Comparison of linear and nonlinear elastic moduli for reservoir rock by use of a granular medium model

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(Received 13 July 1993; accepted for publication 22 August 1995)

The variations of linear and nonlinear elastic parameters as a function of initial stress and material structure are considered using a model of a granular medium with fluid pore filling. Examples of such variations for some geological conditions interesting for seismoprospecting are presented. It is demonstrated that the nonlinear parameter may be used in exploration seismology as a much more sensitive characteristic compared with conventionally exploited linear moduli. © *1996 Acoustical Society of America.*

PACS numbers: 43.25.Dc

INTRODUCTION

Conventional seismoacoustical methods used in exploration seismology are based on measuring linear seismoacoustic parameters such as sound velocity and coefficients of wave reflection and damping in the medium.[1,2] Recently, some investigations of nonlinear effects of elastic waves propagation in earth soils and rocks have been performed.[3–6] Measurements of nonlinear elastic moduli of natural rocks and soils (which always contain defects, discontinuities, and other structural inhomogeneities) have shown that they may exceed by two or three orders of magnitude the nonlinear moduli of similar homogeneous materials containing no defects, while linear characteristics remain relatively similar. These results indicate that nonlinear moduli can be much more sensitive to changes in medium structure than the linear parameters. Thus it seems worthwhile to evaluate the possibility of using nonlinear acoustoelastic parameters in geophysics, e.g., for applications in exploration seismology or engineering seismics.[7] In exploring the possibility of using nonlinear parameters for these purposes it is necessary to investigate the origin and magnitude of medium nonlinearity and its connection with the material structure not only empirically, but to provide some theoretical models of the phenomenon.

This article presents some results of comparative analysis of linear (longitudinal sound velocity) and the nonlinear parameter in the frame of a model of fluid-filled granular material[8] using some field data.[1,2,9] The aim of this research is to demonstrate the high sensitivity of the nonlinear parameter to geological conditions (the influence of initial precompression, the presence of fluid filling) that may be used for elaboration of nonlinear seismoacoustical exploration methods.

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I. NONLINEAR PROPERTIES OF GRANULAR MEDIA

We consider the granular medium as continuous; i.e., the waves in the medium are long compared to the sizes of granules. For the case of random packing of grains the material can be treated as isotropic, which enables one to characterize the longitudinal deformations by a scalar stress–strain relation $\sigma=\sigma(\epsilon)$, which often can be represented as a power series expansion:

$$\tilde{\sigma}=\sigma'_{\epsilon}(\epsilon_0)\tilde{\epsilon}+\frac{1}{2!}\,\sigma''_{\epsilon\epsilon}(\epsilon_0)\tilde{\epsilon}^2+\frac{1}{3!}\,\sigma'''(\epsilon_0)\tilde{\epsilon}^3+\cdots\,, \quad (1)$$

where $\tilde{\sigma}=\sigma(\epsilon)-\sigma(\epsilon_0)$, $\tilde{\epsilon}=\epsilon-\epsilon_0$, and ϵ_0 is initial prestrain. The first term in the right-hand side corresponds to the linear Hooke law. Using the expansion (1) the following linear and nonlinear parameters are usually introduced:

$$M=\sigma'_{\epsilon}(\epsilon_0), \quad (2)$$

which is the linear elastic longitudinal modulus which is related to the sound velocity $c_p=[M/\rho]^{1/2}$ (ρ is the medium density), and the quadratic and cubic nonlinear parameters that determine nonlinear vibroacoustic effects in the material:

$$\Gamma^{(2)}=\sigma''_{\epsilon\epsilon}(\epsilon_0)/\sigma'_{\epsilon}(\epsilon_0), \quad (3)$$

$$\Gamma^{(3)}=\sigma'''_{\epsilon\epsilon\epsilon}(\epsilon_0)/\sigma'_{\epsilon}(\epsilon_0). \quad (4)$$

Consider a granular medium sample composed of a large number N of particles of radius R, the properties of which are described by the constants K_s (the bulk compression modulus) and E_s (Young's modulus). Let α be the fraction of empty (porous) space per unit volume of the aggregate and $\bar{n}$ the average number of contacts between the particles. The values of α and $\bar{n}$ for a system of randomly packed identical particles have been found experimentally: $\alpha=0.392$, $\bar{n}=8.84$.[7] (These values range from $\alpha=0.477$ in the case of simple cubic packing to $\alpha=0.23$ in the case of the tightest packing of identical granules.) For the description of the pore fluid filling the voids the following parameters are chosen: the fluid compressibility K_f and the fluid density ρ_f.

