On the differences between "adhesion-type" and "friction-type" hysteresis: theoretical description and experimental indications

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Summary

An efficient tool for theoretical modeling of hysteretic nonlinearities provides the so-called Preisach approach, in which hysteretic response of a material is modeled as superposition of responses of a huge ensemble of elementary hysteretic units. Although these elements are not assumed to directly correspond to some physical microstructural features in the material, real frictional phenomena at microcontacts in microstructured solids may be considered to support the model. Indeed, in its conventional form, for each hysteretic unit, the stress-strain dependence looks like for an elementary "frictional machine" (rectangular loop with vertical jumps of fixed magnitude between two strain-independent stress levels). For acoustic (that is relatively small) strains, the distribution of the hysteretic units over their parameters can be approximated as uniform. This results in piecewise-quadratic, convexo-convex ("pea pod-shaped") antisymmetric macroscopic hysteresis loops with elastic modulus jumps at the turning points. Inhomogeneous distribution of the units allows for modeling quite arbitrary forms of hysteretic dependencies. Here we report a modified Preisach-type model with more general shape of hysteretic trapezoidal units (up to triangular ones). Physically, such units may be viewed as "elementary adhesion machines" implying adhesion (and tearing off) effects at contacting asperities, when the jumps in stress are accompanied by the jumps in modulus. Even at uniform density of these elements (which is a reasonable approximation for acoustic strains), the resultant macroscopic hysteresis strongly differs from the conventionally discussed "friction-type" antisymmetric piece-wise quadratic loops. The adhesional loops are asymmetrical and are rather peace-wise cubic. This affects the ratio of the hysteretic losses and modulus defect, modifies the character of harmonic generation, etc. Some experimental data supporting these conclusions are known.

Main features of the hysteresis model based on "friction-type" units

In order to better show similarities and differences between the two mentioned types of hysteresis we briefly recall here basic assumptions and results obtained using the Preisach-Krasnoselsky-Mayergoyz (PKM) [1] approach to hysteresis description based on consideration of large ensembles of elementary hysteretic units. These units are supposed to be embedded into linearly-elastic matrix material. Conventionally, for individual elements the simplest hysteretic stress-strain function of a rectangular shape is

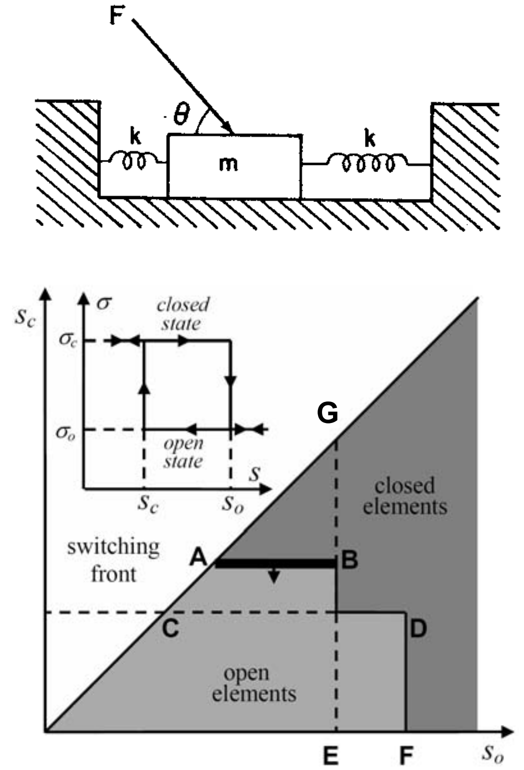

Figure 1: (a) Elementary "friction machine" with rectangular "stress-displacement" dependence proposed in [2] and (b) rectangular hysteresis loop of an individual hysteretic unit (in the inset) and the PKM-plane (s_c , s_o) characterising distribution of a large ensemble of the units over their controlling strains. Straining of the material corresponds to movement of either vertical (at $\partial s/\partial t>0$) or horizontal (at $\partial s/\partial t<0$) "switching front" (see segment AB). After the front the units switch to the opposite state.

