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Dissipation and Dispersion Properties of Microinhomogeneous Media

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Abstract—The propagation of a harmonic elastic wave in a microinhomogeneous (defect-containing) medium is considered in the framework of the rheological model that reperesents the medium in the form of a one-dimensional chain of masses connected by purely elastic elements and by Kelvin–Voigt viscoelastic elements. Analytical expressions are derived for the dissipation and dispersion characteristics of this medium for various distributions of the parameters of the viscoelastic elements. The dissipation and dispersion properties are found to obey the Kramers–Kronig relations. It is also shown that the damping decrement of the wave is almost constant, and the phase velocity monotonically increases in a sufficiently wide range of parameters of the viscoelastic elements in a wide frequency band. The derived expressions for the dispersion and dissipation are used to simulate the propagation of broadband pulses in this kind of medium. © *2000 MAIK "Nauka/Interperiodica".*

INTRODUCTION

It has been established experimentally that, in a wide frequency band, most microinhomogeneous (defect-containing) solids (in particular, rocks and metals) are described by an almost constant Q-factor [1–3]. The Kramers–Kronig relations [1, 4], which relate the dispersion and attenuation characteristics, show that these media must also exhibit noticeable dispersion. This conclusion is corroborated by experiments. They show that, in particular, the waveforms of propagating seismic pulses are asymmetric [1]. The problem of an adequate analytical description of the dissipation and dispersion properties of such media as functions of frequency is still under discussion [1, 3, 5, 6], because phenomenological approximations of these dependencies often do not fit each other and violate the causality principle.

A rheological model of a microinhomogeneous medium was proposed in our previous publications [7, 8]. This model explains why the Q-factor of such a medium is almost frequency-independent. We calculated the Q-factor on the basis of the energy approach, which allows for the absorption of the elastic wave energy by soft dissipative inclusions (defects) of different compliance. Clearly, this approach cannot directly determine the dispersion properties of the medium, though they can be found from the dissipation dependencies provided by the Kramers–Kronig relations.

In this paper, we derive the dissipation and dispersion properties of the microinhomogeneous medium in the framework of the rheological model [7, 8]. At first, we address the propagation of a harmonic elastic wave in this medium and find its attenuation factor and propagation velocity. Then, we show that the dissipation and dispersion properties thus derived satisfy the Kramers–Kronig relations. Finally, we study the propagation of broadband pulses by numerical simulation with the use of these dissipation and dispersion relations.

MODEL OF A MICROINHOMOGENEOUS MEDIUM AND THE BASIC EQUATIONS

Consider the propagation of a harmonic elastic wave in the framework of the model that represents the microinhomogeneous medium as a chain of masses and elastic and viscoelastic elements [7, 8] (Fig. 1). As in [7], we assume that the elastic elements of the chain are characterized by the elasticity coefficient K, and the dissipation effects are associated with the Kelvin–Voigt viscoelastic elements whose equations of state have the form

$$\sigma = k_j X + g\dot{X}, \tag{1}$$

where k_j and g are the elasticity and viscosity coefficients of the jth element ($k_j = \zeta_j K$, $\zeta_j \ll 1$), X is the variation of the defect length, and $\dot{X} = \partial X/\partial t$. Assume that a unit length of this chain contains a total of N elements of the length H, the number of viscoelastic elements being equal to N_1, so that the ratio $\nu = N_1/N$ characterizes the defect density. Also assume that the length of the elastic wave $\lambda = 2\pi/k$ (where k is the wave number) is much greater than the defect length H, so that $kH \ll 1$. These conditions allow one to regard the homogeneous regions of the chain as a continuum and assume that the elastic wave subjects each element to quasistatic deformations. Then, one can consider the interaction of the

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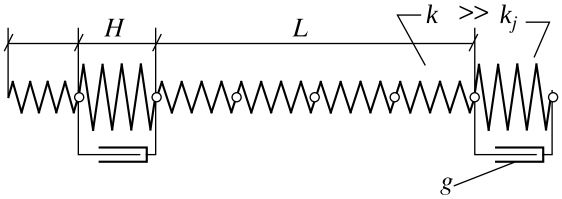

Fig. 1. Rheological model of a microinhomogeneous medium.