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Using the approach[8] based on the energy balance equation that equates the work of given external pressure p_{ext} to the energy accumulated at intergrain contacts and stored in the pore fluid and in the volume of grains compressed by the surrounding fluid, one can derive a stress–strain relation $\sigma=\sigma(\epsilon)$ for such a medium (for details of the derivation see the Appendix):

$$\sigma_{\mathrm{eff}}(\epsilon)=\frac{K_f}{\alpha+(1-\alpha)K_f/K_s}\,\epsilon+\frac{\bar{n}(1-\alpha)E_s}{3\pi(1-\nu_s^2)}\,\epsilon^{3/2}. \tag{5}$$

The first linear term in the right-hand side of Eq. (5) is due to the presence of the pore fluid and vanishes when the fluid is absent (that is, $K_f=0$). The second nonlinear term is determined by the intergrain contacts. Note that in the presence of fluid the left-hand side of the stress–strain relation is written (as is generally accepted[1,9]) in the form of the effective stress equal to the difference of load pressure and pressure of the pore fluid:

$$\sigma_{\mathrm{eff}}=p_{\mathrm{ext}}-kp_f \tag{6}$$

(where k is the unloading coefficient such that $k=1$ in the case of nonconsolidated media). For grainy rocks (which contain some amount of intergrain cement) there are empirical data available which show that the unloading coefficient k belongs to the range $0.85\leqslant k\leqslant 1$.[9]

According to the definitions (2)–(4) the following expressions for linear and nonlinear parameters are derived from the stress–strain relation (5):

$$M=\frac{K_f}{\alpha+(1-\alpha)K_f/K_s}+\frac{\bar{n}(1-\alpha)E_s}{2\pi(1-\nu_s^2)}\,\epsilon^{1/2}, \tag{7}$$

$$\Gamma^{(2)}=\left(\frac{\bar{n}(1-\alpha)E_s}{4\pi(1-\nu_s^2)}\,\epsilon_0^{-1/2}\right)\Big/ M, \tag{8}$$

$$\Gamma^{(3)}=\left(\frac{n(1-\alpha)E_s}{8\pi(1-\nu_s^2)}\,\epsilon_0^{-3/2}\right)\Big/ M. \tag{9}$$

Note that the appearance of the nonlinear term in (5) is not due to nonlinearity of the grain material itself or to nonlinear properties of the fluid component, but is caused by the presence of highly nonlinear "soft," stress-concentrating, intergrain contacts.

We also do not consider here "kinematic" nonlinearity associated with the difference between Lagrangian and Eulerian descriptions. Both these simplifications are due to specific properties of the considered class of media in which the "structural" nonlinearity parameters may exceed by several orders of magnitude those of the "classical" nonlinearity of homogeneous media—gases, liquids, monocrystals, etc. Experiments with samples of rocks, sand, etc., confirm this. On the other hand, the terms of a classical origin can be added to the equation of motion of the type of (13) below.