used as shown in the inset in Fig. 1. Such rectangular loops (that sometimes were superimposed to the linear Hooke law yielding parallelograms) were often discussed when hysteresis was physically attributed to friction phenomena at crack interfaces and intergrain contacts [2,3]. It is convenient to assume identical variation $\Delta\sigma=\sigma_c - \sigma_o$ of threshold stresses σ_c and σ_o corresponding to threshold strains s_o and s_c for each unit. The difference between the elements is described by different controlling threshold values for the material strain s, that is s_o at which the elements are switched from the "closed" state to the "open" one during the positive (with $\partial s/\partial t>0$) phase of straining, and s_c, at which the element switches back to the "closed" state during the negative phase of straining (with $\partial s/\partial t<0$). Since by definition $s_c \le s_o$ (see Fig. 1), the hysteretic elements on the plane (s_c , s_o) are located only below the diagonal $s_c = s_o$. The plane (s_c , s_o) of the controlling parameters is called PKM-plane (or PKMspace). The hysteretic correction to stress can be geometrically interpreted as the area or, more rigorously, the "weight" of triangles (like triangle *ABG*), for which the "switching front" plays the role of the base. Consequently, the length (or rather the "weight") of the switching front *AB* corresponds to the current hysteretic correction to the elastic modulus.

Note also that it is possible to represent the elementary hysteretic units in terms of controlling parameters (σ_c ,σ_o), assuming that elements' switching is controlled by the material stress, rather than strain. The two representations are equivalent (as is elucidated in [4]), provided the hysteretic correction to the stress-strain relationship is small and in the first approximation the material can be described by the linear Hooke's law, the latter condition being practically always valid in nonlinear acoustics.

We limit ourselves to the conventional assumption of instantaneous switching between different states of the element.

In order to obtain stress-strain relationship for the material containing a large ensemble of such elements, it is convenient to evaluate first the hysteretic correction to the elastic modulus. The latter is equal to the derivative $\partial\sigma_H/\partial s$ of the hysteretic correction to stress given by summation of contributions of the elements,

$$\partial\sigma_H/\partial s = \sum_{\mathrm{M}} \partial\sigma_M/\partial s\,, \quad (1)$$

and elementary $\partial\sigma_M/\partial s$ are readily found by differentiating the curve plotted in the inset in Fig. 1:

$$\frac{\partial\sigma_M}{\partial s}(s,s_o,s_c) = -\begin{cases} \Delta\sigma\cdot\delta(s\text{-}s_0),\ \text{if}\ \partial s/\partial t>0, \\ \text{and the element is initially closed} \\ \Delta\sigma\cdot\delta(s\text{-}s_c),\ \text{if}\ \partial s/\partial t<0, \\ \text{and the element is initially opened} \end{cases}, \quad (2)$$

Summation (1) can be performed as integration over the distribution $f(s_c, s_o)$ of the elements over their parameters (s_c, s_o). Assuming periodic acoustic strain with amplitude *A* and using the approximation $f(s_c, s_o)\approx f_o$ one obtains :

$$\partial\sigma_H/\partial s = \begin{cases} hE(\varepsilon - A), & \partial\varepsilon/\partial t<0 \\ -hE(\varepsilon + A), & \partial\varepsilon/\partial t>0 \end{cases} \quad (3)$$

$$\sigma_H = \begin{cases} \sigma_H(A) + \dfrac{h\,E}{2}(\varepsilon - A)^2, \quad \partial\varepsilon/\partial t<0 \\ \sigma_H(-A) - \dfrac{h\,E(\varepsilon+A)^2}{2} = \\ \sigma_H(A) - \dfrac{h\,E}{2}\left[(\varepsilon + A)^2 - 4A^2\right], \partial\varepsilon/\partial t>0 \end{cases} \quad (4)$$

where $h_H = \Delta\sigma f_o$ and *E* is the elastic modulus for the matrix material containing hysteretic units. Equations (4) describe asymmetrical piece-wise quadratic in strain hysteretic loops with modulus jumps 2 $h_H EA$ in both turning points. Graphically the current hysteretic correction to modulus equals to the « weight » of the "switching front" or simply to its length for $f(s_c, s_o)\approx$const. (see segment AB in Fig. 1). Inhomogeneous functions $f(s_c, s_o)$, in principle, can describe rather arbitrary shapes of hysteretic loops. For example, since the "weight" of the switching front is always zero when it departs from the turning point (since the initial front