elastic wave with the defect of number j in terms of only the incident, reflected, and transmitted waves outside this defect and describe the deformation of the defect by the equations

$$\begin{gathered}\sigma_i + \sigma_r\big|_{x = x_j} = \sigma_t\big|_{x = x_j + H},\\ U_i - U_r\big|_{x = x_j} = U_1,\\ U_t\big|_{x = x_j + H} = U_2,\\ \sigma_t\big|_{x = x_j + H} = k_j(U_2 - U_1) + g(\dot{U}_2 - \dot{U}_1),\end{gathered} \quad (2)$$

where $x = x_j$ and $x = x_j + H$ are the coordinates of the defect boundaries, and the stress and strain, σ_i, σ_r, σ_t and U_i, U_r, U_t, in the incident, reflected, and transmitted waves, respectively, are related through the elasticity coefficient K

$$\sigma_{i, r, t} = K\partial U_{i, r, t}/\partial x. \quad (3)$$

DISPERSION RELATION FOR THE MICROINHOMOGENEOUS MEDIUM

For a harmonic wave $\exp(-i\omega t \pm ikx)$ propagating in the homogeneous part of the chain in the positive (incident and transmitted waves) and negative (reflected wave) directions, respectively, equations (3) take the form

$$\sigma_i = ikKU_i, \quad \sigma_r = -ikKU_r, \quad \sigma_t = ikKU_t, \quad (4)$$

where $k = \omega/c_0$ and c_0 is the elastic wave velocity in a defect-free medium. Introduce the transmission T_j and reflection R_j factors for the jth defect:

$$\sigma_i\big|_{x = x_j} = R_j\sigma_r, \quad \sigma_t\big|_{x = x_j + H} = T_j\sigma_i. \quad (5)$$

Substituting expressions (4) and (5) into the system of equations (2), we obtain

$$R_j = \exp(2ikx_0)\left[\frac{2(\zeta - i\omega/\Omega)}{ikH} - 1\right]^{-1}, \quad (6)$$

$$T_j = \exp(-ikH)\left[1 - \frac{ikH}{2(\zeta - i\omega/\Omega)}\right]^{-1}, \quad (7)$$

where $\Omega = K/g$.

These expressions show that, when $kH \ll \zeta_j < 1$, the magnitudes of the transmission and reflection factors tend to unity ($|T_j| \longrightarrow 1$) and zero ($|R_j| \ll 1$), respectively. Under the assumption that reflection from each defect is negligible, the microinhomogeneous medium can be considered as an equivalent homogeneous (on average, on a scale much larger than the distance between the defects and covering a great number of them) medium with effective (spatially averaged) parameters [9]. In order to calculate the effective parameters of the equivalent homogeneous medium, which characterize the wave attenuation and the phase shift gained in the course of the propagation through the chain of the length $L = NH$ containing a large number of defects, let us find their effect on the wave amplitude and phase. Clearly, for this purpose one should find a product of the transmission factors of all elements of the chain. As a result, we obtain the following formula for the total transmission factor T_{total} of the wave transmitted through the chain of length L:

$$T_{\text{total}} = \exp(-ikL)\prod_{j = 1}^{N_1}\tilde{T}_j, \quad (8)$$

where the factor $\exp(-ikL)$ describes the phase delay of the wave transmitted through the chain of length $L = NH$ consisting of N elements, N_1 of which contain defects, and the factors $\tilde{T}_j$ are given by the expression

$$\tilde{T}_j = \left[1 - \frac{ikH}{2(\zeta_j - i\omega/\Omega)}\right]^{-1}. \quad (9)$$

By equating expression (8) to the transmission factor of the equivalent homogeneous medium defined as $T_0 = \exp(-i\tilde{k}L)$, we obtain an equation for the complex wave number $\tilde{k} = \tilde{k}(\omega)$ of the homogeneous medium

$$\exp(-ikL)\prod_{j = 1}^{N_1}\tilde{T}_j = \exp(-i\tilde{k}L), \quad (10)$$

which yields

$$(\tilde{k} - k)L = -i\ln\left(\prod_{j = 1}^{N_1}\tilde{T}_j\right) = -i\sum_{j = 1}^{N_1}\ln(\tilde{T}_j). \quad (11)$$

