional strain dependence and in magnitude).

nonlinear-relaxation mechanism discussed above. It follows from the above that the strain dependences of dissipation and modulus defect caused by the nonlinear-relaxation mechanism should coincide with each other and with that of hysteretic losses (see (22)). Moreover, hysteretic and nonhysteretic losses can be also comparable in magnitude, at least within a certain frequency range (see below). To illustrate this idea, we consider an example of these nonlinear viscoelastic soft defects which show the behavior equivalent to the hysteresis of the form of (21) in terms of nonlinear elasticity, but lack hysteretic losses. The “equivalent” elastic nonlinearity of the defects is given by the odd quadratic function $F(\epsilon)$ of the form

$$F(\epsilon) = -\gamma\epsilon^2 \operatorname{sgn}(\epsilon). \quad (24)$$

Using the same procedure as above for quadratic and cubic nonlinear defects we obtain the equations for the modulus defect and the nonhysteretic part of dissipation in a material containing defects with odd-type quadratic nonlinearity (24), for the case of a self-action of a periodic perturbation with the amplitude A :

$$(\Delta E/E) \approx -\frac{\nu}{\zeta} \frac{1}{1+\bar{\omega}^2} - \frac{8}{3\pi} \frac{\nu\gamma A(1-\bar{\omega}^2)}{\zeta^2(1+\bar{\omega}^2)^{5/2}}, \quad (25)$$

$$\theta = \theta_{\text{lin}} + \theta_{\text{nl}} = \pi \frac{\nu}{\zeta} \frac{\bar{\omega}}{1+\bar{\omega}^2} + \frac{16}{3} \frac{\gamma A \bar{\omega}}{\zeta^2(1+\bar{\omega}^2)^{5/2}}. \quad (26)$$

Equations (25) and (26) indicate that the amplitude dependent behavior of nonlinear dissipation and modulus defect for the equivalent nonhysteretic nonlinearity (24) does functionally coincide with their counterparts (22) obtained for quadratic hysteresis (21). Then it is easy to ensure the quantitative coincidence of nonlinear modulus defect (25) with $(\Delta E/E)_H = h_H A$ corresponding to (21). Comparison with (25) shows that the same modulus defect (in the low-frequency limit) is provided at $h_H = 8\nu\gamma/(3\pi\zeta^2)$. The resulting equivalent nonhysteretic nonlinear stress-strain dependence is plotted as a dashed line in Fig. 11.

The quantitative relations are illustrated in Fig. 12 and 13 by strain dependent modulus defect (25) and the nonlinear-relaxation component of decrement (26) for the equivalent nonhysteretic nonlinearity and the similar values given by (22) for hysteretic nonlinearity of the type of (21). At the same concentration ($\nu = 2 \cdot 10^{-6}$) and softness ($\zeta = 10^{-4}$) of defects as in the previous sections, the linear quality factor is $Q \sim 100$. Further, choosing the nonlinearity parameter of the defects $\gamma = 8$ corresponding to the equivalent hysteretic nonlinearity of the medium yields $h_H = 8\nu\gamma/(3\pi\zeta^2) \approx 1400$, which is likewise typical of rocks [1, 12, 13] and provides a nonlinear change in

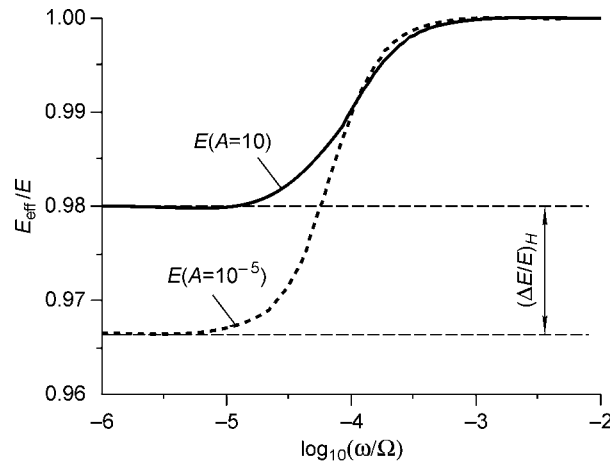


Fig. 12. Strain dependent modulus defect for hysteretic nonlinearity (21) and equivalent elastic nonlinearity (24) that produces a similar modulus defect in low-frequency limit.

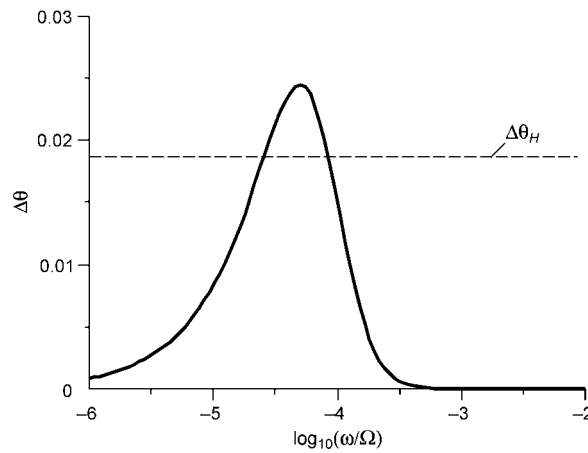


Fig. 13. Nonhysteretic strain dependent dissipation for equivalent elastic nonlinearity and hysteresis contribution to dissipation corresponding to modulus defect in Fig. 12.