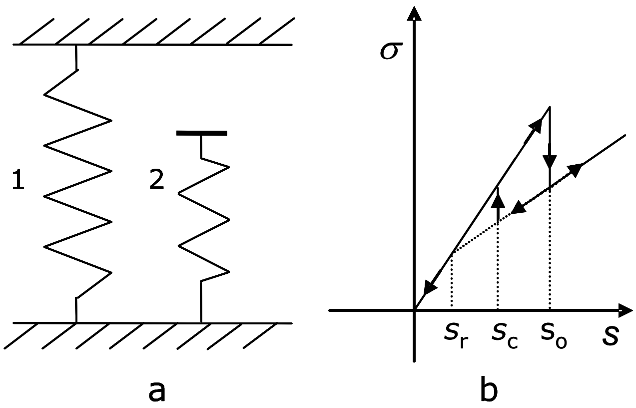

Figure 2: (a) Schematically shown adhesion-type hysteretic element in which part 2 may lost and restore adhesion contact. Plot (b) represents for such an element its stress-strain loop of a trapezoidal form that reduces to triangular one in the ultimate case when characteristic strains s_c and s_r coincide, so that parameter $\rho=(s_c\text{-}s_r)/(s_c\text{-}s_r)=0$.

length is zero as is clear from Fig.1), the absence of modulus jump means that its "weight" should be zero when it approaches the turning point. The latter means that function $f(s_c, s_o)$ is strongly inhomogeneous, so that it becomes zero before this turning point. Even if this might be provided at a given average static strain, at another static strain this condition might break, which should result in the hysteresis shape change. In this sense, for smooth (in scale of the oscillation amplitude) function $f(s_c, s_o)$, the resultant loop of the form (4) is self-similar at any initial strain. Other loop shapes may require essential adjusting the distribution $f(s_c, s_o)$ for different initial strains, which means that such a PKM-plane does not reflect physical structure of the material, but is rather a formal description which is satisfactory in a rather limited strain range.

The consideration below demonstrates that appropriately chosen another shape of elementary units on the basis of some physical arguments may simplify the description of hysteretic loops, which are essentially different from (4) and exhibit important features well correlated with some experimental data.

Main features of the hysteresis model based on "adhesion-type" units

Along with physically verisimilar « friction-type » [1-3] rectangular hysteretic nonlinearity of defects in rocks and similar microstructured materials, adhesion is another likely physical cause of hysteresis [5]. In this respect, by analogy with elementary « friction » machine, elementary « adhesion-machine » can be considered (see Fig. 2). Break and restoration of adhesion bonds for a single hysteretic unit may be viewed as elementary trapezoidal (or triangular in the ultimate case) stress-strain loop. Such elements are characterized as earlier by thresholds strains s_o and s_c and by parameter ρ describing the difference of the trapezoid shape from the complete triangle. At ρ=0 trapezoids transform into triangles, and "closing" strain s_c coincides with "return" strain s_r as shown in Fig.2). To simplify mathematics, equal jumps in the elastic modulus are assumed for the trapezoids (triangles) by analogy with equal jumps in stress as it was assumed in the case of rectangular units in the previous section. The distribution of the adhesion-type elements again is described by distribution function $f(s_c, s_o)$ at the PKM-plane. Analytically summation of contributions of the trapezoidal elements can be made much like in the case of rectangular elements (although differentiation of elementary σ_M over strain s is convenient to make twice in order to intermediately obtain delta-functions arising from both jumps in the stress and in the elastic modulus). Using such a procedure, the following expressions for macroscopic hysteretic corrections $\partial^2\sigma_H/\partial s^2$, $\partial\sigma_H/\partial s$ and σ_H are readily obtained assuming again the approximation of uniform distribution of the elements $f(s_c, s_o)\approx f_o$:

$$\partial^2\sigma_H/\partial s^2 = \begin{cases} hE(\varepsilon - A)\dfrac{1-2\rho}{(1-\rho)^2}, & \partial\varepsilon/\partial t < 0 \\ -hE(\varepsilon + A)\dfrac{2-\rho}{(1-\rho)^2}, & \partial\varepsilon/\partial t > 0 \end{cases} \quad (5)$$

$$\frac{\partial\sigma_H}{\partial s} = \begin{cases} \left\{(\varepsilon - A)^2\dfrac{1-2\rho}{2} - 2A^2(1-\rho)\right\}\dfrac{hE}{(1-\rho)^2}, & \dfrac{\partial\varepsilon}{\partial t} < 0 \\ -(\varepsilon + A)^2\dfrac{2-\rho}{2(1-\rho)^2}hE, & \dfrac{\partial\varepsilon}{\partial t} > 0 \end{cases} \quad (6)$$

$$\sigma_H = \begin{cases} \left\{(\varepsilon - A)^3\dfrac{1-2\rho}{6} - 2A^2(s-A)(1-\rho) - \dfrac{4A^3(2-\rho)}{3}\right\}\dfrac{hE}{(1-\rho)^2}, & \dfrac{\partial\varepsilon}{\partial t} < 0 \\ -(\varepsilon + A)^3\dfrac{2-\rho}{6(1-\rho)^2}hE, & \dfrac{\partial\varepsilon}{\partial t} > 0 \end{cases} \quad (7)$$

These equations demonstrate essential differences between the « adhesion-type » hysteresis and « friction-type » hysteretic response described by Eqs. (3) and (4).