Next, we take into account that, for the part of defects that are described by the same parameter ζ_j, $\sum_{j = 1}^{N_j} H/L = \nu_j$ characterizes their linear concentration. When the number of defects is great, the distribution function $\nu = \nu(\zeta)$ can be introduced, so that $\nu(\zeta)d\zeta$ is the number of defects belonging to the interval $[\zeta, \zeta + d\zeta]$ per unit length of the chain. Then, in view of (9), expression (11) takes the form

$$\tilde{k} - k = i\int_0^1 \ln\left[1 - \frac{ikH}{2(\zeta - i\omega/\Omega)}\right]\nu(\zeta)d\zeta. \quad (12)$$

When $kH \ll \zeta < 1$, this equation yields the dispersion relation for the microinhomogeneous medium:

$$\tilde{k} - k = \frac{kH}{2}\int_0^1 \left[\frac{\zeta}{\zeta^2 + (\omega/\Omega)^2} - \frac{i(\omega/\Omega)}{\zeta^2 + (\omega/\Omega)^2}\right]\nu(\zeta)d\zeta. \quad (13)$$

The first summand in the integrand determines the frequency-dependent phase correction, and the second one determines the wave attenuation. Using the relationship $\theta(\omega) = \alpha(\omega)\lambda$ between the wave attenuation factor $\alpha(\omega)$ and the damping decrement $\theta(\omega)$ in equation (13), we obtain

$$\theta = \pi\int_0^1 \nu(\zeta)\frac{\omega/\Omega}{\zeta^2 + (\omega/\Omega)^2}d\zeta. \quad (14)$$

(A similar expression was obtained in [7, 8] by summing the losses due to individual inclusions.) Equation (13) can be used to obtain the dispersion correction to the phase velocity $\Delta c(\omega) = c(\omega) - c_0$ (when $\Delta c/c_0 \ll 1$):

$$\frac{\Delta c(\omega)}{c_0} = -\frac{1}{2}\int_0^1 \nu(\zeta)\frac{\zeta}{\zeta^2 + (\omega/\Omega)^2}d\zeta. \quad (15)$$

According to [7, 8], the independence of the Q-factor from frequency can be explained under the assumption that the defect distribution in the parameter ζ is wide, which is likely to be valid for actual microinhomogeneous media. (Taking additionally into consideration the defect distribution in the parameter Ω does not change the main conclusions. Therefore, for the time being, we can assume that the parameter Ω is constant.) Let the defect distribution in the parameter ζ in equations (14) and (15) be described by a Π-shaped function

$$\begin{aligned} \nu(\zeta) &= \nu_0 \ \text{ for } \ \zeta \in [a, b], \quad a \ll \zeta \ll b, \\ \nu(\zeta) &= 0 \ \text{ for } \ \zeta \notin [a, b]. \end{aligned} \quad (16)$$

In this case, integral (14) yields

$$\theta = \pi\nu_0 \arctan\left[\left(\frac{b\Omega}{\omega} - \frac{a\Omega}{\omega}\right)\Big/\left(1 + \frac{ba\Omega^2}{\omega^2}\right)\right]. \quad (17)$$

From expression (17), we obtain

$$\theta \approx \pi\nu_0(\omega/2a\Omega), \ \text{ for } \ \omega/\Omega < a, \quad (18)$$

$$\theta \approx \pi^2\nu_0/2, \ \text{ for } \ a < \omega/\Omega < b, \quad (19)$$

$$\theta \approx \pi\nu_0(b\Omega/2\omega), \ \text{ for } \ \omega/\Omega < b. \quad (20)$$

In a similar manner, we use expression (15) for the dispersion correction to obtain

$$\frac{\Delta c(\omega)}{c} \approx -\frac{\nu_0}{4}\ln\frac{b^2 + (\omega/\Omega)^2}{a^2 + (\omega/\Omega)^2}. \quad (21)$$

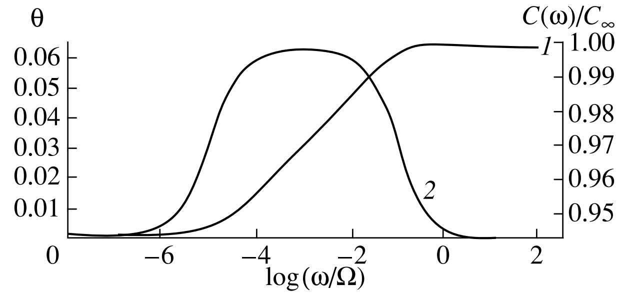

Fig. 2. (*1*) Dispersion and (*2*) dissipation characteristics of the medium for $a = 10^{-5}$, $b = 10^{-1}$, $\Omega = 10^8$ Hz, $\nu = 1.3 \times 10^{-3}$, and $c_0 = 3000$ m/s.