Young's modulus of the order of 1.5% at the strain amplitude $A = 10^{-5}$ (Fig. 12). Figure 13 shows that the nonlinear-relaxation contribution to decrement can be comparable to or even greater than the nonlinear hysteretic decrement estimated by (22) at the given modulus defect. Note that quasi-static hysteresis estimates (22) are overestimated as relaxational freezing of the defect response should decrease (relative to the estimates by (22)) both the modulus defect and the area of the hysteresis loop corresponding to hysteretic losses. Therefore, as the elastic wave (or periodic) frequency approaches the relaxation frequency of defects, the role of hysteretic losses decreases whereas the nonlinear-relaxation contribution to decrement, on the contrary, increases to a maximum and then decreases again. This decrease in hysteretic nonlinearity (indicated by a decrease in modulus defect) at a high excitation frequency was prominent in the experiment reported by Nazarov et al. [14]. Note that experimental investigation of the frequency dependent behavior is especially difficult for the nonlinear-relaxation contribution to dissipation. First, the latter cannot be straightforwardly discriminated from hysteretic losses because of identical strain-amplitude dependences (in the case of wave self-action). Second, separating the contributions from different mechanisms is also complicated in the case of a pumping wave that influences a weaker probe wave because the decrement of the latter shows an essentially nonmonotone frequency dependence even at a frequency independent hysteresis loop of the form of (21) [37]. Moreover, comparison with real materials requires a realistic estimate of the relaxation frequency distribution of defects. A more detailed discussion of these issues being beyond the scope

of this paper, we limit ourselves to model illustrations (Figs. 12 and 13). Even these simplified examples demonstrate that the nonlinear-relaxation contribution can exhibit the same strain dependent behavior as hysteresis and can cause manifold changes to the observed magnitude of total nonlinear losses. This inference, as well as the possibility for the nonlinear correction to Young's modulus to change its sign at a certain frequency indicates that the ratio of nonlinear losses to nonlinear variations in Young's modulus can be arbitrarily far from the true hysteretic value of the Read parameter r which can be reliably constrained only in the quasi-static limit.

CONCLUSIONS

This study, which is a development of [25], demonstrates that the joint effect of purely elastic nonlinearity and linear relaxation can cause prominent strain dependence of dissipation in microinhomogeneous materials with linear viscous and nonlinear elastic defects. Dissipation can change strongly under both quasi-static and symmetrical periodic loading (if defects have an odd nonlinearity component). The relative change of dissipation (decrement) can be an order of magnitude greater than variations in Young's modulus.

This is the basic difference from homogeneous viscoelastic materials which do not show this effect and have relative changes in dissipation and elasticity of the same order. We found out that the mechanism [25] of effective dissipative nonlinearity in a viscoelastic microinhomogeneous medium is much similar to the effect of rapid elastic nonlinearity growth at slight companion variations in linear elastic properties studied earlier using similar models of microinhomogeneous solids [8–10]. Indeed, in both cases soft defects themselves exhibit no anomalous nonlinear properties (when strain is measured in the scale of their thickness). The key role belongs to their relative softness: local strain can approach unity at soft defects the average strain being orders of magnitude lower. The locally high strain causes a very notable local deviation from the linear deformation behavior, so that average macroscopic nonlinearity increases strongly if concentration of defects is not too small. Furthermore, the same high local strain is responsible for macroscopic dissipative nonlinearity. Inasmuch as each local viscoelastic defect can have local strain about unity, even at a moderate average strain, the defects show large but still comparable local relative variations in elasticity and dissipation: $(\Delta\theta/\theta)_{loc} \approx (\Delta E/E)_{loc}$ (see the beginning of the first section). However, the macroscopic manifestations of these comparable variations are drastically different. Indeed, the contribution of soft defects to macroscopic Young's modulus of a medium is normally rather small, and even a quite strong change $(\Delta E/E)_{loc}$ causes a minor effect on the macroscopic elasticity: $(\Delta E/E)_{macro} \ll (\Delta E/E)_{loc}$. On the contrary, macroscopic dissipation in a microinhomogeneous medium is originally mostly due to soft defects, and local strong dissipation change at defects can thus cause a comparable change in the total macroscopic dissipation, i.e., $(\Delta\theta/\theta)_{macro} \sim (\Delta\theta/\theta)_{loc}$. This is the physical meaning of the mechanism we discuss. We note additionally that exponential accumulation of losses with distance can cause still greater variations in wave attenuation (due to manifold changes in the resulting amplitude even at moderate decrement variations).

Thus, the combined effect of linear viscous losses and nonlinear elasticity can provide an essential contribution to the experimentally observed behavior related to strain-amplitude dependent dissipation along with the known hysteretic nonlinear mechanisms and eventual genuine nonlinear dissipation (e.g., due to nonlinear viscous losses). The combined nonhysteretic mechanism acts simultaneously with hysteretic nonlinearity related to adhesion and friction effects and should be thus taken into account in interpretation of experiments. Namely, estimates of the Read parameter (r) from acoustic measurements can be strongly biased as the observed ratio of nonlinear dissipation to modulus variations can be strongly affected by the nonlinear-relaxation component. Thus the nonlinear acoustic data can differ considerably from the true r values expected for a quasi-static hysteresis. In other cases, like that of an amplitude dependent velocity increase of an elastic wave [38], the relaxational sign change in the nonlinear correction to Young's modulus at a high-frequency excitation can work along with the conventional hysteretic mechanism discussed in [38].

Note finally that the nonlinear-relaxation correction to dissipation can likewise change its sign at different frequencies, for instance, due to thermoelastic relaxation at strip-like nonlinear contacts in cracks [24, 39]. The reason is that the nonlinear shift of the relaxation peak at these contacts can occur at an almost invariable peak height. As a result, dissipation can decrease on one peak slope and increase on the other depending on observation frequency. Thus, the possible superposition of hysteretic nonlinearity, elastic nonlinearity, and nonlinear-relaxation effects should be taken into account in interpretation of nonlinear acoustic and seismic data.

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