The first essential difference is that summation of contributions of the adhesion-type units results in piece-wise cubic stress-strain dependence (7) rather than piece-wise quadratic shape (4) obtained for "friction-type" units in the same approximation $f(s_c, s_o)\approx f_o$. Figure 3 illustrates the shapes for hysteretic corrections to the elastic modulus $\partial\sigma_H/\partial s$ and hysteretic stress-strain loops σ_H of both types. Cases ρ=1/2 and ultimate case ρ=0 (when the elementary adhesional trapezoids reduce to triangles) are shown. Comparison indicates that, even in the small-amplitude limit, when for the hysteretic units the uniform-density approximation $f(s_c, s_o)\approx f_o$ is valid, the adhesion-type hysteresis is essentially asymmetric. Namely, the $\sigma_H(s)$ shape described by Eq. (7) is convexo-concave "banana-like" unlike antisymmetric convexo-convex "peapod-like" loop for friction type hysteresis.

Under sinusoidal excitation, this asymmetry of the adhesion-type loop results in generation of both even- and odd-order harmonics which all are cubic in amplitude in contrast to quadratic in amplitude harmonics (and odd-orders only!) in the case of antisymmetrical friction-type hysteresis (4). Further, at the left turning point (corresponding to compression) the adhesion hysteresis in the ultimate case ρ=0 (corresponding to triangular units) does not exhibit modulus jump at all. At ρ≠0 (for trapezoidal elementary

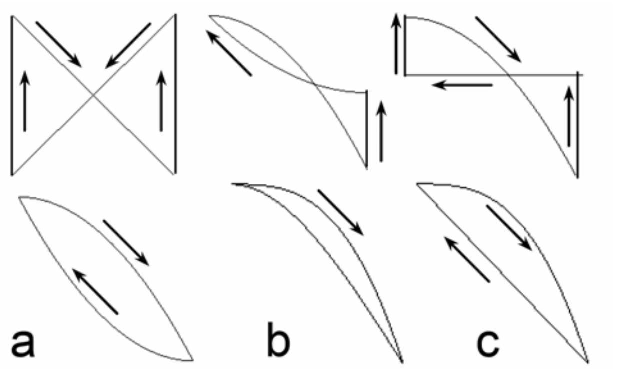

Figure 3: Hysteresis loops for the elastic modulus $\partial\sigma_H/\partial s$ (upper row) and for stress-strain correction $\sigma_H(s)$ (lower row) in the case of friction-type hysteresis (a) and for adhesion-type hysteresis: (b) – parameter ρ=0 (elementary triangle), and (c) parameter ρ=1/2 (elementary trapezoid).

loops) the modulus jump occurs at both turning points, but it is essentially asymmetrical, and at the left side it remains smaller than that for the right turning point. In the loop for the modulus $\partial\sigma_H/\partial s$ this asymmetry manifests itself both in unequal vertical jumps and in asymmetry in slopes for compression and tension half-cycles.

Another important characteristic of hysteretic loops is the ratio between the hysteretic energy losses over one oscillation period and the period-averaged correction to the elastic modulus due to the hysteretic nonlinearity. For piece-wise power-type loops this ratio does not depend on amplitude [6]. Functional amplitude behavior of the nonlinear losses and the modulus variation is different as follows from Eqs. (4) and (7): proportional to oscillation amplitude for friction-type quadratic hysteresis (4) and proportional to the amplitude squared for piece-wise cubic adhesion-type hysteresis (7). The aforementioned differences are summarized in the following Table:

	Piece-wise quadratic friction-type hysteresis, Eq. (4)	Piece-wise cubic adhesion-type hysteresis, Eq. (7) at ρ=0
Nonlinear losses over period $\theta_H=\Delta W_H/(EA^2)$	$(4/3)hA$	$(2/3)\,hA^2$
Nonlinear variation in elasticity $\Delta E_H/E$	$-hA$	$-(21/16)\,hA^2$
Read parameter $r=\theta_H/\lvert\Delta E_H/E\rvert$	4/3	32/63
Amplitude behavior of higher harmonics	$\sim A^2$ odd-orders only	$\sim A^3$ odd and even