This correction is negative and tends to zero as $\omega \longrightarrow \infty$; therefore, c_0 has the meaning of the high-frequency limit of the phase velocity in the microinhomogeneous medium. Figure 2 displays the damping decrement and the phase velocity of the elastic wave against frequency. These plots show that a wide defect distribution in elasticity leads to a wide frequency region $a < \omega/\Omega < b$ where the damping decrement is almost constant and the phase velocity gradually increases. It is of interest that, in this region, the decrement is independent of the viscosity of the defects and is determined by their density alone. The relative change in the wave phase velocity $\Delta c/c_0 = [c(\omega \gg b\Omega) - c(\omega \ll a\Omega)]/c_0 \approx (\nu_0/2)\ln(b/a)$ is also determined by the defect density and by the range $d = b/a$ where $\theta \approx$ const.

ALLOWANCE FOR THE DEFECT DISTRIBUTION IN VISCOSITY

Expressions (17) and (21) were derived under the assumption that the defects possess identical viscous properties, i.e., that the parameter Ω is the same for all defects. In real microinhomogeneous media, the viscoelastic inclusions are not only distributed in their elastic parameters, but also in their viscous properties. It can be expected that these distributions are statistically independent, i.e.,

$$\nu = \nu(\zeta, \Omega) = \nu(\zeta)\nu(\Omega). \quad (22)$$

In this case, in order to find the contribution of all defects, the integrals in (14) and (15) should be taken with respect to both parameters ζ and Ω:

$$\theta = \pi\iint \nu(\zeta, \Omega)\frac{\omega/\Omega}{\zeta^2 + (\omega/\Omega)^2}d\Omega d\zeta, \quad (23)$$

$$\frac{\Delta c(\omega)}{c_0} = -\frac{1}{2}\iint \nu(\zeta, \Omega)\frac{\zeta}{\zeta^2 + (\omega/\Omega)^2}d\Omega d\zeta. \quad (24)$$

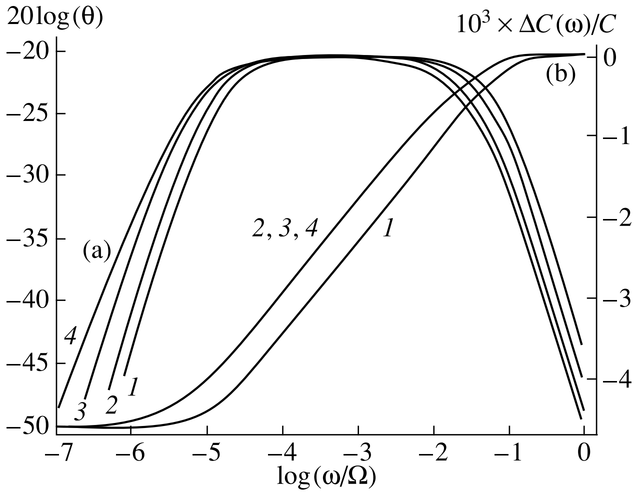

Fig. 3. (a) Dissipation and (b) dispersion characteristics of the medium for Ω_b/Ω_a = (*1*) 1, (*2*) 2, (*3*) 10, and (*4*) 10^5.

For a Π-shaped distribution function $\nu = \nu(\zeta, \Omega)$ in the form

$$\begin{aligned} \nu(\zeta, \Omega) &= \nu_0 \ \text{ for } \ \zeta \in [a, b], \quad \Omega \in [\Omega_a, \Omega_b], \\ \nu(\zeta, \Omega) &= 0 \ \text{ for } \ \zeta \notin [a, b], \quad \Omega \notin [\Omega_a, \Omega_b]. \end{aligned} \tag{25}$$

expressions (23) and (24) yield

$$\begin{aligned} \theta(\omega) &= \pi\nu_0\Omega\left[\arctan\left(\frac{b\Omega}{\omega}\right) - \arctan\left(\frac{a\Omega}{\omega}\right)\right] \\ &+ \frac{\pi\nu_0\omega}{2}\left[\frac{1}{a}\ln\left(\left(\frac{a\Omega}{\omega}\right)^2 + 1\right) - \frac{1}{b}\ln\left(\left(\frac{b\Omega}{\omega}\right)^2 + 1\right)\right]\Bigg|_{\Omega = \Omega_a}^{\Omega = \Omega_b}, \end{aligned} \tag{26}$$

