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Tomography of elastic nonlinear parameters of rocks in problems of seismology and seismic prospecting

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Abstract. The paper discusses the potentialities of measuring nonlinear seismic parameters of rocks by methods based on observation of phase modulation of a weak sounding wave under the action of a different, more intensive field of deformations. The time required for signal stacking is estimated for several variations of configuration and length of sounding ray paths. The schemes considered are promising both for monitoring problems in earthquake-prone areas and seismic prospecting.

1. Introduction

The vast majority of seismoacoustic sounding methods are based on measurements of linear seismoacoustic parameters of a medium (seismic velocities and coefficients of attenuation and reflection). Conclusions made on geological and rock structure in a sounding area are drawn from empirical or theoretically derived relationships between measured linear seismoacoustic parameters of rock and its structural features of interest (e.g., porosity, fracturing, and liquid and gas content).

A new tendency in development of seismic sounding methods is connected with the use of nonlinear parameters of a medium [*Nikolayev,* 1981; *Nikolayev and Galkin,* 1987]. Available experimental data (see for example, *Groshkov et al.* [1990, 1991] and *Johnson and Shankland* [1987]) and theoretical calculations [*Belyayeva et al.,* 1993; *Dunin,* 1989] strongly support the close connection between nonlinear seismoacoustic parameters of a medium and its structure, properties, and magnitude of internal stresses. We should emphasize here that the range of variation of nonlinear parameters can be essentially higher (sometimes by a few orders of magnitude) than a momentary change of linear characteristics (numerous experimental proofs of this can be found in the monograph by *Gamburtsev* [1992] and references therein, while numerical modeling results are reviewed, for instance, by *Belyayeva et al.* [1994]). Thus one may expect that the use of nonlinear parameters as informative characteristics in seismoacoustic sounding would be advantageous in providing valuable additional information.

The purpose of the present work is to analyze potentialities of remote techniques applied for determination of nonlinear parameters of rocks based on measurement of phase modulation of a weak sounding wave subject to the action of a more intensive strain field produced by a harmonic source. Such methods were first proposed by Japanese scientists for measuring nonlinearity in biologic tissues [*Ichida et al.,* 1983; *Sato et al.,* 1985]. *Belyayeva and Sutin* [1992] have discussed some possibilities of their seismic application using a pulse pump wave. *Nazarov* [in press] applied a similar one-dimensional pulse scheme of tomography for studying dissipative nonlinearity, with attenuation of a weak probe wave depending upon the presence of a high-power pump wave.

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2. Quantitative Characteristics of Nonlinear Material Properties

The main feature of a nonlinear medium in connection with seismic waves consists in that parameters controlling propagation of the waves (seismic velocity and attenuation coefficient) depend upon deformation in a medium and the wave itself. We note that nonlinearity can be both conservative (when an amplitude-dependent characteristic is, for instance, the velocity of wave propagation) and dissipative (amplitude-dependent losses). Below, we restrict ourselves with analysis of nonlinearity of conservative type which is usually observed in experiments [*Groshkov et al.,* 1990, 1991; *Johnson and Shankland,* 1987].

To understand interaction effects of seismoacoustic waves and relevant methods of tomography of nonlinear parameters, we confine ourselves to a model of an isotropic medium and consider interaction of longitudinal waves only. Interaction of shear waves is of less interest for us, since, due to symmetry of displacements in such waves, their interaction does not give rise to quadratic nonlinear effects which do underlie the method offered. We will assume a medium to be isotropic and comply with the equation of state $\sigma = \sigma(\varepsilon)$, where σ is a stress and ε relative longitudinal deformation. We consider comparatively weak waves which admit series expansion of the equation in the parameter $\tilde{\varepsilon} = \varepsilon - \varepsilon_0$:

$$\tilde{\sigma} = \sigma'_{\varepsilon}(\varepsilon_0)\tilde{\varepsilon} + \frac{1}{2!}\sigma''_{\varepsilon\varepsilon}(\varepsilon_0)\tilde{\varepsilon}^2 + \frac{1}{3!}\sigma'''(\varepsilon_0)\tilde{\varepsilon}^3 + \cdots, \quad (1)$$

where $\tilde{\sigma} = \sigma(\varepsilon) - \sigma(\varepsilon_0)$.