Although the results discussed above were obtained based on phenomenologically introduced shapes of elementary hysteretic units, the main properties of these units can be readily recognized in main peculiarities of adhesion and friction phenomena that occur at real microstructural features of a solid (at crack interfaces, at contacting asperities, etc.), for example, in rocks and other microstructured materials. From this point of view, even if quite an arbitrary hysteretic loop (including convexo-concave loops of the adhesion type) might be modelled using inhomogeneous distribution of classical rectangular (friction-type) hysteretic units, choosing physically more adequate shape of the elementary units may significantly simplify the description. Eqs. (5)-(7) demonstrate that simple uniform distribution of adhesion-type units may be used instead of strongly localized distribution $f(s_c, s_o)$ of classical rectangular units, which should be assumed in order to model the absence of the modulus jump at the turning point (see elucidation in the previous section). Such a strongly localized $f(s_c, s_o)$ can be provided at certain area of the PKM-plane corresponding to a specific average (static) strain in the material. However, at another static strain this condition may break (so that the hysteresis loop would exhibit the modulus jump at the turning point). In contrast, for adhesion-type triangular units at any initial strain and at sufficiently small oscillatory strain the locally distribution $f(s_c, s_o)$ remains near-constant. This results in formation of similar loops with the intrinsic absence of the modulus jump at one even for strongly different initial strains (different initial locations on the PKM-plane). Thus observation of the type of the loops (convexo-convex or convexo-concave), comparison of the shapes at different static strains and different amplitudes of the oscillatory strains as well as comparison of other functional dependencies (see the Table) may be used for identification of the hysteresis type. Certainly, microstructural features with different types of hysteretic responses may coexist in the material. In such a situation due to different amplitude behavior it is reasonable to expect that, at smaller amplitudes, contribution of quadratic frictional-type hysteresis may dominate, whereas at higher amplitudes cubic adhesion-type contribution can become dominant.

Experimental data are available which support the conclusions obtained above. Note first, that at strains larger than 10^{-5} direct observations of quasistatic hysteretic stress-strain loops indicate that their shape is not "peapod-type" antisymmetric, but indeed is rather convexo-concave (e.g. see [2,3,7,8], where numerous examples for strains in the range 10^{-4}-10^{-3} are presented). At smaller strains below 10^{-4} the loop shapes are not sufficiently resolved, but they are clearly rather convexo-convex at the same average strains. When making comparison between experiments and theory one should take into account that theoretical curves (like those shown in Fig. 3) represent only the nonlinear correction due to do hysteretic units embedded into the matrix material. In experiments, the contribution of the linear elasticity of the matrix strongly dominates in the observed stress-strain relationship. In this sense, more direct comparison can be made with the experimentally obtained loops for the elastic modulus, since the "background" linear term does not affect their shapes. In ref. [8], for the modulus, the experimentally obtained butterfly-type loops are essentially asymmetrical (not like case (a), but closer to case (c) in Fig.3). It is important that this convexo-concave shape persists at different initial strains and thus is not only typical of one specific state of the material.

At acoustic amplitudes below 10^{-5} direct stress-strain hysteresis observations are not yet available, so that more indirect indications (like amplitude dependencies presented in the Table) should be used for hysteresis identification. There are numerous data indicating that amplitude-dependent losses and complementary modulus variation are both linear in amplitude [9,10]. In such cases, higher harmonics are predominantly odd-orders and are quadratic in amplitude (3^{rd} harmonic is quadratic in excitation amplitude), which agree well with piece-wise quadratic "friction-type" model. However, even for quite small acoustic amplitudes 10^{-7} -10^{-5} there are known examples (marble, sandstone samples) [10,11] that exhibit clearly quadratic-in-amplitude nonlinear losses and the complementary modulus variation, and, respectively, cubic amplitude dependence for the 3^{rd} harmonic. The variety of these data consistently agrees well with the cubic character of hysteretic stress-strain curve for these samples. Possible influence of strongly inhomogeneous element density for classical rectangular PKM-elements physically looks very unlikely for rather weak strains 10^{-7} -10^{-5} in experiments [10-11]. In contrast, the obtained above conclusions for piece-wise cubic hysteresis based on "adhesion-type" elementary units quite naturally correlate with such data under physically more natural assumption of locally near-uniform density $f(s_c, s_o)$.

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