$$\begin{aligned} \frac{\Delta c(\omega)}{c_0} &= -\frac{\Omega\nu_0}{4}\ln\left(\frac{b^2 + (\omega/\Omega)^2}{a^2 + (\omega/\Omega^2)}\right) \\ &+ \frac{\omega\nu_0}{2b}\arctan\left(\frac{b\Omega}{\omega}\right) - \frac{\omega\nu_0}{2a}\arctan\left(\frac{a\Omega}{\omega}\right)\Bigg|_{\Omega = \Omega_a}^{\Omega = \Omega_b}. \end{aligned} \tag{27}$$

Figure 3 shows the frequency dependencies (26) and (27) at various Ω_b/Ω_a and a fixed total defect density ν_t:

$$\begin{aligned} \nu_t &= \int \nu(\zeta, \Omega)\, d\Omega\, d\zeta \\ &\approx (b - a)(\Omega_b - \Omega_a)\nu_0 = \text{const}. \end{aligned} \tag{28}$$

These plots show that, first, a wide region where the damping decrement (or the Q-factor of the medium) is almost constant is present in every curve. Second, at a fixed total density ν_t, the decrement remains almost unchanged in this region, and the frequency boundaries of this region are weakly sensitive to the parameter Ω_b/Ω_a; namely, when Ω_b/Ω_a changes from 1 to 10^4 (curves *1–4* in Fig. 3a), the boundaries of the frequency band move towards lower frequencies (approximately by half an order of magnitude in frequency for curve *4*, which has the maximum displacement). Dispersion curves *1–4* plotted in Fig. 3b for the same values of Ω_b/Ω_a exhibit a similar behavior. These results once again show that the distribution of the elastic properties of the soft defects is the key factor that primarily determines the frequency dependencies of attenuation and dispersion.

DISSIPATION AND DISPERSION PROPERTIES AND THE KRAMERS–KRONIG RELATIONS

It is well known that the requirement for the causality principle to be fulfilled leads to the integral Kramers–Kronig relations [1, 4], which relate the frequency behavior of the real and imaginary parts of the wave number $\tilde{k}(\omega) = k(\omega) + i\alpha(\omega)$, i.e., the dissipation and the dispersion. In the case under study, it is convenient to introduce the following notation: $\tilde{k}(\omega) = \frac{\omega}{c_0} + k'(\omega) + ik''(\omega)$, where $k'(\omega) = k(\omega) - \frac{\omega}{c_0}$ is the dispersion correction and $k''(\omega) = \alpha(\omega)$. In terms of this notation, the Kramers–Kronig dispersion relations have the form [1]

$$k'(\omega) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{k''(\omega')}{\omega - \omega'} d\omega', \tag{29}$$

$$k''(\omega) = -\frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{k'(\omega)}{\omega - \omega'} d\omega'. \tag{30}$$

By virtue of integral (29) and expression (13), one obtains the relation

$$\begin{aligned} \frac{\omega}{c(\omega)} - \frac{\omega}{c_0} &= \frac{1}{2\pi c_0}\int_{-\infty}^{+\infty}\int_a^b \frac{\nu(\zeta)(\omega'/\Omega)^2}{(\omega'/\Omega)^2 + \zeta^2}\frac{1}{(\omega' - \omega)} d\zeta\, d\omega' \\ &= \frac{1}{2c_0}\omega\int_a^b \frac{\nu(\zeta)\zeta}{\zeta^2 + (\omega/\Omega)^2} d\zeta, \end{aligned}$$

which is equivalent to dispersion relation (21) when $\Delta c(\omega)/c_0 \ll 1$. In a similar manner, it can also be shown that the second Kramers–Kronig relation (30) is identically satisfied.

