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Detection of Acoustic Pulses in River Sand. Theory

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Abstract—Nonlinear generation and propagation of video-pulse signals produced as a result of the detection of high-frequency acoustic pulses in dry and water-saturated river sand is studied theoretically. The waveform of the video pulses, their amplitude against the amplitude of the primary wave, and the propagation velocity versus static pressure are determined. The theoretical results are contrasted with experimental data to determine the acoustic parameters of sand. These parameters are compared with the data on the nonlinear and dissipation properties of granular media described in the literature.

PARAMETRIC GENERATION AND PROPAGATION OF VIDEO-PULSES

In order to describe the processes of generation and propagation of video-pulse signals under the conditions of the experiment described in our earlier paper [1] (such theoretical description being the subject of this paper) we assume that river sand is an isotropic medium and the elastic waves are longitudinal and propagate along the z axis.

The equation of state of a nonlinear isotropic medium can be presented in the form [2, 3]

$$\sigma_{ik} = KU_{jj}\delta_{ik} + 2\mu(U_{ik} - (1/3)U_{jj}\delta_{ik}) + G_{ik}(U_{ik}), \quad (1)$$

where K and μ are the omnidirectional compression and shear moduli, respectively; $U_{jj} = U_{xx} + U_{yy} + U_{zz}$; and the functions $G_{ik}(U_{ik})$ describe the nonlinear relationship between the stress (σ_{ik}) and strain (U_{ik}) tensors. Generally, these functions should be determined theoretically on the basis of a model of the medium or experimentally from some measured dependences, for example, the elastic-wave velocity against static pressure.

If planar strain is applied to an infinite medium in the z direction, the transverse components U_{xx} and U_{yy} of the strain tensor are zero. In our experiments, we dealt with quasiplane waves, therefore, one can assume that the condition $U_{zz} \gg U_{xx}, U_{yy}$ was met. We also take into account the fact that, under shear stress and stain, an isotropic medium exhibits no even elastic nonlinearity, while the odd nonlinearity does not provide the detection process [3]. Thus, one should set $G_{ik}(U_{ik}) = 0$ at $i \neq k$ in equation (1). Additionally, we take into account that, in granular media, Poisson's ratio is very small (almost zero) [4–6]. Therefore, one can assume that $\sigma_{zz} \gg \sigma_{xx}, \sigma_{yy}$ and $G_{zz} \gg G_{xx}, G_{yy}$ and ignore the functions G_{xx} and G_{yy} in equation (9). By contrasting equation (1) with the equations of state for dry and water-saturated sand [1] at $g = 0$ (in terms of notations introduced in [1])

$$\sigma(\epsilon) = (3B/2)(-\varepsilon_0)^{1/2}\epsilon + \alpha(-\epsilon)^{3/2}h(-\epsilon), \quad (2)$$

$$\sigma(\epsilon) = (D + (3B/2)(-\varepsilon_0)^{1/2})\epsilon + \alpha(-\epsilon)^{3/2}h(-\epsilon), \quad (3)$$

we obtain an expression for the function $G_{zz}(\epsilon)$

$$G_{zz}(\epsilon) = \alpha(-\epsilon)^{3/2}h(-\epsilon), \quad \epsilon = U_{zz}. \quad (4)$$

Subsequently, we derive a nonlinear equation for an acoustic beam propagating in an elastic medium by using the technique developed in [7].

Represent the components of the vector of particle displacement U in the acoustic beam in the form

$$U_z = \delta U_z(\tau, x', y', z'), \quad U_{x,y} = \delta^{3/2}U_{x,y}(\tau, x', y', z'), \quad (5)$$

where $\tau = t - z/c(z)$, $z' = \delta z$, $x' = \delta^{1/2}x$, $y' = \delta^{1/2}y$, δ is the small dimensionless parameter describing the slow divergence of the acoustic beam [7], z is the depth, and $c(z)$ is the depth-dependent local velocity of longitudinal elastic waves in the medium. Henceforth, in order to simplify our calculations, we assume that $c(z) = c \approx \bar{c}(P_0)$. This approximation will not cause any fundamental or high errors in the final result at $P_0 \geq 2 \times 10^3$ Pa, because, under such conditions, the sound velocity in sand is basically determined by the static pressure P_0 [1].