It is common practice in nonlinear acoustics to introduce linear and nonlinear parameters as follows:

$$M = \sigma'_{\varepsilon}(\varepsilon_0) \quad (2)$$

$$\Gamma = \sigma''_{\varepsilon\varepsilon}(\varepsilon_0)/(2\sigma'_{\varepsilon}(\varepsilon_0)) \quad (3)$$

$$\mathrm{B} = \sigma'''_{\varepsilon\varepsilon\varepsilon}(\varepsilon_0)/(6\sigma'_{\varepsilon}(\varepsilon_0)) \quad (4)$$

or

$$\Gamma = (1/2)\rho V(\partial V/\partial\sigma) \quad (5)$$

$$\mathrm{B} = (1/6)\rho^2 V^2(\partial^2 V/\partial\sigma^2) \quad (6)$$

The value M has a meaning of the linear modulus of longitudinal deformation determining the velocity of longitudinal acoustic wave $V = [M/\rho]^{1/2}$, ρ is a density, and the values Γ and B are responsible for quadratic and cubic effects respectively. Expansion (1) is valid under assumption of small strains ε, when changes of linear parameters are small ($\Gamma\varepsilon$, $\mathrm{B}\varepsilon^2 \ll 1$); this condition is generally met at $\varepsilon \leq 10^{-5}$.

We note that the equation of state $\sigma = \sigma(\varepsilon)$ can be of a more complex, e.g., hysteretic form (a number of representative examples can be found in the work by *Vasilyev* [1991]). Nonlinear effects observed by *Zimenkov and Nazarov* [1993] are also explainable by hysteretic phenomena. However, rougher effective characteristics like the nonlinear parameters (3) and (4) can be introduced in these cases too.

We should emphasize that variability ranges of linear and nonlinear parameters can differ considerably for most of media. Thus sound velocities of nearly all liquids and elastic materials vary generally within a few tens of percents and, occasionally, within an order of magnitude (from several hundreds to several thousands of meters per second), whereas nonlinear parameters range within a few orders of magnitude depending upon the inner structure of a material. Most of measurements are concerned with the quadratic parameter Γ whose values range from 3 to 10 for "ordinary" homogeneous materials such as air, water, and fused quartz, etc. [*Naugol'nykh and Ostrovskiy,* 1990], to 10^3–10^4 for media of "complex" structure, e.g., water with gas bubbles [*Kobelev and Ostrovskiy,* 1980], porous plastisols [*Belyayeva and Timanin,* 1991; *Ostrovskiy,* 1988, 1991], and many rocks [*Gamburtsev,* 1992; *Groshkov et al.,* 1990, 1991; *Johnson and Shankland,* 1987; *Nikolayev and Galkin,* 1987; *Zimenkov and Nazarov,* 1993]. There are theoretical models that explain the observed differences. Thus based on analysis of nonlinear elastic properties of a granular medium saturated with gas and fluid, *Belyayeva et al.* [1994] have shown that nonlinear parameters, as compared with the elastic wave velocity, are much more sensitive to pressure and saturation with gas and liquid. According to results of their calculations, variation of the nonlinear parameter at boundaries of fluid-bearing areas can be many times as high as that of sound velocity, which is of interest for geological exploration.

Estimates of nonlinear parameter variability in periods of seismic activity are quite intriguing. Available data on variation of seismic velocities themselves in the process of earthquake preparation are of diverse nature and are often contradictory [*Gamburtsev,* 1992]. As for the nonlinear parameter (coefficient of strain sensitivity), it is itself characterized by high strain sensitivity [*Gamburtsev,* 1992; *Nikolayev and Galkin,* 1987], and so its variation with rock deformation can serve as a sensitive informative indicator of changes in the stress state of a medium even in those situations when variations of travel times do not exceed background values.

3. Probe Wave Phase Modulation Due to a Pumping Wave

Measurements of elastic nonlinear parameters of a medium are based on the effect of velocity change in a weak acoustic wave (called a signal or probe wave) due to nonlinearity of a medium under the action of a strong field of another wave (called a pumping wave in nonlinear acoustics). By (1)–(4), this relation can be represented in the form

$$\tilde{V} = V(1 + \Gamma\varepsilon_n + \mathrm{B}\varepsilon_n^2 + \cdots) \quad (7)$$

where ε_n is the strain amplitude of the pumping wave and V is unperturbed velocity.