The structure of integrals (29) and (30) shows that, for example, the dispersion correction at a particular frequency is related to the behavior of the attenuation factor in the entire frequency range from zero to infinity, and the requirement that the integral be convergent imposes constraints, which determine the physically admissible frequency behavior of the decrement at low and high frequencies [1, 5]. It is clear that a simple approximation of the decrement, for example, by a constant does not satisfy relationships (29) and (30) and is unsuitable for estimates both at low and at high frequencies. It should be noted that some other more complex approximations of the decrement [10], which agree well with the experiments, also fail to fit the causality principle. Nevertheless, in the frequency range where the Q-factor of the medium is approximately constant, it was found that the following approximate relationship for the ratio of the phase velocities at two frequencies ω_1 and ω_2 is valid [1]:

$$\frac{c(\omega_1)}{c(\omega_2)} \approx 1 + \frac{1}{\pi Q}\ln\left(\frac{\omega_1}{\omega_2}\right). \tag{31}$$

One can easily verify that the relationships derived above agree well with this approximate result. In particular, for $a < \omega/\Omega < b$, expression (21) yields

$$\Delta c(\omega) = -\frac{\nu_0}{4}c_0\ln\frac{b^2 + (\omega/\Omega)^2}{a^2 + (\omega/\Omega)^2} \approx -\frac{\nu_0}{2}c_0\ln\frac{b\Omega}{\omega},$$

which, when used with (19), gives

$$\frac{C(\omega_1)}{C(\omega_2)} \approx \left(1 - \frac{\nu_0}{2}\ln\frac{b\Omega}{\omega_1}\right)/\left(1 - \frac{\nu_0}{2}\ln\frac{b\Omega}{\omega_2}\right) \approx 1 + \frac{1}{\pi Q}\ln\left(\frac{\omega_1}{\omega_2}\right).$$

Thus, expressions for the dispersion and the damping decrement obtained from the solution to the one-dimensional propagation problem satisfy the causality principle. Furthermore, since the same expression for the decrement can be obtained from energy considerations, which do not require that the medium be one-dimensional, the derivation of the dispersion correction from the Kramers–Kronig relations also allows us to extend these results to the three-dimensional medium, for which the propagation problem cannot be solved that easily.

NUMERICAL SIMULATION OF THE PROPAGATION OF BROADBAND PULSES

Let us use expressions (17) and (21) for the attenuation factor and phase velocity of a harmonic elastic wave to study the propagation of broadband pulses in a medium for which the wave number has the form

$$\tilde{k}(\omega) = \frac{\omega}{c_0} + \frac{\nu_0\omega}{4c_0}\ln\left|\frac{b^2 + (\omega/\Omega)^2}{a^2 + (\omega/\Omega)^2}\right| + i\nu_0\frac{\omega}{2c_0}\arctan\left[\left(\frac{b\Omega}{\omega} - \frac{a\Omega}{\omega}\right)/\left(1 + \frac{ba\Omega^2}{\omega^2}\right)\right]. \tag{32}$$

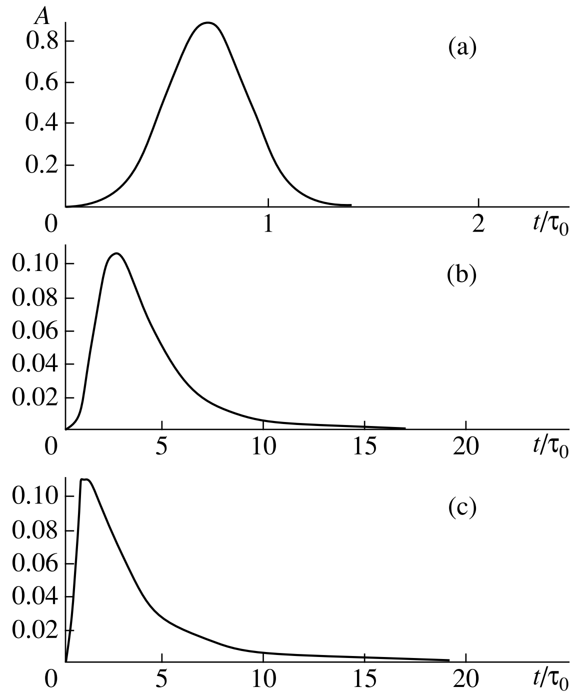

Fig. 4. Pulse waveforms in the medium for $f(t) = \exp(-(t/2\tau_0)^2)$, $x = 400\lambda$, and τ_0 = (a) 5×10^{-2}, (b) 5×10^{-5}, and (c) 10^{-7} s.