Substitute expressions (1) and (5) into the equation describing the elastic wave propagation in solids [2, 3]

$$\rho U_{itt} = \frac{\partial}{\partial x_k}(\sigma_{ik} + \sigma'_{ik}), \quad (6)$$

where U_i is the ith component of the vector of particle displacement and σ'_{ik} is the inelastic stress tensor.

Note that, along with the description of nonlinear elasticity, the description of dissipative (inelastic) prop-

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erties is one of the topical problems of acoustics of inhomogeneous media including granular ones. For homogeneous media, the inelastic stress tensor has the form

$$\sigma'_{ik} = 2\eta(\dot{U}_{ik} - \delta_{ik}\dot{U}_{jj}/3) + \xi\delta_{ik}\dot{U}_{jj}, \quad (7)$$

where η and ξ are the viscosity coefficients, and the sound dissipation factor is a quadratic function of frequency [2]. For many microinhomogeneous media (apparently, including the river sand), the attenuation factor of an elastic wave is proportional to the first power of its frequency [4, 5, 8], which means that the form of the inelastic stress tensor σ'_{ik} must be different from (7). In this case, we describe the processes of the video-pulse generation and propagation as follows. At first, we describe these processes for a medium with the quadratic frequency dependence of the attenuation factor and, then, in the final expression, we take into account that the acoustic attenuation factor in sand linearly depends on frequency. We neglect the geometric nonlinearity and use the linearized strain tensor $U_{ik} = (\partial U_i/\partial x_k + \partial U_k/\partial x_i)/2$ assuming that the nonlinearity of the medium state equation is the dominant one.

Applying manipulations similar to those used in [7] to equations (1) and (5)–(7), we obtain the wave equation for the longitudinal strain $\epsilon = U_{zz}$:

$$\frac{\partial^2\epsilon}{\partial\tau\partial z'} - (c/2)\Delta_\perp\epsilon - \beta\frac{\partial^3\epsilon}{\partial\tau^3} - \frac{1}{2\rho c^3}\frac{\partial^2}{\partial\tau^2}G_{zz}(\epsilon) = 0, \quad (8)$$

where $\beta = (\xi + 4\eta/3)/2\rho c^3$ and $\Delta_\perp$ is the Laplace operator with respect to the transverse coordinates.

We impose the boundary condition at the radiator in the form

$$\epsilon_1(r, z = 0, t) = \mathcal{E}_0\Pi(r/a)\Phi(t/T)\sin\omega t, \quad (9)$$

where $\Pi(r/a) = 1$ for $r/a \le 1$ and $\Pi(r/a) = 0$ for $r/a > 1$, $r^2 = x^2 + y^2$, a is the radiator radius, $\Phi(t/T)$ is the pumping pulse envelope, T is the pumping pulse duration, and $\omega T \gg 1$.

We seek a solution to equation (8) with boundary condition (9) by the perturbation method. We assume that, in (8), $\epsilon(\mathbf{R}, \tau) = \varepsilon_1(\mathbf{R}, \tau) + \varepsilon_2(\mathbf{R}, \tau)$, where $\varepsilon_1(\mathbf{R}, \tau) = \mathcal{E}_1(\mathbf{R}, \tau)\sin\omega t$. We also assume that, near the radiator, where the pumping pulse attenuation and video pulse generation predominantly occur, the relationship $\mathcal{E}_1(\mathbf{R}, \tau) \gg |\varepsilon_2(\mathbf{R}, \tau)|$ is valid. Then, we obtain an equation for the envelopes $\mathcal{E}_1(\mathbf{R}, \tau)$ and $\varepsilon_2(\mathbf{R}, \tau)$ of the pumping and detected pulses, respectively:

$$\frac{\partial}{\partial z}\mathcal{E}_1 - \beta\omega^2\mathcal{E}_1 = 0, \quad (10)$$

$$\frac{\partial^2}{\partial\tau\partial z}\varepsilon_2 - (c/2)\Delta_\perp\varepsilon_2 - \beta\frac{\partial^3}{\partial\tau^3}\varepsilon_2 = Q(\mathbf{R}, \tau), \quad (11)$$

$$Q(\mathbf{R}, \tau) = \frac{1}{2\rho c^3}\frac{\partial^2}{\partial\tau^2}\langle G_{zz}(\varepsilon_1)\rangle, \quad (12)$$

where $\langle\ldots\rangle$ means averaging over the high-frequency wave period. When deriving equation (10), we neglected the diffraction effects assuming that the wave is attenuated within the distance much less than the diffraction wavelength $l_{dif} = \omega a^2/2c$. (In the experiment described in [1], the attenuation length of the pumping wave was less than 1 cm, whereas l_{dif} was about 5 m.)

By solving equation (10) with boundary condition (9), we find an expression for the pumping pulse near the radiator ($4\pi\beta z/T^2 \ll 1$) [9]:

$$\varepsilon_1(\mathbf{R}, \tau) = \mathcal{E}_0\Pi(r/a)\exp(-\beta\omega^2 z)\Phi(\tau/T)\sin\omega\tau. \quad (13)$$

Applying the Fourier transform to equation (11), we obtain the diffraction equation for the spectral components of the video pulse:

$$\partial\tilde{\varepsilon}_2/\partial z + (j/2K)\Delta_\perp\tilde{\varepsilon}_2 = \tilde{Q}(j\Omega, \mathbf{R}), \quad (14)$$

where

$$\tilde{\varepsilon}_2(j\Omega, \mathbf{R}) = (1/2\pi)\exp(-\beta\Omega^2 z)\int_0^\infty \varepsilon_2(\tau, \mathbf{R})\exp(-j\Omega\tau)d\tau,$$

$$K = \Omega/c,$$

and

$$\begin{aligned}\tilde{Q}(j\Omega, \mathbf{R}) &= -(j/2\pi\Omega)\exp(\beta\Omega^2 z) \\ &\times \int_{-\infty}^{\infty} Q(\mathbf{R}, \tau)\exp(-j\Omega\tau)d\tau \\ &= -(j\Omega/4\pi\rho c^3)\exp(\beta\Omega^2 z) \\ &\times \int_{-\infty}^{\infty}\langle G_{zz}[\varepsilon_1(\tau, \mathbf{R})]\rangle\exp(-j\Omega\tau)d\tau.\end{aligned} \quad (15)$$

The solution to equation (14) has the form [9]

$$\begin{aligned}\tilde{\varepsilon}_2(j\Omega, \mathbf{R}) &= \int_0^\infty J_0(\nu r)\nu \\ &\times \int_0^z \tilde{\tilde{Q}}(\nu, z, j\Omega)\exp(j\nu^2(z - z')/2K)dz'd\nu,\end{aligned} \quad (16)$$

where $\tilde{\tilde{Q}}(\nu, z, j\Omega) = \int_0^\infty \tilde{Q}(j\Omega, \mathbf{R})J_0(\nu r)rdr$.

Applying the inverse Fourier transform to equation (16), we obtain the general expression for the detected pulse $\varepsilon_2(\tau, z)$ in the far-field region at the radiator axis:

$$\varepsilon_2(\tau, z) = \frac{a^2}{8\pi z\rho c^4}$$

$$\times\int_0^z\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Omega^2\langle G_{zz}(\mathscr{E}_0\exp[-\beta\omega^2 z']\Phi(\tau'/T)\sin\omega\tau')\rangle \times\exp(-\beta\Omega^2 z-j\Omega(\tau'-\tau))dz'd\tau'd\Omega. \quad (17)$$

When deriving expression (17), we assumed that the parametric radiator was operated in the Westerwelt mode [9], i.e., the dephasing of the secondary sources $Q(\mathbf{R}, \tau)$ across the beam was negligible: $K^*a^2/4L \approx 0.5$, where $K^* = 2\pi F^*/c$ and $F^* = 1/2T^*$ is the characteristic frequency of the video pulse.