For the particular case of dry medium these expressions for nonlinear parameters and the stress–strain relation itself take the simple form[8]

$$\sigma'(\epsilon)=\frac{\bar{n}(1-\alpha)E_s}{3\pi(1-\nu_s^2)}\,\epsilon^{3/2}, \tag{10}$$

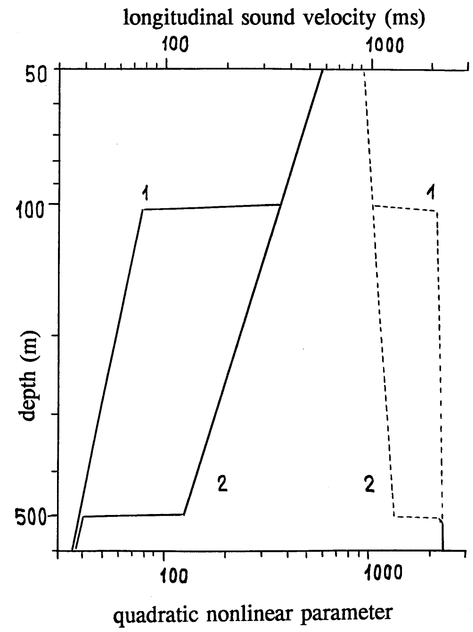

FIG. 1. Dependence of longitudinal sound velocity C_p and quadratic nonlinear parameter $\Gamma^{(2)}$ on depth h in the absence of nonpenetrable dome over fluid-saturated layer. 1—fluid boundary $H_0=100$ m, 2—$H_0=500$ m. Solid line—nonlinear parameter, dashed line—longitudinal sound velocity.

$$\Gamma^{(2)}=1/(2\epsilon_0),\quad \Gamma^{(3)}=1/(6\epsilon_0^2); \tag{11}$$

that is, values $\Gamma^{(2)}$, $\Gamma^{(3)}$ are determined only by the initial prestrain ϵ_0: It is obvious that at small ϵ_0 (of the order of 10^{-3} and less) the nonlinear parameters of grainy medium reach rather high magnitudes, much higher than the nonlinear moduli of the grain material itself (see also Ref. 10, where some model experiments on measurements of grainy medium nonlinearity are described).

II. COMPARATIVE ESTIMATES OF LINEAR AND NONLINEAR PARAMETER CHANGES

Consider some examples demonstrating the high sensitivity of nonlinear parameters to changes in geological conditions. As a first example we choose a crumbly sandlike sediment containing a water layer beginning at depth H_0. Results of the calculations based on Eqs. (5) and (6) are presented in Fig. 1. The following model parameters typical of sand and water were chosen for the calculation: $\alpha=0.2$, $E_s=5.3\times10^{10}$ N/m^2, $\nu=0.2$, $\rho_s=2.65\times10^3$ kg/m^3, $K_f=2.2\times10^9$ N/m^2, and the value of p_{ext} was determined by the weight of the overlaying medium layer. Figure 1 shows that the sound velocity variation at the boundary H_0 of the water layer is about 50%–70%, while the nonlinear parameter $\Gamma^{(2)}$ decreases by 4–5 times (both the changes are caused by the increase of the medium compressibility modulus due to fluid filling). With the increase of H_0, the variations of both sound velocity and nonlinear parameter decrease, although the contrast of nonlinear parameter change remains significantly higher than that of the sound velocity.

Much stronger variations of the nonlinear parameter can be observed in the presence of a nonpenetrable boundary (dome) overlaying a fluid-containing reservoir. In such a case the dome pressure may be shared between the fluid and the

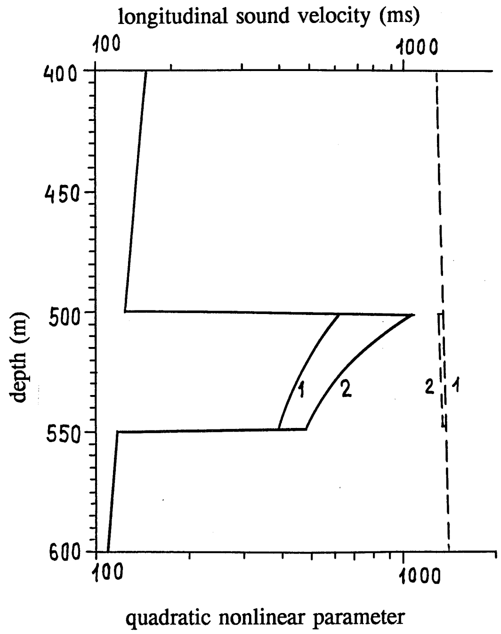

FIG. 2. Dependence of longitudinal sound velocity c_{p} and nonlinear parameter $\Gamma^{(2)}$ on depth h in the presence of nonpenetrable dome over the fluid-saturated layer; $H_L=H_0$. 1—unloading coefficient $k=0.95$, 2—$k=0.985$. Solid line—nonlinear parameter, dashed line—longitudinal sound velocity.