We shall use ray optics approximation for calculation of phase change in a probe wave. We note that this approximation is applicable when the characteristic scale of inhomogeneities is large as compared both with the wavelength λ and cross-sectional size of the Fresnel volume d surrounding a ray path under consideration. If a ray in a homogeneous space connects points 1 and 2, then the cross-sectional diameter of its Fresnel volume is determined by the relationship $d = 2(\lambda r_1 r_2/(r_1 + r_2))^{1/2}$, where r_1 and r_2 are distances from a given point of the ray path to the points 1 and 2 respectively [*Kravtsov and Orlov,* 1980]. In this approximation, the phase increment of the probe wave along a sounding ray path in the interval where the ray coordinate changes from l_0 to L can be expressed in the form

$$\varphi = \omega \left(t - \int_{l_0}^{L} \frac{1}{V(l)} dl \right)$$

Using the smallness condition for velocity increment due to the pump wave and (7) we have for the variation of the probe wave phase

$$\Delta\varphi = \omega \int_{l_0}^{L} \Delta\left(\frac{1}{V}\right) dl = \omega \int_{l_0}^{L} \frac{\Gamma(l)}{V} \varepsilon_n(l,t)\, dl + \\ + \omega \int_{l_0}^{L} \frac{\mathrm{B}(l)}{V^3} \varepsilon_n^2(l,t)\, dl \tag{8}$$

Here, L is the ray coordinate of a point of observation. We shall consider the case of small deformations in the pumping wave when $\mathrm{B}\varepsilon_n \ll 1$ and the main contribution to the expansion of $\tilde{V}$ (7) is given by the quadratic term. From here on we will neglect the second term in (8) that is responsible for cubic nonlinearity.

We consider the phase shift of a portion of the probe wave determined by the ray coordinate $l = l_*$ at the moment $t = 0$. At the moment t its coordinate can be found from the equation

$$t = \int_{l_*}^{L} \frac{dl}{V(l)} \tag{9}$$

In the general case of an inhomogeneous medium, this equation has no analytical solution and hereinafter we shall neglect velocity variation along the ray, i.e., we assume that V is an average velocity. The value of corresponding systematic error in arrival time determination has an order of relative variation of propagation velocity along the ray path which, as a rule, is much lower than unit for real stratifications. In this case (9) gives $l = l_* + Vt$, and the integral (8) can be written in quadratic approximation as

$$\Delta\varphi\left(\frac{L - l_*}{V}\right) = \omega \int_{0}^{L} \frac{\Gamma(l)}{V} \varepsilon_n \left(l, \frac{L - l_*}{V}\right) dl \tag{10}$$

where $(L - l_*)/V$ is the time at which the portion of the probe wave with the initial coordinate l_* arrives at the point of observation.

Thus generally speaking, nonlinear phase variations are connected in an integral manner with spatial distribution of nonlinear parameter(s) of interest for us so that the kernel of integral transformation is determined by the spatial-temporal structure of the pumping field. In other words, we have a typical tomography problem in determining spatial distribution of a value studied (in our case, the nonlinearity parameter Γ) from a set of its integral characteristics provided both analog and digital processing of a set of such integrals is possible. Spatial resolution by an analog method is attainable, for instance, through time gating using a pulse pump waveform. Some aspects of such approach as applied to seismic problems were considered by *Belyayeva and Sutin* [1992]. The present paper is mainly concerned with reconstruction of a nonlinear parameter by means of audio pumping for various spatial field distributions.

4. Conditions for Signal Detection Against Noise

To examine the possibility of signal detection against present noise, we consider conditions at which a signal exceeds noise and which enable measurements of weak modulation of a probe wave. We assume that the probe wave source is highly stable and fluctuations of its parameters are much smaller than the nonlinear variations discussed. Let the strain amplitude in a probe wave under consideration be A, then, at weak phase modulation $\Delta\varphi$, the level of deformation in an useful signal can be estimated from the relationship $\Delta A \approx A\Delta\varphi$. Due to this relationship, the signal to noise intensity ratio S_N can be estimated like it was done by *De Fazio et al.* [1973] and can be written in the form

$$\frac{(\Delta\varphi A)^2}{\langle\sigma_\varepsilon^2\rangle T} = S_N \tag{11}$$

where $\langle\sigma_\varepsilon^2\rangle$ is the level of spectral density of noise in the 1 Hz band observed at deformation measurement and T time of coherent signal stacking. The spectral density in seismics is usually determined from measuring the vibrational velocity. The spectral density of noise of vibrational velocity $\langle\sigma_V^2\rangle$ is connected with that of deformation noise by the relationship $\langle\sigma_\varepsilon^2\rangle = \langle\sigma_V^2\rangle/(2\pi V)^2$. We note that (9) is written under assumption of signal detection by a single receiver; an additional advantage can be gained through the use of a receiver array.