For a wave travelling along the z axis, its strain and acceleration parameters are related as $A(\tau, z) = -c\partial\varepsilon_2(\tau, z)/\partial\tau$. In the experiment, we recorded the acceleration of the free surface of the plate, therefore, the waveform of the pulse received by the accelerometer can be written as

$$A(\tau, z) = -\frac{a^2}{4\pi z\rho c^3}\frac{\partial}{\partial\tau}$$

$$\times\int_0^z\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Omega^2\langle G_{zz}(\mathscr{E}_0\exp[-\beta\omega^2 z']\Phi(\tau'/T)\sin\omega\tau')\rangle \times\exp(-\beta\Omega^2 z-j\Omega(\tau'-\tau))dz'd\tau'd\Omega. \quad (18)$$

From expression (18), it follows that the dependence of the video pulse amplitude A_0 on the amplitude $\mathscr{E}_0$ of the pumping pulse is similar to that obtained in the experiment (see expression (4) in [1]), because, $G_{zz}(\varepsilon_1) = \alpha(-\varepsilon_1)^{3/2}h(-\varepsilon_1) \sim \mathscr{E}_0^{3/2}$. It also follows from formula (18) that when the attenuation of the low-frequency signal is low ($\beta\Omega^2 z \ll 1$), the video pulse waveform $A(t)$ corresponds to the third time derivative of the nonlinear function $\langle G_{zz}[\varepsilon_1(\tau, z)]\rangle$, which depends on the envelope $\Phi(t/T)$ of the pumping pulse. In the case of a finite attenuation, the pulse waveform $A(t)$ will be slightly spread (due to the high-frequency attenuation), but it will still be close to the third time derivative of the nonlinear function $\langle G_{zz}[\varepsilon_1(\tau, z)]\rangle$. Let us illustrate this behavior by examples of pumping pulses with the Gaussian and rectangular envelopes. In these cases, analytical expressions for the detected signals can be derived. Note that, in the experiment, the envelope of the short pumping pulse was close to the Gaussian shape, while the envelope of the long pumping pulse was almost rectangular.

For the pumping pulse with the Gaussian envelope $\Phi(t/T) = \exp(-t^2/T^2)$, formula (18) yields

$$A(t) = -\frac{\gamma H a^2}{z c\beta\omega^2(1+6\beta z/T^2)^{1/2}} \times\mathscr{E}_0^{3/2}\frac{\partial^3}{\partial t^3}\exp\left(-\frac{3t^2/2}{T^2+6\beta z}\right), \quad (19)$$

where $H = \Gamma^2(1/4)/[9(2\pi)^{3/2}]$, $\Gamma(**)$ is the gamma function, and $\gamma = \alpha/\rho c^2 = (2\alpha/3B)(-\varepsilon_0)^{-1/2}$ is the dimensionless nonlinearity parameter. Note that the coefficient $2\alpha/3B$ is determined by the ratio between the numbers of tight and loose contacts; for randomly packed grains, the order of magnitude of this coefficient is equal to unity [10, 11].

Expression (19) shows that the waveform $A(t)$ of the video pulse, produced as a result of detection of the high-frequency pulse with the Gaussian envelope, follows the third time derivative of this envelope extended by a factor of $[(2/3) + 4\beta z/T^2)^{1/2}$. Since an increase in the static pressure P_0 decreases the attenuation factor of elastic waves in a granular medium (and, consequently, the factor β as well) [4, 12, 13], the video pulse duration must also decrease.