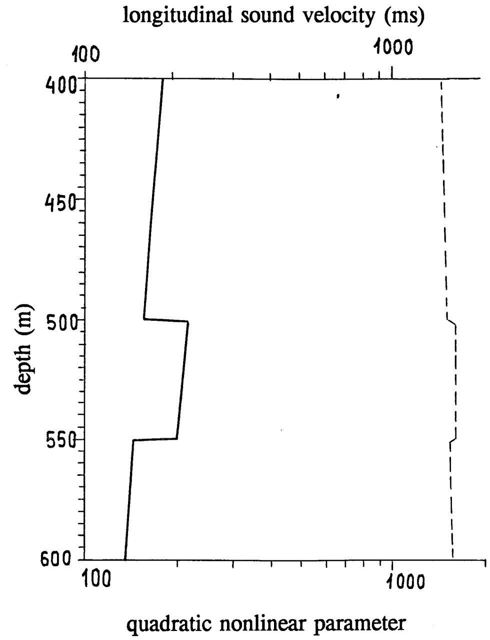

FIG. 3. Dependence of longitudinal sound velocity C_p and quadratic nonlinear parameter $\Gamma^{(2)}$ on depth h in the presence of nonpenetrable dome over the fluid-saturated layer; $H_L=H_0/2$. Unloading coefficient $k=0.96$. Solid line—nonlinear parameter, dashed line—longitudinal sound velocity.

solid skeleton, thus decreasing the strain of the latter, which may cause significant change of its nonlinearity according to (5)–(9). The discovery of the fluid layers covered by nonpenetrable domes is very important for oil exploration.[2] Consider the case of such a structure similar to that in the Ker–Girs oil field in Azerbaijan.[11] The oil boundary depth is $H_0=500$ m. The liquid pressure at the horizon H_0 is determined by the weight of the overlaying rock layer with an effective thickness H_L. The value H_L is less than the total thickness H_0, because the total pressure of the dome is shared between the fluid and the solid skeleton. Let us start from the extreme case when the grain skeleton under the dome is unloaded to maximal extent, and the pore fluid pressure under the dome is determined by the total weight of the upper layer: $H_L=H_0$. Therefore in the range $h>H_0$ one can obtain for initial fluid pressure

$$p_{f0}(h)=(1-\alpha)\rho_s g H_L+\rho_f g(h-H_0), \tag{12}$$

where g is acceleration due to gravity.

In order to find the initial strain ϵ_0, the stress–strain equations (5) and (6) were solved numerically using the expression (12) for $p_{f0}(h)$. The resulting magnitude of ϵ_0 is very sensitive to the choice of the unloading coefficient k in the expression (6) for σ_{eff}, because the anomalous fluid pressure at $k=1$ and $H_L=H_0$ in (12) provides total unloading of the grain "skeleton" ($\epsilon_0=0$) and, consequently, formally leads to infinite values of nonlinear parameters according to (8) and (9). We chose, therefore, more realistic unloading coefficient values $k<1$. Other parameters used in the calculation were as follows: intergrain void content $\alpha=0.38$, liquid compressibility $K_f=1.2\times10^9$ N/m^2, and density $\rho_f=0.85\times10^3$ kg/m^3 (typical of oil). The thickness of the fluid reservoir was taken to be 50 m. The corresponding results are presented in Fig. 2. One can see that at $k=0.95$ (the curves marked by 1) the sound velocity changes are hardly seen, and at $k=0.985$ (the curves marked by 2) the sound velocity variations are about 5%, while the corresponding changes of the nonlinear parameter are two orders of magnitude higher.