5. A Homogeneous Pump Field

The field of natural tidal deformations used as a pump wave appears to afford a simplest realization of the above approach. In this case, even for rather extended ray paths of about tens kilometers, we can consider the pump field to be homogeneous, $\varepsilon_n(l,t) = \varepsilon_n(t)$ and quasi-static within the time interval of signal propagation along the path. The approximation of quadratic nonlinearity being assumed, amount of nonlinear phase variation of a probe wave can be represented in the form

$$\Delta\varphi(t) = \omega\varepsilon_n(t)\int_0^L \frac{\Gamma(l)}{V}\,dl \approx \omega\rho V^2\varepsilon_n L\langle\Gamma\rangle/V \qquad (12)$$

where $\langle\cdots\rangle$ stands for averaging along the ray trace.

Assuming that we know the acoustic velocity field $V(x,y,z)$ and density $\rho(x,y,z)$ (or equivalent distributions $\rho(l)$ and $V(l)$ along the ray trajectory), the amplitude of deformation in the probe wave field can be written at ray approximation as

$$A(l) = \exp(-\delta_c l)A_0([\Sigma(l_0)\rho(l_0)V(l_0)] \times \times [\Sigma(l)\rho(l)V(l)]^{-1})^{1/2} \qquad (13)$$

where $\Sigma(l)$ is the cross section of the ray tube [*Kostrov and Orlov,* 1980], $A(l_0)$ is the amplitude of the signal wave at $l = l_0$ (in what follows we assume, as is the practice, $l_0 = 1$ m), and δ_c is the attenuation constant of the signal wave.

Below, for the sake of preliminary evaluation, we assume ray tubes to be spherically divergent (which is valid for a homogeneous medium as well as for linear dependence of the velocity V on depth and fits well profiles of other types outside the vicinity of ray caustics). Then, deformations at observational points can be estimated from the formula $A(l) = \exp(-\delta_0 l)A_0/l$. Based on the expression for amplitude variations $\Delta A \approx A\Delta\varphi$, with $\Delta\varphi$ defined by (12), we obtain

$$\Delta A \approx A_0\Delta\varphi\exp(-\delta_0 L)/L \approx \approx A_0\omega\varepsilon_n(t)\exp(-\delta_0 L)\langle\Gamma\rangle/V \qquad (14)$$

i.e., in the case of weak attenuation the amplitude of nonlinear variations of a signal wave is virtually independent of distance. Thus the source of a probe wave being sufficiently stable, observations of tidal period nonlinear variations can be used for obtaining a path-averaged value of quadratic nonlinear parameter. *De Fazio et al.* [1973] report tidal variations of this kind observed at the path length $L \sim 300$ m and probe wave frequency 500 Hz. Accuracy required for measuring acoustic velocity variations amounted to $\Delta V/V \sim 10^{-4}$ and was achieved due to coherent signal stacking. On substituting the expression for the effective amplitude of a signal (14) into (11), we obtain for the time of coherent signal stacking:

$$T \approx S_N\frac{\langle\sigma_\varepsilon^2\rangle\exp(2\delta L)}{A_0^2(2\pi)^2 f^2(\Gamma^{(2)}/(2V))^2\varepsilon_n^2} \qquad (15)$$

where S_N is the signal to noise intensity ratio and L limiting distance of observation. The following values have been used for our estimates: nonlinearity parameter $\Gamma \sim 10^4$ determined by *De Fazio et al.* [1973] (*Gamburtsev* [1992] reports similar values Γ and even those which are by an order of magnitude higher), sound velocity $V \sim 3\times10^3$ mps, spectral density level of noise $\langle\sigma_\varepsilon^2\rangle \sim 6\times10^{-27}$ Hz^{-1}, magnitude of tidal deformations $\varepsilon_n \sim 10^{-8}$, characteristic density $\rho \sim 3\times10^3$ kg m^{-3}, and level of probe wave source field $A_0 \approx 2\times10^{-7}$ m (which corresponds to emission of a single pulse of acoustic power about 100 W). Powerful vibrators are able to produce deformations of similar levels, although their drawback is difficulty in obtaining stable frequency of emission. High-stability emission is attainable through the use of electrodynamic emitters produced for hydroacoustic investigations; they develop acoustic power to a few kilowatts [*Levushkin and Penkin,* 1993].