From equation (19), the characteristic duration T^* and amplitude A_0 of the video pulse $A(t)$ (see Figs. 2, 3 in [1]) can be calculated:

$$T^* = T(1+6\beta z/T^2)^{1/2}, \quad (20)$$

$$A_0 = \frac{3(3/2)^{1/2}[1-(1/6)^{1/2}]^{1/2}\gamma H a^2}{z c\beta\omega^2 T^3(1+6\beta z/T^2)^2}\mathscr{E}_0^{3/2}. \quad (21)$$

Similarly, one can obtain an expression for the video pulse $A(t)$ produced as a result of the detection of the high-frequency pulse with the rectangular envelope $\Phi(t/T) = h(t/T) - h(t/T-1)$:

$$A(t) = -\frac{\gamma H a^2}{c\omega^2(\beta z)^{3/2}} \times\mathscr{E}_0^{3/2}\frac{\partial^2}{\partial t^2}\left[\exp\left(-\frac{t^2}{4\beta z}\right)-\exp\left(-\frac{(t-T)^2}{4\beta z}\right)\right], \quad (22)$$

where $H = \Gamma^2(1/4)2^{1/2}/36\pi$.

In this expression, the summands $\exp(-t^2/4\beta z)$ and $-\exp[-(t-T)^2/4\beta z]$ result from the single time differentiation of the leading and trailing edges of the initially rectangular envelope of the pumping pulse and from the subsequent high-frequency filtering due to attenuation in the medium. Thus, in this case, the video pulse waveform $A(t)$ is also close to the third time derivative of the pumping pulse envelope.

When a periodic sequence of high-frequency pulses is generated, the spectrum of the detected signal contains the pulse repetition frequency F and its higher harmonics. Substituting the modulating function

$$\Phi(t/T) = \begin{cases}\sin(2\pi t/T), & 0\le t\le T/2\\ 0, & T/2\le t\le NT/2\end{cases} \quad (23)$$

into equation (18), we obtain an expression for the spectral components $A(pF)$ of the detected signal:

$$A(pF) = \frac{\pi^2\Gamma(1/4)\gamma a^2(pF)^3}{12zc\beta\omega^2} \times \frac{\sin(\pi p/N)\exp[-\beta(2\pi pF)^2 z]}{N\Gamma(7/4+p/N)\Gamma(7/4-p/N)}\mathscr{E}_0^{3/2}, \quad (24)$$

where $F = 1/NT$ and N is the duty factor.

CALCULATION OF THE PARAMETERS OF THE EQUATIONS OF STATE FOR DRY AND WATER-SATURATED SAND

Let us try to estimate the attenuation length $l_m = (\beta\omega^2)^{-1}$ of the pumping wave and the nonlinearity parameter γ_m for dry sand at the maximum static pressure $P_0 = P_m \simeq 9 \times 10^3$ Pa ($|\varepsilon_0| = \varepsilon_m \simeq 10^{-4}$) by using experimental results [1] and expressions (20) and (21), derived under the assumption that the elastic wave attenuation factor is a quadratic function of frequency. We set the following values of other parameters involved in these expressions: $T \simeq 40$ μs; $T^* \simeq 73$ μs; $z = L = 10$ cm; $P \simeq 2.5 \times 10^3$ Pa ($\mathscr{E}_0 = P/\rho c^2 \simeq 2.85 \times 10^{-5}$); $c \simeq 2.5 \times 10^4$ cm/s; $\rho \simeq 1.4$ g/cm^3; $A_0 \simeq 1.2 \times 10^2$ cm/s^2 ($\mathscr{E}_d = A_0/2\pi cF^* \simeq 10^{-7} \ll \mathscr{E}_0$).

Computations show that the attenuation length of the pumping wave is $l_m \simeq 1.2 \times 10^{-2}$ cm and the nonlinearity parameter is $\gamma_m \simeq 9 \times 10^2$.