The analogous dependences were calculated for a more realistic case when the fluid pressure is determined by a part of the upper layer weight. Namely, the calculation was made for the effective thickness $H_L H_0/2$ in (12) and the coefficient $k=0.96$ in (6) (see Fig. 3). The contrast between the changes of the nonlinear parameter and the sound velocity decreases due to smaller unloading, but the nonlinear parameter variations are still an order of magnitude higher. Note that the presence of a gas component may also be taken into account, and it can be shown that it leads (even for a rather small gas cubic content of about 10^{-2}) to an additional nonlinear parameter increase without significant influence on the sound velocity.

III. DISCUSSION

An important aspect of the previous analysis is how realistic the model of contacting spheres is, and how it relates to actual crumbly rocks. First of all, it is worthy of notice that practically the same models were already successfully used for explanation of experimental data on linear elastic properties of grainy rocks.[1] It is essential that the Hertz law used as the basis of the theory is applicable not only to ideal spherical grains, but to arbitrary contacting surfaces of the second-order curvature as well,[12] and the discrepancy between measurements and theoretical estimates usually does not exceed 50%.[1] The nonlinear parameters introduced by the expressions (3) and (4) are normalized to the linear elasticity modulus, and in the frames of the model considered they are determined by elastic properties of singular contacts. Experimental measurements of the quadratic and cubic nonlinear parameters for equal-size spheres have demonstrated good agreement with theoretical estimates according to Eqs. (11).[10] Real packages evidently contain particles of different sizes and, consequently, part of the contacts may be initially significantly unloaded, which may lead to further increase of the material nonlinearity. Experiments with such multisize packages[13] have demonstrated this additional growth of nonlinear parameters even in comparison with the estimates obtained from (11). This fact confirms that the nonlinearity of real crumbly rocks may be even higher than that predicted by the above model.

The relations derived above can be used for derivation of the wave equations describing the propagation of acoustic (seismic) waves in the medium. Such a derivation may be done by substituting the stress–strain relation (5) or (10) into the Newton law written for a material element:[14]

$$\rho \frac{\partial u}{\partial t} = \frac{\partial \sigma}{\partial x}, \tag{13}$$

where u is the displacement of the element from its initial position. (As was mentioned above, we neglect the kinematic nonlinearity.) Introducing the strain $\epsilon = \partial u / \partial x$ we obtain the nonlinear wave equation from Eq. (13) and the stress–strain relation. The influence of attenuation and dispersion can readily be taken into account by a common procedure of inserting into the equation the corresponding additive terms that can be found into the linear approximation.[15,16] It is evident that the attenuation can significantly decrease the level of sounding waves. The role of sound dispersion seems to be less important as it should influence the propagation of rather high-frequency waves, whose wavelength is comparable with the characteristic grain size $\lambda \sim R$.

The analysis presented above does not address the question of collecting and interpreting field data. The problem is outside the scope of the present consideration, but we discussed it elsewhere.[17,18] Here we restrict ourselves to the following brief remarks. For obtaining the information on spatial distribution of the medium nonlinearity one may use, for example, tomographic methods based on observation of the waves transmitted through the medium.[17] Another approach (that may be preferable in many cases) based on the effect of nonlinear sound backscattering[19] for seismic problems was discussed in Ref. 18. There are some laboratory experimental data available that confirm applicability of the mentioned principles to spatial reconstruction of nonlinear parameters.[19–21] Extension of these methods to the seismic problems requires special experimental investigations. As far as we know, the field data on successful separation of source and material nonlinear effects are available only for the case of transmitted signals and for the path averaged values of the nonlinear parameters.[22,23] Apparently, conventional apparatus and techniques used in routine problems of seismoprospecting are not quite appropriate for such measurements because of intrinsic nonlinearity of existing sources (such as vibroseis) and the nonlinearity of conventional detector electronics. On the other hand, there is a successful experience in radiation of intensive and stable sound waves, and in processing weak signals (including the ones generated due to the medium nonlinearity) in hydroacoustic problems (see, e.g., Refs. 24–26). It seems possible that extension of this nonlinear hydroacoustic experience to seismics may be fruitful.