Available experimental data on the attenuation constant δ indicate that it grows roughly proportional to the frequency [*Gurvich and Nomokonova,* 1981]:

$$\delta = \theta/\lambda = \theta f/V \qquad (16)$$

where θ averages to 7×10^{-3} for hard rocks that are of greatest interest for monitoring of regions of seismic danger [*Gurvich and Nomokonova,* 1981]. As is clear from (12) and (13), at a fixed distance L, there exists an optimum frequency of the probe wave f which minimizes the required processing time:

$$f = V/(\theta L) \qquad (17)$$

As a result, we obtain for values of L chosen as 1, 10, and 20 km $T_{\rm opt} \approx 2\times10^{-1}$ s (at $f_{\rm opt} = 420$ Hz), $T_{\rm opt} \approx 20$ s (at $f_{\rm opt} = 42$ Hz), and $T_{\rm opt} \approx 1$ min (at $f_{\rm opt} = 21$ Hz), respectively. The optimum frequency 4.2 Hz at the distance $L = 100$ km appears to be hardly attainable, and the processing time estimate for this distance at $f \approx 20$ Hz amounts to $T \approx 30$–40 hour. Thus the use of tidal deformations as a pump wave allows one to obtain distance-averaged values of the nonlinearity parameter and in this way to implement “nonlinear” seismic monitoring on characteristic scales up to hundreds of kilometers and at processing times of about several hours.

6. Harmonic Pump Field Along Probe Wave Direction

Another possibility for obtaining information about spatial distribution of the nonlinear parameter consists in using an artificial harmonic source of the pump field, with a signal emitted at the frequency Ω that is much lower than the probe wave frequency ω_0. This field is no longer spatially homogeneous, so while using it calculation of integral expressions of the type (10) for variations of the probe wave phase would involve kernels that are different from those of (10), i.e., give an additional information on spatial distribution of the nonlinear parameter.

The pump wave field can be calculated from given values of sound velocity $V(x, y, z)$ and density $\rho(x, y, z)$ with the aid of an expression similar to (13). If the problem of creating a stationary ray path arises with the purpose of long-term monitoring of nonlinear parameters variations, then the spatial distribution of linear characteristics (V and ρ) should be determined from preliminary measurements by conventional (remote or contact) methods. Below, with the view of preliminary estimation, we roughly assume, as we did earlier in discussing (13), that ray tubes are spherically divergent, i.e.,

$$\varepsilon_n(l) = \exp(-\delta_n l)(\varepsilon_0/l) \tag{18}$$

where ε_0 is the strain amplitude in the pump source field reduced to 1 m.

The mutual position of pump and probe wave sources can be chosen in many ways. A simple case is represented by codirectional propagation of both waves when positions of both sources coincide. Then, by (10) we obtain the following two integrals for opposite endpoints of the trajectory where coincident sources of a pump field and probe waves are alternately located

$$\Delta\varphi = \omega \int_1^L \frac{\varepsilon_0}{l} \frac{\Gamma(l)}{V} \exp(-\delta_n l)\, dl \tag{19}$$

$$\Delta\varphi = \omega \int_L^1 \frac{\varepsilon_0}{L-l} \frac{\Gamma(l)}{V} \exp[-\delta_n (L-l)]\, dl \tag{20}$$

where the ray coordinate l in both cases is measured from the same path endpoint. As distinct from (12), this phase modulation has the pump wave period $2\pi/\Omega$. It is noteworthy that generally speaking, the integrals (19) and (20) contain different information on distribution of the nonlinear parameter. In fact, any function $f(l)$ can be represented as a sum of even and odd components (with respect to the midpoint of the path $l = L/2$). Contributions of the even component into (19) and (20) evidently will be the same, and those of the odd component will be different.