However, these estimates are implausible, because this attenuation length is too small (much less than the grain size of sand) and the nonlinearity parameter is too large, more than one order of magnitude higher than the expected value $\gamma = (-\varepsilon_0)^{-1/2} \simeq 10^2$ (see the above discussion on expression (19)). This result contradicts the condition that the amplitude of the detected pulse is small with respect to the pumping wave amplitude ($\mathscr{E}_d \ll \mathscr{E}_0$), which was fulfilled in the experiment. This condition allows one to assume that the nonlinear term in equation of state (2) is much less than the linear one (i.e., $\gamma(\mathscr{E}_0)^{-1/2} \ll 1$) and to solve the problem by the perturbation method. If the value $\gamma = \gamma_m \simeq 9 \times 10^2$ is used, one obtains $\gamma(\mathscr{E}_0)^{-1/2} \simeq 5 \gg 1$.

For water-saturated sand, the estimates of the attenuation length of the pumping wave and those of the nonlinearity parameter lead to similar discrepancies. Computations show that, at $T^* \simeq 83$ μs, $P_0 = P_m \simeq 9 \times 10^3$ Pa ($\varepsilon_m \simeq 10^{-4}$), $\mathscr{E}_0 = 2.85 \times 10^{-5}$, $A_0 \simeq 34$ cm/s^2 ($\mathscr{E}_d = A_0/2\pi cF^* \simeq 3 \times 10^{-7} \ll \mathscr{E}_0$), $c \simeq 4 \times 10^4$ cm/s, and $\rho \simeq 1.86$ g/cm^3, the attenuation length of the pumping wave is $l_m \simeq 9 \times 10^{-3}$ cm and the nonlinearity parameter is $\gamma_m \simeq 9.5 \times 10^2$. In this case, $\gamma(\mathscr{E}_0)^{-1/2} \simeq 5$, which disagrees with the condition $\mathscr{E}_d \ll \mathscr{E}_0$.

Thus, these estimates of the attenuation length and nonlinearity parameter, calculated for both dry and water-saturated sand in the framework of the model of the square frequency dependence of the attenuation factor, disagree with the conditions of the experiment.

Next, let us try to describe the processes of the detection and propagation of acoustic pulses on the assumption that the attenuation factor for the elastic waves in sand (as in many other rocks) is a linear function of frequency [4, 8, 13]. The dissipative properties of such media are commonly characterized by the dimensionless frequency-independent decrement θ, the attenuation of an elastic wave of frequency ω being determined as $\exp[-\tilde{\beta}\omega z]$, where $\tilde{\beta} = \theta/2\pi c$. For this type of the elastic wave attenuation, an expression for the detected pulse can be obtained by replacing the coefficients $\beta\omega^2$ and $\beta\Omega^2$ in (18) by $\tilde{\beta}\omega$ and $\tilde{\beta}\Omega$, respectively:

$$A(\tau, z) = -\frac{a^2}{4\pi z\rho c^3}\frac{\partial}{\partial\tau} \times \int_0^z\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Omega^2\langle G_{zz}(\mathscr{E}_0\exp[-\tilde{\beta}\omega z']\Phi(\tau'/T)\sin\omega\tau')\rangle \times \exp(-\tilde{\beta}\Omega z - j\Omega(\tau'-\tau))dz'd\tau'd\Omega. \quad (25)$$

When analyzing this integral, it is convenient to approximate the pumping pulse envelope by the Lorentz-type function $\Phi(\tau/T) = [1 + (\tau/T)^2]^{-2/3}$. In this case,

$$A(t) = -\frac{\gamma Ha^2T}{zc\tilde{\beta}\omega(T+\tilde{\beta}z)}\mathscr{E}_0^{3/2}\frac{\partial^3}{\partial t^3}\left[\frac{1}{1+t^2/(\tilde{\beta}z+T)^2}\right], \quad (26)$$

where $H = \Gamma^2(1/4)/9(2\pi)^{3/2}$ and $\gamma = \alpha/\rho c^2$. For the characteristic pulse duration T^* and pulse amplitude A_0 (see Figs. 2a, 4a in [1]), the following expressions can be obtained:

$$T^* = T + \tilde{\beta}z, \quad (27)$$

$$A_0 = \frac{75(1-2/5^{1/2})^{1/2}\gamma Ha^2T}{(1-5^{1/2})^4 cz\tilde{\beta}\omega(T+\tilde{\beta}z)^4}\mathscr{E}_0^{3/2}. \quad (28)$$