IV. CONCLUSION

The presented analysis of elastic properties of fluid-filled grainy media demonstrates that the nonlinear parameters can serve as rather useful informative characteristics in exploration seismology. Thus the nonlinear parameter profiling, especially by means of remote methods, that has been successfully tested in medical and industrial applications,[27] may open new potentialities in resource prospecting.

ACKNOWLEDGMENTS

This work was supported by Tesla Laboratories, Inc., La Jolla, California, within the research and development project "Acoustical Physics and Technology," and by a grant of the International Science Foundation and Russian Government (Project No. R8U300).

APPENDIX

Assuming that the porous space between particles is filled with a fluid under pressure P_f we find that the total volume of the "particle+fluid" system amounts to

$$V_t = \frac{4}{3} \frac{\pi N R^3}{1-\alpha}. \tag{A1}$$

Suppose that the whole aggregate of N particles is subjected to quasistatic compression due to pressure P_{ext} that causes particle deformation. Therefore, the contact point transforms to a circle and the particle centers draw closer to a distance $2 \cdot \tilde{\Delta}$ which is related to the force F acting at the contact by the Hertz formula[12]

$$\tilde{\Delta} = \left(\frac{3(1-\nu_s^2)F}{4E_s R^{1/2}} \right)^{2/3} \tag{A2}$$

that is valid at relatively small strains $\Delta/R \ll 1$. Here, ν_s is the Poisson coefficient of the particle material. The variation of total volume V_t to the first order of $\tilde{\Delta}$ is equal to

$$\delta V_t = -b \frac{4\pi N R^2 \tilde{\Delta}}{1-\alpha}, \tag{A3}$$

where $b=1$ in the case of bulk compression and $b=1/3$ for longitudinal strains.

This variation includes the decrease of the porous volume as the particle centers draw together, the decrease of the spherical granule volume due to the all-round compression by fluid, and, finally, the particle volume variation in the contact region. It can be shown that the latter correction is of the order of $\tilde{\Delta}^2$ and is therefore negligible (whereas the main terms are of order $\tilde{\Delta}$).

To ascertain the relationship between the force F acting on the contacts between particles and the given external pressure P_{ext} we make use of the energy balance equation:

$$W_{\text{ext}} = W_f + W_s + W_c \tag{A4}$$

where W_{ext} is the energy required for the quasistatic compression of the aggregate by external pressure, W_f is the energy of pore fluid, W_s the elastic energy of the particle matter due to the bulk compression by the fluid, and W_c the energy stored by the particles on the contacts.

The energy spent on reducing the aggregate volume by δV_t is determined by the expression

$$W_{\mathrm{ext}}=\int_{\delta V_t} P_{\mathrm{ext}}\, dV_t\,. \tag{A5}$$

The energy accumulated by a pore fluid is given by

$$W_f=\int_{\delta V_t}(P_f+\delta P_f)dV_f\,. \tag{A6}$$

where $\delta \rho_f$ is the excess pressure due to the variation of the fluid volume δV_f.

The energy stored in the volume of fluid-compressed granules is given by

$$W_s=\int_{\delta V}(P_f+\delta P_f)dV_s\,, \tag{A7}$$

where δV_s is the variation of the spherical granule volume under the action of compressing excess pressure δP_f from the pore fluid side.