Thus source configurations considered above give three integrals $\Delta\varphi_k$ for phase variations determined by (19), (20), and (12) (as has been discussed earlier, the latter is calculated from observations of tidal phase variations in a probe wave). This set of integrals allows one not only to estimate average values of the parameter $\Gamma(l)$, but also to solve the problem of reconstructing its distribution along the ray path (though in a fairly rough approximation). For instance, this can be done by representing an unknown distribution $\Gamma(l)$ as a sum of certain functions F_i ("reasonably" chosen in accordance with the expected type of a distribution to be reconstructed):

$$\Gamma(l) = \sum_i \alpha_i F_i(l) \tag{21}$$

Substitution of (21) into (12), (19), and (20) leads to the system of linear equations:

$$M_{kj}\alpha_j = \Delta\varphi_k \tag{22}$$

where

$$M_{1j} = \omega \int_0^L (\varepsilon_n/V) F_j(l)\, dl$$

$$M_{2j} = \omega \int_1^L \varepsilon_0 \frac{\exp(-\delta_n l)}{lV} F_j(l)\, dl$$

$$M_{3j} = \omega \int_L^1 \varepsilon_0 \frac{\exp[-\delta_n(L-l)]}{(L-l)V} F_j(l)\, dl$$

Solution of (22) in unknown coefficients of expansion α_i can be, for example, obtained by means of regularization methods that are generally used in tomography problems [*Levushkin and Penkin,* 1993]. In principle, resolution of (19) can be enhanced through the use of more than three components, which, on the other hand, leads to underdifiniteness of the problem solved. Validity of the choice of the solution in such a form should be checked on by preliminary numerical modeling with regard to the expected form of an inhomogeneity to be reconstructed [*Pikalov and Preobrazhenskiy,* 1983].

Now, we estimate now the possible level of a useful signal that can be detected in terms of the discussed scheme. With the assumption of a homogeneous distribution of the nonlinear parameter $\Gamma = \Gamma_0$, the phase variation $|\Delta\varphi|$ can be estimated from (19) as follows:

$$|\Delta\varphi| = \omega\varepsilon_0\Gamma_0[\mathrm{Ei}[\delta_n L(M)] - \mathrm{Ei}[\delta_n \cdot 1M]]/V \tag{23}$$

where Ei is the integral exponent, and the expression in parentheses has the asymptotics $\log(L^*)$, where $L^* = L$ at $\delta_n L \ll 1$ or $L^* = L_3/\gamma$ at $\delta_n L \gg 1$ ($L_3 = 1/\delta_n$ is the attenuation length of the pump wave and $\gamma \approx 1.78$ Euler constant). Proceeding in the same manner as in deriving (14), we obtain that (23) corresponds to amplitude modulation of the probe wave:

$$|\Delta A| \approx A_0 \Delta\varphi \exp(-\delta_0 L)/L \approx \approx \omega A_0 \varepsilon_0 \Gamma^{(2)} \exp(-\delta_0 L) \log[L^*(M)]/(LV) \tag{24}$$

where δ_0 is the attenuation constant of the probe wave.

As distinct from (14), the value ΔA remains dependent upon the path length even if attenuation is neglected. In this case the expression for the time required for coherent stacking assumes the form

$$T \approx S_N \frac{\langle\sigma_\varepsilon^2\rangle \exp(2\delta_0 L) L^2}{A_0^2 (2\pi)^2 f^2 (\Gamma/(2V))^2 \varepsilon_0^2 [\log(L^*)]^2} \tag{25}$$

The values of material parameters and noise level used for estimating the value T are the same as in the analysis of (12). We also assume that acoustic powers of pump and probe wave sources are 1000 W and 100 W, respectively. As a result, optimum frequencies (17) chosen for the probe wave give $E \approx 1$ s for the distance $L = 1$ km and $T =$ 1–2 hours for $L = 10$ km (also, the pump frequency is supposed to be low enough to ignore attenuation, $L^* \approx L$).

7. Harmonic Pump Field in Perpendicular and Counter Directions

To enhance the spatial resolution, additional integrals of type (10) are required which would provide independent information on an inhomogeneity to be reconstructed. For this purpose, several different pump frequencies could be employed since they imply differences in the gradient of the factor in (18), but this means is constrained by the requirement that the frequency of pumping should be much lower than that of a probe wave. An alternate approach is related with an alternate relative position of probe and pump wave sources. In a general case, mutual orientation of interacting waves can be arbitrary. If we suppose that the spatial distribution of the linear characteristics V and ρ is known from preliminary measurements then we are able to calculate the path shape of the probe wave and spatial distribution of the pump field $\varepsilon_n(\mathbf{r}, t)$ (or $\varepsilon_n(l, t)$ in terms of the ray coordinate defined by the probe wave). The latter should be substituted into (10) for a chosen configuration of sources. However, assuming an arbitrary source position, such calculation requires fairly detailed preliminary information about linear parameters, which is difficult to realize in practice. In this respect more advantageous appears to be a scheme in which pump wave propagation is normal to the probe wave direction, and the pump field amplitude is set to be the same all along the signal path. Such a scheme is realized when the pump source is on the side of the path under investigation, at a distance much greater than the path length. In this approximation, expression (10) that defines the probe wave phase shift due to interaction with pumping takes the form