The shape of the pulse that passed a distance x in the medium can be found from the equation [4]

$$f'(t, z) = \int_{-\infty}^{+\infty} \tilde{f}(\omega)\exp(-i[\omega t - \tilde{k}(\omega)z])d\omega = \int_{-\infty}^{+\infty} \tilde{f}'(\omega)\exp(-i\omega t)d\omega, \tag{33}$$

where $\tilde{f}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(t)\exp(i\omega t)dt$ is the spectrum of the initial pulse $f = f(t)$ and $\tilde{f}'(\omega) = f'(\omega)\exp(i\tilde{k}(\omega)z)$ is the spectrum of the pulse in the medium.

Below, we present the results of the simulation for initial pulses of three types.

In the first case, we consider a Gaussian unipolar initial pulse: $f(x) = \exp(-(t/2\tau_0)^2)$. Figure 4 presents the pulse waveforms in the medium calculated for three durations of the initial pulse. The long pulse whose spectrum completely falls into the dispersion-free region of the phase velocity is symmetrically broadened due to attenuation (Fig. 4a). The pulse whose spectrum covers part of the region where the Q-factor is constant experiences dispersion effects: its leading edge becomes steeper, while the trailing edge is spread (Fig. 4b). The short pulse whose spectrum completely covers the region of the constant Q-factor experiences

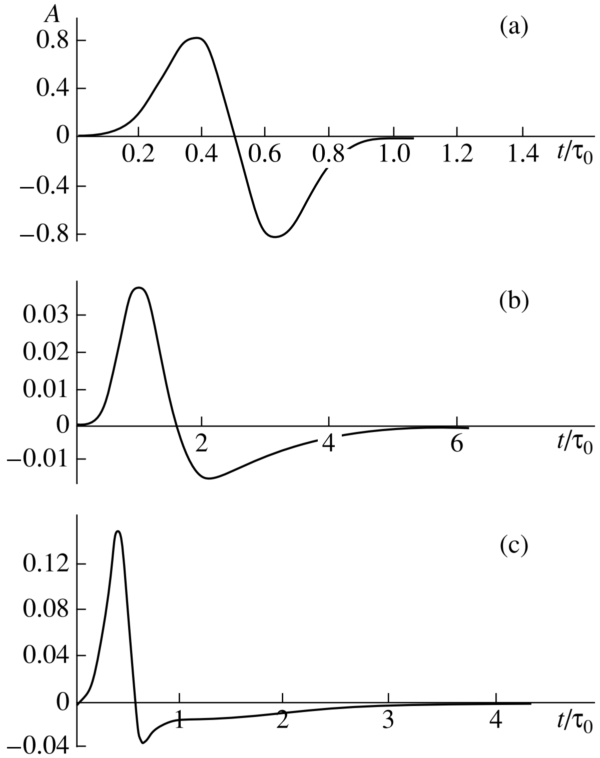

Fig. 5. Pulse waveforms in the medium for $f(t) = (-\tau_0 \exp(0.5)) \frac{d}{dt}\exp(-(t/2\tau_0)^2)$, $x = 400\lambda$, and τ_0 = (a) 5×10^{-2}, (b) 5×10^{-5}, and (c) 10^{-7} s.

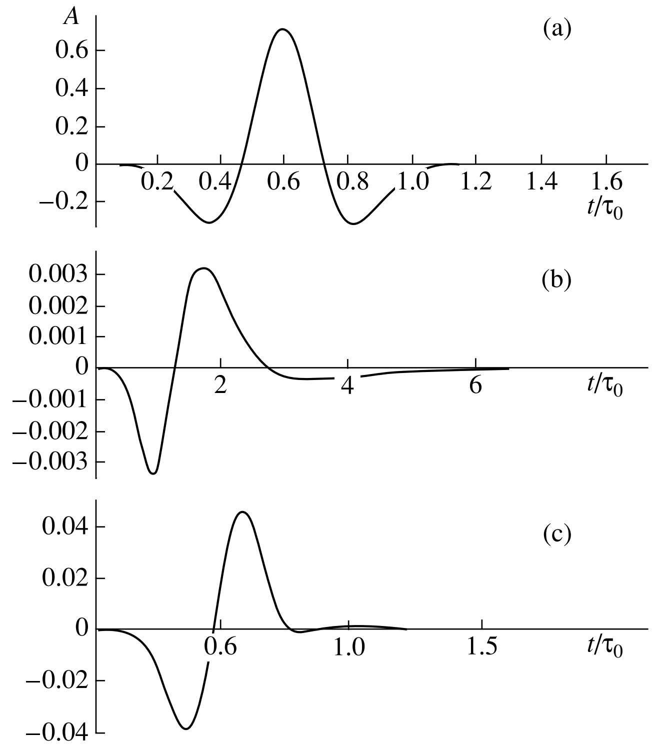

Fig. 6. Pulse waveforms in the medium for $f(t) = \tau_0^2 \frac{d^2}{dt^2}\exp(-(t/2\tau_0)^2)$, $x = 400\lambda$, and τ_0 = (a) 5×10^{-2}, (b) 5×10^{-5}, and (c) 10^{-7} s.

similar but more pronounced dispersion effects (an essentially asymmetric pulse shape, as shown in Fig. 4c).