For the long pulse with the rectangular envelope, one obtains

$$A(t) = -\frac{\gamma Ha^2}{zc\tilde{\beta}}\mathscr{E}_0^{3/2}\frac{\partial^2}{\partial t^2} \times \left[\frac{1}{(\tilde{\beta}z)^2+t^2} - \frac{1}{(\tilde{\beta}z)^2+(t-T)^2}\right]. \quad (29)$$

Here, as in the case of the quadratic frequency dependence of the attenuation factor, the video pulse $A(t)$ waveform also follows the third time derivative of the pumping pulse envelope.

When a periodic sequence of the high-frequency pulses is radiated, an expression for the amplitudes

$A(pF)$ of the detected signal harmonics can be obtained similarly to (24) in the form

$$A(pF) = \frac{\pi^2\Gamma^2(1/4)\gamma a^2(pF)^3}{12zc\tilde{\beta}\omega} \times \frac{\sin(\pi p/N)\exp[-\tilde{\beta}(2\pi pF)z]}{N\Gamma(7/4+p/N)\Gamma(7/4-p/N)}\mathscr{E}_0^{3/2}. \quad (30)$$

Let us estimate the pumping wave attenuation length $l_m = (\tilde{\beta}_m\omega)^{-1}$ and the nonlinearity parameter γ_m for dry and water-saturated sand at the maximum static pressure $P_0 = P_m = 9 \times 10^3$ Pa by using expressions (27) and (28). As a result, we obtain that $l_m \simeq 2.3 \times 10^{-2}$ and $l_m \simeq 2 \times 10^{-1}$ cm, which corresponds to the decrements $\theta_m \simeq 0.61$ and 1.1 for the dry and water-saturated sand, respectively. These results agree well with the decrement measured in sand by other researchers [13, 14]. The nonlinearity parameter calculated from (28) is $\gamma_m \simeq 84$ and 74 for dry and water-saturated sand, respectively, which is in good agreement with the expected value $\gamma_m \simeq (\varepsilon_m)^{-1/2} \simeq 10^2$. With these values of the nonlinearity parameter, one obtains $\gamma_m(\varepsilon_0)^{-1/2} \simeq 0.4 < 1$. The nonlinearity parameter estimated by formula (30) using the amplitudes of the first three harmonics of the pulse repetition frequency is also on the order of $\gamma_m \simeq 10^2$ (between 80 and 230). Apparently, such scatter occurs, because the modulation function used in the experiment slightly differs from theoretical approximation (23).

Subsequently, we use the experimental results (see Fig. 6 in [1]) to determine the parameter $\tilde{\beta}$ as a function of the static pressure P_0 for dry and water-saturated sand. Expression (27) yields

$$T^* - T = \tilde{\beta}z. \quad (31)$$

The figure shows $\log(T^* - T) = \log(\tilde{\beta}z)$ versus $\log P_0$ for dry and water-saturated sand. One can see that, at $P_0 \geq 2 \times 10^3$ Pa, $\tilde{\beta}$ as a function of P_0 has the form