$$\Delta\varphi = \omega \int_0^L \frac{\Gamma(l)}{V} \varepsilon_n(L_1) \exp(iKl)\, dl \tag{26}$$

where $K = \Omega/V$ is the mean wave vector of the pump wave (its variation along the path is ignored) and $\varepsilon_n(L_1) = \varepsilon_0/L_1$. Thus by varying the pump frequency, we obtain a set of the integrals (which are in essence components of the Fourier series of the nonlinearity parameter $\Gamma(l)$). The reconstruction is attainable either by obtaining and solution of the system of type (22) or by applying the inverse Fourier transformation.

To estimate the level of an useful signal, we assume that the pump wave is generated by a source with power of about 1000 W and operating frequency of 50 Hz which is located at the distance $L_1 = 3$ km from a probe wave path, the wavelength being $L = 1$ km. The following expression, instead of (12), is obtained for the coherent stacking time:

$$T \approx S_N \frac{\langle\sigma_\varepsilon^2\rangle \exp(2\delta_0 L) 4 L_1^2 L^2}{A_0^2 (2\pi)^2 f^2 (\Gamma^{(2)}/(2V))^2 \varepsilon_0^2 \Lambda^2} \tag{27}$$

where $\Lambda = 2\pi/K$. Hence we have $T \approx 100$ hour.

Also, another limiting case is sufficiently simple for description and it is represented by pump and probe wave sources located at opposite endpoints of the path. Both waves in such configuration propagate along the same trajectory (though in counter directions) so that in solving the problem on reconstruction of the nonlinear parameter distribution along the ray path we do not need so detailed preliminary information that is required in a general case. In much the same way as (26), the probe wave phase shift is now written in the form

$$\Delta\varphi = \omega \int_1^L \left(\frac{\varepsilon_0}{l}\right) \exp(-\delta_n l) \frac{\Gamma(l)}{2V} \exp(i2Kl)\, dl \tag{28}$$

Thus the cases of orthogonal and counter propagation of waves at different pump frequencies (of wave vectors) give a set of integrals of the type (28). Use of this set (in addition to (12), (19), and (20)) for reconstruction of the nonlinear parameter distribution enhances resolution whose limiting scale is defined by the pump wavelength. This can be implemented, for instance, in terms of the above-described approach, with a greater number of components being chosen in the decomposition of a reconstructed image (21) to increase dimensionality of the system (22). However, decrease in pump wavelength evidently leads to decrease in the absolute magnitude of the phase variation ((26) and (18)), and so desired accuracy of measurements requires higher stacking time. Therefore in practice, it would be hardly reasonable to choose the value KL in excess of a few tens. The time required for signal stacking will amount from a few hours to a few days if pump sources in use have acoustic power of about 1 kW, frequencies of about one half hundred hertz, and sounding range from one to several kilometers.

We note that the problem of identification of trajectories of interacting waves is of importance in the above schemes based on the use of audio signals. For instance, surface (Rayleigh) waves and the like, beside the deep path of interest, can contribute to a general signal received. So, in practice, identification of trajectories can require the use of pulse signals with synchronized envelope and carrier.

8. Conclusions

Thus the above discussion has demonstrated the possibility to observe nonlinear phase variations of a probe wave subject to the action of another (pumping) wave which can be represented either by a wave emitted by an artificial source or, for instance, by the field of tidal deformations. The effect can be applied to measuring the value and spatial distribution of the nonlinear parameter along a sounding ray path. Since such measurements, particularly at great distances, require considerable time for their coherent processing, they appear to be most advantageous for implementation of long-term monitoring of nonlinear rock properties at stationary ray paths with the view of detecting changes of internal stresses connected with preparing processes of earthquakes. A higher level of signal, comparatively smaller processing times and higher spatial resolution are attainable at smaller distances, and in this case methods of nonlinear parameter tomography can be applied to seismic exploration (mineral prospecting, monitoring of oil and gas fields, etc.).

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