In the second and third cases, we consider bipolar pulses in the form of the first and second time-derivatives of the Gaussian pulse:

$$f(t) = (-\tau_0 \exp(1/2))\frac{d}{dt}\exp(-(t/2\tau_0)^2),$$

$$f(t) = \tau_0^2 \frac{d^2}{dt^2}\exp(-(t/2\tau_0)^2).$$

(Such pulses are generated as a result of the self-demodulation of the high-frequency pulses in nonlinear media and can be radiated by parametric acoustic sources operating in the Berktay and Westervelt modes [11].)

The respective pulse waveforms in the medium calculated for three characteristic durations of the initial pulse are plotted in Figs. 5 and 6. For long pulses whose spectrum is localized in the region of the square-law frequency behavior of the attenuation factor, only the attenuation and the corresponding pulse broadening are significant (Figs. 5a, 6a). The pulses whose major part of spectrum falls into the region of the constant Q-factor experience noticeable dispersion effects: as they travel through the medium, their leading edge becomes steeper, and the trailing edge becomes flatter (Figs. 5b, 6b). For a short pulse, similar but more pronounced dispersion effects are observed, which make the pulse asymmetric, so that the negative part of the trailing edge almost disappears (Figs. 5c, 6c).

CONCLUSIONS

The origin of the frequency-independent decrement observed for the elastic wave attenuation in various microinhomogeneous media had been a problem under discussion for a long time [1–3, 5]. A number of phenomenological models of this effect (the best of them, Gurevich's [12] and Liu's [13] models, are in essence identical and based on the introduction of the spectrum of relaxation times of the type $1/\tau$) left open the question of their physical realization and, therefore, the question of the relationship between the model parameters and the microstructure characteristics of the medium.

The rheological model proposed above for describing a microinhomogeneous medium occupies an intermediate position between the physical models of elastic and dissipative properties of media and the phenomenological approximations that describe the empirical data. This model is based on the assumption that the elastic properties of the defects are characterized by a wide distribution, which, in terms of the relaxation times, is equivalent to the phenomenological models [5, 13]. The rheological model proposed above allows us to assign the physical meaning to the parameters introduced earlier and used in the phenomenological approaches [1, 5, 13, 14] and to relate these parameters to the microstructural characteristics of the medium. An important result of our analysis is the conclusion that, for the Q-factor to be almost frequency-independent, only a small number of microinhomogeneities are necessary. In fact, the only important factor is the presence of highly compliant defects whose size is small as compared to the wavelength of the elastic waves and whose elastic parameters are characterized by a wide distribution. It was found that, in the frequency region where the Q-factor is almost constant, its value is primarily determined by the geometric properties of the microstructure and is almost independent of both the effective viscosity and the elasticity of the defects (though these parameters determine the boundaries of this frequency region). Clearly, such defects are typical of a wide class of microinhomogeneous media; therefore, it is no wonder that materials, which at first glance appear to be wildly different (for example, many metals and rocks), exhibit similar dissipation and dispersion properties. The effective viscosity introduced in the model can be associated with thermal loss at the defects due to high temperature gradients (because the defects are small [15]) and the high rate of material deformation near the defects (intergranular contacts, cracks, etc.).

Our results agree well with the known experimental data on the dissipation and dispersion properties of microinhomogeneous elastic media [1, 2, 5, 6]. In particular, the simulation of the propagation of broadband pulses has shown that the pulse edges are asymmetric due to the dispersion distortions, which is known to be typical of real seismic pulses [1]. Finally, our results (unlike some phenomenological approximations [1, 5]) do not violate the physical limitations imposed on the relationship between attenuation and dispersion: the consequences of the physically realizable model proposed above satisfy the Kramers–Kronig relations, which directly follow from the causality principle.

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