$$\tilde{\beta}(P_0) \simeq \tilde{\beta}_m(P_0/P_m)^b, \quad (32)$$

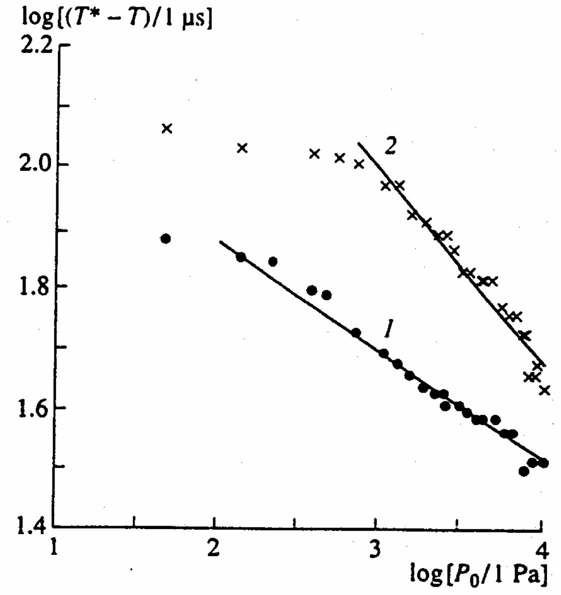
where $b \simeq -1/6$ and $-1/3$ for dry and water-saturated sand, respectively.

According to (32) and the dependence $c(P_0) \sim P_0^m$ ($m \simeq 1/6$ for $P_0 > 2 \times 10^3$ Pa) established experimentally [1], the decrement $\theta = 2\pi\tilde{\beta}c$ is determined as

$$\theta(P_0) \simeq \theta_m(P_0/P_m)^{-q}, \quad (33)$$

where $q \simeq 0$ and $\theta_m \simeq 0.61$ for dry sand, and $q \simeq 1/3$ and $\theta_m \simeq 1.1$ for water-saturated sand.

Using the experimental dependences of the video pulse velocity and amplitude versus pressure P_0 [1] and relationship (32), we can determine the nonlinearity



$\log(T^* - T)$ versus $\log P_0$ for (*1*) dry and (*2*) water-saturated sand. The straight lines show the exponential functions: (*1*) $\tilde{\beta} \simeq P_0^{-1/6}$ and (*2*) $\tilde{\beta} \simeq P_0^{-1/3}$.

parameter γ as a function of static pressure P_0 from expression (28):

$$\gamma(P_0) \simeq \gamma_m(P_0/P_m)^{-r}, \quad (34)$$

where $r \simeq 1/5$ and $\gamma_m \simeq 84$ for dry sand and $r \simeq 1/2$ and $\gamma_m \simeq 74$ for water-saturated sand.

As we mentioned above [see the note after formula (19)], the nonlinearity parameter is determined by the value (α/B), which has the meaning of the ratio between the numbers of tight and loose contacts: $\gamma = \alpha/\rho c^2 = (2\alpha/3B)(-\varepsilon_0)^{-1/2}$. In view of expression (34), we obtain that this ratio only slightly depends on the static pressure: $\alpha/B \sim P_0^s$, where $s \simeq 2/15$ for dry sand and $s \simeq 1/6$ for water-saturated sand.

Thus, the assumption that the attenuation factor characterizing the propagation of elastic waves in dry and water-saturated sand is a linear function of frequency allows one to adequately interpret the experimental results and to obtain reasonable estimates of the decrements and nonlinearity parameters for these media at various static pressures.

SUMMARY

In this paper, the results of the experimental study of the parametric generation and propagation of low-frequency video-pulse signals in dry and water-saturated river sand [1] are analyzed theoretically. The waveform of the detected pulses observed in the experiment and the dependence of their amplitude on the pumping

wave amplitude as well as the dependence of their propagation velocity on static pressure are explained.

The nonlinear equations of state of sand are proposed. Their parameters and the dissipative sand characteristics are calculated on the basis of the comparison between the theoretical results and the experimental data. It is shown that the description of the acoustic pulse detection and propagation, in terms of the quadratic frequency dependence of the attenuation factor for elastic waves in sand, disagrees with the conditions of the experiment. However, the assumption that the attenuation factor is a linear function of frequency provides an adequate interpretation of the experimental results.

The experimental data obtained and their theoretical description can be used for analyzing seismic signals and for the development of seismoacoustic sounding methods—first of all, methods based on the use of nonlinear effects in diagnostics.

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