The energy due to the work on the contacts of deformed particles can be written as

$$W_c=b\bar{n}N\int_0^{\tilde{\Delta}} F\, dr. \tag{A8}$$

where the relationship between $\tilde{\Delta}$ and δV_t is given by (A3). The quantity b in this formula describes the effective number of "active" (those that accumulate energy) contacts. It is obvious that in the case of all-round compression, the particle centers are shifted with respect to each other only along the normal to the contact surface. Note that all contacts make equal contribution owing to isotropy of the problem and one should put $b=1$ in (A8). In the case of plane deformation, granule centers are also shifted along contact normals (i.e., in radial directions). However, in this case the average effective contribution to the elastic energy of contact W_c is made by one third of the total contact number, since the granule centers do not shift along two coordinate axes (which are orthogonal to the plane deformation axis), the contact orientation is isotropic, and each contact deformation work is independent. [The independence of each contact work means actually that the Poisson coefficient of the grainy skeleton is equal to zero. This, in turn, is due to the fact that most point contacts owing to their geometry are much "softer" compared with the grains matter. The simple estimation, in particular, shows that displacement Δ of the grain boundary at the contact causes the corresponding grain size change in the orthogonal direction of the higher order, namely, proportional to $\Delta^{3/2}$.] Therefore one should put in (A8) the factor $b=1/3$.

The pressure and volume variations of the fluid and the granules are related via elastic constants by

$$\delta p_f=-K_f\,\frac{\delta V_f}{V_f}, \tag{A9}$$

$$\delta p_f=-K_s\,\frac{\delta V_s}{V_s}. \tag{A10}$$

Thus, for the unknowns δp_f, δV_f, and δV_s we have a set of linear equations (A4), (A9), and (A10) that give the expressions of the unknowns quantities via $\tilde{\Delta}$:

$$\delta p_f=\frac{(3\tilde{\Delta}/R)bK_f}{\alpha+(1-\alpha)\eta}, \tag{A11}$$

$$\delta V_f=-\frac{\alpha b}{\alpha+(1-\alpha)\eta}\,\frac{4\pi NR^2}{1-\alpha}\,\tilde{\Delta}, \tag{A12}$$

$$\delta V_s=-\frac{\gamma b}{\alpha+(1-\alpha)\eta}\,4\pi NR^2\tilde{\Delta}, \tag{A13}$$

where $\eta=K_f/K_s$.

In these expressions $\tilde{\Delta}$ is actually equal to $(\Delta-\delta R_s)$, where 2Δ is the change that would take place in the center-to-center distance of the granules without the pore fluid, and δR_s is the change in the radius of the granules under the hydrostatic pressure exerted by the pore fluid [note that $(\delta R_s/R_s)\approx(\delta V_s/V_s)/3$], i.e., from (A13), we have

$$\tilde{\Delta}=\Delta\left(1-\frac{\eta b}{\alpha+(1-\alpha)\eta}\right). \tag{A14}$$

Substituting these expressions into the energy balance equation (A4) and bearing in mind that the resultant equality should be valid at any arbitrary $\tilde{\Delta}$ (so that we can equate the integrand to zero) we obtain

$$\frac{4\pi R^2}{1-\alpha}\left\{P_{\mathrm{ext}}-\frac{\alpha}{\alpha+(1-\alpha)}\,P_f\right\}-\pi F$$
$$-\frac{12\pi RK_fb\tilde{\Delta}}{(1-\alpha)(\alpha+(1-\alpha)\eta)}=0, \tag{A15}$$

where F, according to (A2), is equal to

$$F=\frac{4E_sR^{1/2}}{3(1-\nu_s^2)}\,\tilde{\Delta}^{3/2}. \tag{A16}$$

Equation (A15) relates, in fact, the effective pressure in the medium and the strain of the system under the action of this pressure. Consider the case of pure longitudinal strain which corresponds to the longitudinal wave propagation and put $b=1/3$ in (A15). Introducing the effective strain $\sigma_{\mathrm{eff}}=P_{\mathrm{ext}}-P_f$ and the longitudinal strain $\epsilon=\delta l_{11}/l_{11}=\delta V_t/V_t$, we obtain the "nonlinear Hooke law" for a fluid saturated granular medium:

$$\sigma_{\mathrm{eff}}=\frac{K_f}{\alpha+(1-\alpha)\eta}\,\epsilon+\frac{\bar{n}(1-\alpha)E_s}{3\pi(1-\nu_s^2)}\,\epsilon^{3/2}. \tag{A17}$$

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